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DEGENERATE POLYEXPONENTIAL FUNCTIONS AND POLY-EULER POLYNOMIALS

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Abstract. Degenerate versions of the special polynomials and numbers since they have many applications in analytic number theory, combinatorial analysis and p-adic analysis. In this paper, we define the degenerate poly-Euler numbers and polynomials arising from the modified polyexponential functions. We derive explicit relations for these numbers and polynomials. Also, we obtain some identities involving these polynomials and some other special numbers and polynomials.

1. Introduction

Throughout this paper, $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N}_0$ denotes the set of nonnegative integers and $\mathbb{R}$ denotes the set of real numbers. We begin by introducing the following definitions and notations ([1–15]).

The classical Bernoulli polynomials $B_n(x)$, the classical Euler polynomials $E_n(x)$ and the classical Genocchi polynomials $G_n(x)$ are defined by the following generating functions ([2–15]) respectively;

$$\sum_{n=0}^{\infty} B_n(x)\frac{t^n}{n!} = \left(\frac{t}{e^t-1}\right)e^{xt},\ |t|<2\pi, \tag{1}$$

$$\sum_{n=0}^{\infty} E_n(x)\frac{t^n}{n!} = \left(\frac{2}{e^t+1}\right)e^{xt}\ ,\ |t|<\pi \tag{2}$$

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and

$$\sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!} = \left(\frac{2t}{e^t + 1} \right) e^{xt}, \; |t| < \pi, \tag{3}$$

when $x = 0$, $B_n(0) = B_n$, $E_n(0) = E_n$ and $G_n(0) = G_n$ are called the Bernoulli numbers, the Euler numbers and the Genocchi numbers, respectively.

For $(\lambda \neq 0) \in \mathbb{R}$, the degenerate exponential function is defined by ([4–13]);

$$e_\lambda^x(t) = (1 + \lambda t)^{x/\lambda} = \sum_{n=0}^{\infty} (x)_{n,\lambda} \frac{t^n}{n!}, \; e_\lambda(t) = e_\lambda^1(t) = (1 + \lambda t)^{1/\lambda}, \tag{4}$$

where $(x)_{0,\lambda} = 1$, $(x)_{n,\lambda} = x(x - \lambda)(x - 2\lambda) \cdots (x - (n-1)\lambda)$, $n \geq 1$.

Carlitz ([2,3]) considered the degenerate Bernoulli polynomials $B_{n,\lambda}(x)$ and the degenerate Euler polynomials $E_{n,\lambda}(x)$ which are given by, respectively,

$$\frac{t}{e_\lambda(t) - 1} e_\lambda^x(t) = \frac{t}{(1 + \lambda t)^{1/\lambda} - 1} (1 + \lambda t)^{x/\lambda} = \sum_{n=0}^{\infty} \mathcal{B}_{n,\lambda}(x) \frac{t^n}{n!} \tag{5}$$

and

$$\frac{2}{e_\lambda(t) + 1} e_\lambda^x(t) = \frac{2}{(1 + \lambda t)^{1/\lambda} + 1} (1 + \lambda t)^{x/\lambda} = \sum_{n=0}^{\infty} \mathcal{E}_{n,\lambda}(x) \frac{t^n}{n!}, \tag{6}$$

when $x = 0$, $B_{n,\lambda}(0) := B_{n,\lambda}$ and $E_{n,\lambda}(0) := E_{n,\lambda}$ are called the degenerate Bernoulli numbers and the degenerate Euler numbers, respectively.

Lim [14] considered the degenerate Genocchi polynomials which are given by

$$\frac{2t}{e_\lambda(t) + 1} e_\lambda^x(t) = \sum_{n=0}^{\infty} \mathcal{G}_{n,\lambda}(x) \frac{t^n}{n!}, \tag{7}$$

when $x = 0$, $G_{n,\lambda}(0) := G_{n,\lambda}$ are called the degenerate Genocchi numbers.

From (6) and (7), we get

$$G_{0,\lambda}(x) = 0 \text{ and } E_{n,\lambda}(x) = \frac{G_{n+1,\lambda}(x)}{n+1}, \; n \geq 1.$$

The degenerate Stirling numbers of the first kind $S_{1,\lambda}(n,k)$ are defined by ([7,9,11])

$$\frac{1}{k!} (\log_\lambda (1 + t))^k = \sum_{n=k}^{\infty} S_{1,\lambda}(n,k) \frac{t^n}{n!}, \; k \geq 0. \tag{8}$$

Note here that $\lim_{\lambda \to 0} S_{1,\lambda}(n,l) = S_1(n,l)$, where $S_1(n,l)$ are the Stirling numbers of the first kind given by ([4,11])

$$\frac{(\log(1 + t))^k}{k!} = \sum_{n=k}^{\infty} S_1(n,k) \frac{t^n}{n!}, \; k \geq 0. \tag{9}$$

The degenerate Stirling numbers of the second kind are defined by ([7,9,11])

$$\frac{(e_\lambda(t)-1)^k}{k!}=\sum_{n=0}^{\infty}S_{2,\lambda}(n,k)\frac{t^n}{n!}. \tag{10}$$

Observe that $\lim_{\lambda\to 0}S_{2,\lambda}(n,l)=S_2(n,l)$, where $S_2(n,l)$ are the Stirling numbers of the second kind given by ([4,11])

$$\frac{(e^t-1)^k}{k!}=\sum_{n=k}^{\infty}S_2(n,k)\frac{t^n}{n!},\ k\geq 0. \tag{11}$$

The degenerate Bernoulli polynomials of the second kind are given by [8]

$$\frac{t}{\log_\lambda(1+t)}(1+t)^x=\sum_{n=0}^{\infty}b_{n,\lambda}(x)\frac{t^n}{n!}. \tag{12}$$

Note that $\lim_{\lambda\to 0}b_{n,\lambda}(x)=b_n(x)$, where $b_n(x)$ are the Bernoulli polynomials of the second kind given by

$$\frac{t}{\log(1+t)}(1+t)^x=\sum_{n=0}^{\infty}b_n(x)\frac{t^n}{n!}. \tag{13}$$

2. Degenerate poly-Euler numbers and polynomials

In this section, we consider the modified degenerate polyexponential functions. We give explicit relations for the modified degenerate polyexponential functions. By using the modified degenerate polyexponential functions, we introduce the degenerate poly-Euler numbers and polynomials.

Also, we give some relations and identities for these polynomials.

The polyexponential functions are defined by the following generating functions ([6–8,10,13]);

$$\mathrm{Ei}_k(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^k(n-1)!},\ k\in\mathbb{Z}. \tag{14}$$

For $k=1$, $\mathrm{Ei}_1(x)=e^x-1$.

The modified degenerate polyexponential function are given by ([6–8,10,13]);

$$\mathrm{Ei}_{k,\lambda}(x)=\sum_{n=1}^{\infty}\frac{(1)_{n,\lambda}\ x^n}{n^k(n-1)!},\ \lambda\in\mathbb{R}. \tag{15}$$

Note that,

$$\mathrm{Ei}_{1,\lambda}(x)=\sum_{n=1}^{\infty}(1)_{n,\lambda}\frac{x^n}{n!}=e_\lambda(x)-1.$$

For $k\in\mathbb{Z}$ and by means of the modified degenerate polyexponential function. We define the degenerate poly-Euler polynomials by the following generating functions:

$$\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(k)}(x)\frac{t^n}{n!}=2\frac{\mathrm{Ei}_{k,\lambda}(\log_\lambda(1+t))}{t(e_\lambda(t)+1)}e_\lambda^x(t). \tag{16}$$

When $x=0$, $\mathcal{E}_{n,\lambda}^{(k)}:=\mathcal{E}_{n,\lambda}^{(k)}(0)$ are called the degenerate poly-Euler numbers where $\log_\lambda(t)=\frac{1}{\lambda}\left(t^\lambda-1\right)$ is the compositional inverse of $e_\lambda(t)$ satisfying $\log_\lambda(e_\lambda(t))=e_\lambda(\log_\lambda(t))=t$.

For $k=1$, we get

$$\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(1)}(x)\frac{t^n}{n!}=\frac{2}{e_\lambda(t)+1}e_\lambda^x(t).$$

From (16), we can easily deduce the following relationships involving the modified degenerate poly-Euler polynomials

$$\mathcal{E}_{n,\lambda}^{(k)}(x)=\sum_{m=0}^{n}\binom{n}{m}\mathcal{E}_{m,\lambda}^{(k)}\ (x)_{n-m,\lambda}, \tag{i}$$

$$\begin{aligned}\mathcal{E}_{n,\lambda}^{(k)}(x+y)&=\sum_{m=0}^{n}\binom{n}{m}\mathcal{E}_{m,\lambda}^{(k)}(x)(y)_{n-m,\lambda}\\&=\sum_{m=0}^{n}\binom{n}{m}\mathcal{E}_{m,\lambda}^{(k)}(y)(x)_{n-m,\lambda}\end{aligned} \tag{ii}$$

and

$$\mathcal{E}_{n,\lambda}^{(k)}(x+y)=\sum_{m=0}^{n}\binom{n}{m}\mathcal{E}_{m,\lambda}^{(k)}\ (x+y)_{n-m,\lambda}. \tag{iii}$$

By (8) and (15), we get

$$\begin{aligned}\mathrm{Ei}_{k,\lambda}(\log_\lambda(1+t))&=\sum_{n=1}^{\infty}\frac{(1)_{n,\lambda}(\log_\lambda(1+t))^n}{(n-1)!n^k}\\&=\sum_{n=1}^{\infty}\frac{(1)_{n,\lambda}}{n^{k-1}}\frac{1}{n!}(\log_\lambda(1+t))^n\\&=\sum_{n=1}^{\infty}\frac{(1)_{n,\lambda}}{n^{k-1}}\sum_{m=n}^{\infty}S_{1,\lambda}(m,n)\frac{t^n}{n!}\\&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{(1)_{n,\lambda}}{n^{k-1}}S_{1,\lambda}(m,n)\frac{t^m}{m!}\\&=t\sum_{m=0}^{\infty}\left(\sum_{n=1}^{m+1}\frac{(1)_{n,\lambda}}{n^{k-1}}\frac{S_{1,\lambda}(m+1,n)}{m+1}\right)\frac{t^m}{m!}.\end{aligned} \tag{17}$$

Using (16) and (17), we write

$$\begin{aligned}
&\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(k)}(x)\frac{t^n}{n!}\\
&=\frac{2}{t(e_\lambda(t)+1)}e_\lambda^x(t)\,\mathrm{Ei}_{k,\lambda}(\log_\lambda(1+t))\\
&=\sum_{l=0}^{\infty}\mathcal{E}_{l,\lambda}^{(k)}(x)\frac{t^l}{l!}\sum_{m=0}^{\infty}\frac{1}{m+1}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}S_{1,\lambda}(m+1,j)\frac{t^m}{m!}\\
&=\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}\frac{1}{m+1}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}S_{1,\lambda}(m+1,j)\,\mathcal{E}_{n-m,\lambda}(x)\right)\frac{t^n}{n!}. \qquad (18)
\end{aligned}$$

Therefore by (18), we obtain the following theorem.

Theorem 2.1. *For $n\geq 0$, we have*

$$\mathcal{E}_{n,\lambda}^{(k)}(x)=\sum_{m=0}^{n}\frac{\binom{n}{m}}{m+1}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}S_{1,\lambda}(m+1,j)\,\mathcal{E}_{n-m,\lambda}(x).$$

From (16) and (17), we write as

$$\begin{aligned}
&\sum_{n=0}^{\infty}\left(\mathcal{E}_{n,\lambda}^{(k)}(x+1)+\mathcal{E}_{n,\lambda}^{(k)}(x)\right)\frac{t^n}{n!}\\
&=\frac{2}{t}\,\mathrm{Ei}_{k,\lambda}(\log_\lambda(1+t))\,e_\lambda^x(t)\\
&=2\sum_{m=0}^{\infty}\frac{1}{m+1}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}S_{1,\lambda}(m+1,j)\frac{t^m}{m!}\sum_{l=0}^{\infty}(x)_{l,\lambda}\frac{t^l}{l!}\\
&=2\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}\frac{1}{m+1}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}S_{1,\lambda}(m+1,j)\,(x)_{n-m,\lambda}\right)\frac{t^n}{n!}. \qquad (19)
\end{aligned}$$

Therefore, by comparing the coefficients of $\frac{t^n}{n!}$ on both sides of (19), we have the following theorem.

Theorem 2.2. *For $n\geq 0$, we have*

$$\mathcal{E}_{n,\lambda}^{(k)}(x+1)+\mathcal{E}_{n,\lambda}^{(k)}(x)=2\sum_{m=0}^{n}\frac{\binom{n}{m}}{m+1}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}S_{1,\lambda}(m+1,j)\,(x)_{n-m,\lambda}.$$

From (16) and (17), we write

$$\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(k)}(x)\frac{t^n}{n!}(e_\lambda(t)+1)=\frac{2}{t}\,\mathrm{Ei}_{k,\lambda}(\log_\lambda(1+t))$$

and

$$\sum_{m=0}^{\infty}\left(\sum_{l=0}^{m}\binom{m}{l}\mathcal{E}_{m-l,\lambda}^{(k)}(x)(1)_{l,\lambda}+\mathcal{E}_{m,\lambda}^{(k)}(x)\right)\frac{t^m}{m!}$$

$$=2\sum_{m=0}^{\infty}\left(\sum_{n=1}^{m+1}\frac{(1)_{n,\lambda}}{n^{k-1}}\frac{S_{1,\lambda}(m+1,n)}{(m+1)}\right)\frac{t^m}{m!}. \tag{20}$$

By (20), we obtain the following theorem.

Theorem 2.3. *For $n\geq 0$, we have*

$$\sum_{l=0}^{m}\binom{m}{l}\mathcal{E}_{m-l,\lambda}^{(k)}(x)(1)_{l,\lambda}+\mathcal{E}_{m,\lambda}^{(k)}(x)=2\sum_{n=1}^{m+1}\frac{(1)_{n,\lambda}}{n^{k-1}}\frac{S_{1,\lambda}(m+1,n)}{m+1}.$$

For $x=0$, and by replacing t by $e_\lambda(t)-1$ in (16), we get

$$\sum_{m=0}^{\infty}\mathcal{E}_{m,\lambda}^{(k)}\frac{(e_\lambda(t)-1)^m}{m!}=2\frac{\mathrm{Ei}_{k,\lambda}(\log_\lambda(e_\lambda(t)))}{(e_\lambda(t)-1)(e_\lambda(e_\lambda(t)-1)+1)}$$

and thus

$$\sum_{n=0}^{\infty}\sum_{m=0}^{n}\mathcal{E}_{m,\lambda}^{(k)}\,S_{2,\lambda}(n,m)\frac{t^n}{n!}=\frac{1}{t^2}\frac{t^2}{(e_\lambda(t)-1)^2}\frac{2(e_\lambda(t)-1)}{e_\lambda(e_\lambda(t)-1)+1}\mathrm{Ei}_{k,\lambda}(t). \tag{21}$$

The left hand side of above equation (21) as

$$\sum_{n=0}^{\infty}n(n-1)\sum_{m=0}^{n-2}\mathcal{E}_{m,\lambda}^{(k)}\,S_{2,\lambda}(n-2,m)\frac{t^n}{n!},\text{ with } n\geq 2. \tag{22}$$

The right hand side of above equation (21) as

$$\sum_{m=0}^{\infty}\mathcal{B}_{m,\lambda}^{2}\frac{t^m}{m!}\sum_{j=1}^{\infty}\frac{(1)_{j,\lambda}}{j^{k-1}}\frac{t^j}{j!}\sum_{l=0}^{\infty}\mathcal{G}_{l,\lambda}\frac{(e_\lambda-1)^l}{l!}$$

$$=\sum_{r=1}^{\infty}\sum_{j=1}^{r}\binom{r}{j}\mathcal{B}_{r-j,\lambda}^{2}\frac{(1)_{j,\lambda}}{j^{k-1}}\frac{t^r}{r!}\sum_{l=0}^{\infty}\mathcal{G}_{l,\lambda}\sum_{i=l}^{\infty}S_{2,\lambda}(i,l)\frac{t^i}{i!}$$

$$=\sum_{n=1}^{\infty}\sum_{r=1}^{n}\binom{n}{r}\sum_{j=1}^{r}\binom{r}{j}\mathcal{B}_{r-j,\lambda}^{2}\frac{(1)_{j,\lambda}}{j^{k-1}}\sum_{l=0}^{i}\mathcal{G}_{l,\lambda}S_{2,\lambda}(i,l)\frac{t^n}{n!},\text{ with } n\geq 2. \tag{23}$$

By (22) and (23), we have the following theorem.

Theorem 2.4. *For $n\geq 2$, we have*

$$n(n-1)\sum_{m=0}^{n-2}\mathcal{E}_{m,\lambda}^{(k)}\,S_{2,\lambda}(n-2,m)$$

$$=\sum_{r=1}^{n}\binom{n}{r}\sum_{j=1}^{r}\binom{r}{j}\mathcal{B}_{r-j,\lambda}^{2}\frac{(1)_{j,\lambda}}{j^{k-1}}\sum_{l=0}^{i}\mathcal{G}_{l,\lambda}S_{2,\lambda}(i,l).$$

It is known that the following an equation

$$\frac{-2}{\left(e_{\lambda}\left(t\right)+1\right)e_{\lambda}\left(t\right)}=\frac{2}{e_{\lambda}\left(t\right)+1}-\frac{2}{e_{\lambda}\left(t\right)}.$$

From here, we write as

$$\begin{aligned}&\frac{-2\,\mathrm{Ei}_{k,\lambda}\left(\log_{\lambda}\left(1+t\right)\right)e_{\lambda}^{x}\left(t\right)}{t\left(e_{\lambda}\left(t\right)+1\right)e_{\lambda}\left(t\right)}\\ &=\frac{2}{t}\frac{\mathrm{Ei}_{k,\lambda}\left(\log_{\lambda}\left(1+t\right)\right)e_{\lambda}^{x}\left(t\right)}{e_{\lambda}\left(t\right)+1}-\frac{2}{t}\frac{\mathrm{Ei}_{k,\lambda}\left(\log_{\lambda}\left(1+t\right)\right)e_{\lambda}^{x}\left(t\right)}{e_{\lambda}\left(t\right)}.\end{aligned}$$

It follows from (16), (18) and (19) that

$$\begin{aligned}&\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(k)}\left(x+1\right)\frac{t^{n}}{n!}-2\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{m}\sum_{j=1}^{m+1}\frac{(1)_{j,\lambda}}{j^{k-1}}\frac{S_{1,\lambda}\left(m+1,j\right)}{m+1}\left(x\right)_{n-m,\lambda}\right)\frac{t^{n}}{n!}\\ &=-\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(k)}\left(x\right)\frac{t^{n}}{n!}.\end{aligned}\tag{24}$$

By (24), comparing the coefficients of $\frac{t^n}{n!}$, we have this recovers Theorem 2.2 again.

From (15), we note that

$$\frac{d}{dx}\mathrm{Ei}_{k,\lambda}\left(x\right)=\frac{d}{dx}\sum_{n=1}^{\infty}\frac{(1)_{n,\lambda}}{(n-1)!n^{k}}x^{n}=\frac{1}{x}\mathrm{Ei}_{k,\lambda}\left(x\right).\tag{25}$$

Thus by (25), we get

$$\begin{aligned}\mathrm{Ei}_{k,\lambda}\left(x\right)&=\int_{0}^{x}\frac{1}{t}\mathrm{Ei}_{k-1,\lambda}\left(t\right)dt=\underbrace{\int_{0}^{x}\frac{1}{t}\int_{0}^{t}\cdots\frac{1}{t}\int_{0}^{t}\frac{1}{t}}_{(k-2)\text{ times}}\mathrm{Ei}_{1,\lambda}\left(x\right)\underbrace{dt\cdots dt}_{(k-2)\text{ times}}\\ &=\underbrace{\int_{0}^{x}\frac{1}{t}\int_{0}^{t}\cdots\frac{1}{t}\int_{0}^{t}\frac{1}{t}}_{(k-2)\text{ times}}\left(e_{\lambda}\left(t\right)-1\right)\underbrace{dt\cdots dt}_{(k-2)\text{ times}},\end{aligned}$$

where k is a positive integer with $k\geq 2$.

From (12), (16) and (25), for $k=2$;

$$\begin{aligned}\sum_{n=0}^{\infty}\mathcal{E}_{n,\lambda}^{(2)}\left(x\right)\frac{t^{n}}{n!}&=\frac{2}{t\left(e_{\lambda}\left(t\right)+1\right)}\int_{0}^{t}\frac{t}{\log_{\lambda}\left(1+t\right)}\left(1+t\right)^{\lambda-1}dt\\ &=\frac{2}{t\left(e_{\lambda}\left(t\right)+1\right)}\sum_{m=0}^{\infty}\frac{b_{m,\lambda}\left(\lambda-1\right)}{m+1}\frac{t^{m}}{m!}\\ &=\sum_{l=0}^{\infty}\mathcal{E}_{l,\lambda}^{(k)}\frac{t^{l}}{l!}\sum_{m=0}^{\infty}\frac{b_{m,\lambda}\left(\lambda-1\right)}{m+1}\frac{t^{m}}{m!}\end{aligned}\tag{26}$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{n} \binom{n}{m} \mathcal{E}_{n-m,\lambda}^{(k)} \frac{b_{m,\lambda}(\lambda-1)}{m+1} \frac{t^n}{n!}.$$

From (26), we have the following theorem.

Theorem 2.5. *For $n \geq 0$, we have*

$$\mathcal{E}_{n,\lambda}^{(2)}(x) = \sum_{m=0}^{n} \binom{n}{m} \mathcal{E}_{n-m,\lambda}^{(k)} \frac{b_{m,\lambda}(\lambda-1)}{m+1}.$$

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