# INDEPENDENT TRANSVERSAL DOMINATION NUMBER IN COMPLEMENTARY PRISMS 

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#### Abstract

A set $D \subseteq V(G)$ is an independent transversal dominating set of $G$ if $D$ is a dominating set and also intersects every maximum independent set in $G$. The minimum cardinality of such a set is equal to the transversal domination number, denoted by $\gamma_{i t}(G)$. This paper is devoted to the computation of the independent transversal domination number of some complementary prism.


## 1. Introduction

Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$, and of order $n=|V(G)|$ and size $m=|E(G)|$. For two vertices $u, v \in V(G), u$ and $v$ are neighbors if $u$ and $v$ are adjacent, that is, if there is an edge $e=u v$. The distance $d_{G}(u, v)$ between $u$ and $v$ is the length of a shortest $(u, v)$-path in $G$. The diameter $\operatorname{diam}(G)$ of $G$ is the maximum distance between vertices of $G$. The degree of a vertex $v$ is $\operatorname{deg}_{G}(v)=|N(v)|$. A vertex of degree zero is an isolated vertex or an isolate. A leaf or an end vertex is a vertex of degree one and its neighbor is called a support vertex. The minimum degree of $G$ is $\delta(G)=\min \left\{\operatorname{deg}_{G}(v) \mid v \in V(G)\right\}$. A set $I \subseteq V(G)$ is independent if every two vertices of $I$ are non-adjacent. While the independence number $\beta(G)$ of $G$ is the cardinality of the largest independent set in $G$, the maximum independent set is the independent set with the cardinality $\beta(G)$ in $G . \beta(G)$-set is a maximum independent set in $G[13,6]$.

A dominating set of a graph $G$ is a set $D \subseteq V(G)$ such that every vertex in $V(G)-D$ is adjacent to at least one vertex in $D$. The domination number of $G$, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of $G$.

[^0]A $\gamma(G)$-set is a dominating set of cardinality $\gamma(G)$. Furthermore, if every vertex in $V(G)$ is adjacent to at least one vertex in $D$, then the set $D$ is a total dominating set of $G$. The total domination number of $G$ is the shortest size of any total dominating set in $G$ and is denoted by $\gamma_{t}(G)$. For more information on domination and total domination see the book $[13,15]$ and the survey [16].

Dominating and independent sets have been studied in graph theory throughout history $[13,6,15]$. An independent dominating set in $G$ consists of a combination of the concepts of domination and independence. This set is both dominating and independent in $G$. The independent domination number of $G$, $i(G)$, is the cardinality of a minimum independent dominating set in $G$ [7].

Given a graph $G$ and a collection of subsets of its vertices, if it interrupts every subset of the collection, then the subset of $V(G)$ is called a transversal of $G$. A high level of attention has been given to transversal graphs in recent years. In $[10,3]$ transversal of different kinds of vertex subsets of a graphs, such as an independent or dominating set, have been studied. There are many transversal examples of the independence number of graphs in the literature. More recently, independence transversal domination number as some new style of transversal has been defined by Hamid [11]. Although it is a new parameter, there is not much work in the literature on the parameter. Only a few interesting results about this have been made by Hamid and Abdollahzadeh Ahangar et al. $[11,1]$.

A set $D \subseteq V(G)$ is an independent transversal dominating set (ITD-set) of $G$ if and only if each $D$ has the following two properties:

- $P_{1}$ : is a dominating set of $G$
- $P_{2}$ : intersects every maximum independent set in $G$.

The minimum cardinality of a $I T D$-set is equal to the independent transversal domination number (ITD-number) of $G$, and denoted by $\gamma_{i t}(G)$. A ITD-set which gives the value $\gamma_{i t}(G)$ is called a $\gamma_{i t}(G)$-set [11].

The complementary product is a generalization of the cartesian product [12]. Complementary prisms of a graph $G$, denoted as $G \bar{G}$, is the subset of complementary products. Let $\bar{G}$ be the complement of $G$. The complementary prism $G \bar{G}$, is formed from a copy of $G$ and a copy of $\bar{G}$ by adding a perfect matching between corresponding vertices. For each $v \in V(G)$, let $\bar{v}$ denote the vertex $v$ in the copy of $\bar{G}$. Formally, $G \bar{G}$ is formed from $G \cup \bar{G}$ by adding the edge $v \bar{v}$ for every $v \in V(G)$. As shown in Fig. 1, the complementary prism graph $C_{5} \bar{C}_{5}$ is also Petersen graph.

Thus, complementary prisms that generalize the concept of graph products and graphs such as the Petersen graph form an interesting family of graphs.

Therefore, it is important to study the parameters for these graphs. In this paper we consider independence transversal domination number of complementary prisms. To aid the discussion of complementary prisms, we will use the following terminology: For vertex partitions $V$ and $\bar{V}$ of $G \bar{G}$, let $V=V(G)$ and $\bar{V}=V(\bar{G})$.

As an example, we can observe the graph $G_{10}$ of Fig. 1 where $\beta\left(G_{10}\right)=4$. We can check that there are five possible $\beta\left(G_{10}\right)$-set: $\left\{v_{1}, v_{4}, \bar{v}_{2}, \bar{v}_{3}\right\},\left\{v_{3}, v_{5}, \bar{v}_{1}, \bar{v}_{2}\right\}$, $\left\{v_{2}, v_{4}, \bar{v}_{1}, \bar{v}_{5}\right\},\left\{v_{2}, v_{5}, \bar{v}_{3}, \bar{v}_{4}\right\}$ and $\left\{v_{1}, v_{3}, \bar{v}_{4}, \bar{v}_{5}\right\}$. And ten possible $\gamma(G)$-set $:\left\{v_{2}, v_{4}, \bar{v}_{3}\right\},\left\{v_{1}, \bar{v}_{3}, \bar{v}_{4}\right\},\left\{v_{1}, v_{3}, \bar{v}_{2}\right\},\left\{v_{2}, \bar{v}_{4}, \bar{v}_{5}\right\},\left\{v_{2}, v_{5}, \bar{v}_{1}\right\},\left\{v_{1}, v_{4}, \bar{v}_{5}\right\}$, $\left\{v_{3}, v_{5}, \bar{v}_{4}\right\},\left\{v_{3}, \bar{v}_{1}, \bar{v}_{5}\right\},\left\{v_{4}, \bar{v}_{1}, \bar{v}_{2}\right\}$ and
$\left\{v_{5}, \bar{v}_{2}, \bar{v}_{3}\right\}$. None of the dominating sets intersect all these five $\beta\left(G_{10}\right)$-sets, and as a consequence $\gamma_{i t}(G) \geq 4$. Now take $D_{1}=\left\{v_{2}, v_{4}, \bar{v}_{3}\right\} \cup\left\{v_{3}\right\}$. Since $D_{1}$ intersects all $\beta\left(G_{10}\right)$-sets $D_{1}$ is an ITD-set and $\gamma_{i t}(G)=4$.

In this paper we studied the independent transversal dominating number of complementary prisms of some of the well known graphs.


Figure 1. The Petersan Graph $C_{5} \overline{C_{5}}$

## 2. Known results

Theorem 2.1. [11] The independent transversal domination number of
(a) the complete graph $K_{n}$ of order $n$ is $\gamma_{i t}\left(K_{n}\right)=n$;
(b) the complete bipartite graph $K_{n, m}$ of order $m+n$ is $\gamma_{i t}\left(K_{n, m}\right)=2$;
(c) the star graph $S_{n}$ of order $n$ is $\gamma_{i t}\left(S_{n}\right)=2$;
(d) the path graph $P_{n}$ of order $n$ is

$$
\gamma_{i t}\left(P_{n}\right)= \begin{cases}2, & n=2,3 \\ 3, & n=6 \\ \left\lceil\frac{n}{3}\right\rceil, & \text { otherwise }\end{cases}
$$

(e) the cycle graph $C_{n}$ of order $n$ is

$$
\gamma_{i t}\left(C_{n}\right)= \begin{cases}3, & n=3,5 \\ \left\lceil\frac{n}{3}\right\rceil, & \text { otherwise }\end{cases}
$$

(f) the wheel graph $W_{n}$ of order $n$ is

$$
\gamma_{i t}\left(W_{n}\right)= \begin{cases}2, & n=5 \\ 3, & n \geq 7 \text { and is odd or } n=6 \\ 4, & \text { otherwise }\end{cases}
$$

Theorem 2.2. [11] If $G$ has an isolated vertex, then $\gamma_{i t}(G)=\gamma(G)$.
Theorem 2.3. [11] For any graph $G$ of order $n$, we have $1 \leq \gamma_{i t}(G) \leq n$. Further $\gamma_{i t}(G)=n$ if and only if $G=K_{n}$.

Theorem 2.4. [11] For any graph $G$ of order n, we have $\gamma(G) \leq \gamma_{i t}(G) \leq \gamma(G)+\delta(G)$.

Lemma 2.5. [11] If $G$ is a non-complete connected graph on $n$ vertices, then $\gamma_{i t}(G) \leq\left\lceil\frac{n}{2}\right\rceil$.

Proposition 2.6. [14] If $G=t K_{2}$, then $\gamma(G \bar{G})=t+1$.

## 3. ITD-number in complementary prisms

Some of the theoretical graph parameters have been applied to the complementary prisms. Haynes et al. study domination and total domination number for complementary prisms [14]. Furthermore, Gongora et al. determine independent domination domination number in complementary prisms [8]. On the other hand, Aytaç and Turacı obtain strong weak domination number of complementary prisms [5]. In [19], Mojdeh et al. concern with the strong roman domination number of complementary prisms. In [18], Kazemi study $k$-tuple total domination in complementary prisms. There are many more studies on
complementary prisms in the literature $[17,2,20,4,9]$. We, in this section, determine ITD-number of complementary prisms of some graphs.

Theorem 3.1. Let $G \cong P_{n}$ be the path graph with $n$ vertices. Then we have the equality $\gamma_{i t}\left(P_{n} \overline{P_{n}}\right)=\gamma\left(P_{n} \overline{P_{n}}\right)$ if one of the following cases hold,

$$
\begin{gathered}
n>9 \text { and } n \equiv 0(\bmod 3) \\
\text { or } \\
n>7 \text { and } n \equiv 1 \text { or } 2(\bmod 3) .
\end{gathered}
$$

Proof. Let $V=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$ and $\bar{V}=\left\{\bar{v}_{1}, \bar{v}_{2}, \bar{v}_{3}, \ldots, \bar{v}_{n}\right\}$. Let $n \equiv$ $0(\bmod 3)$. The set $D=\left\{\bar{v}_{1}\right\} \bigcup\left\{v_{3 i-1}: 1 \leq i \leq \frac{n}{3}\right\}$ is $\gamma$-set. Further $<$ $V\left(P_{n} \overline{P_{n}}\right)-D>=\left(\frac{n}{3}-1\right) K_{2} \cup 2 K_{1} \cup \overline{P_{n-1}}$ and hence every independent set in $V\left(P_{n} \overline{P_{n}}\right)-D$ contains at most $\frac{n}{3}+3$ vertices. If $n>9$, then we have

$$
\frac{n}{3}+3<\left\lceil\frac{n-3}{2}\right\rceil+3=\left|\beta\left(P_{n} \overline{P_{n}}\right)\right| .
$$

It follows that $V\left(P_{n} \overline{P_{n}}\right)-D$ contains no $\beta$-set and hence $\gamma_{i t}\left(P_{n} \overline{P_{n}}\right)=\gamma\left(P_{n} \overline{P_{n}}\right)$.
Now consider $n \equiv 1(\bmod 3)$. The set $D=\left\{\bar{v}_{1}, v_{n}\right\} \cup\left\{v_{3 i-1}: 1 \leq i \leq \frac{n-1}{3}\right\}$ is $\gamma$-set. Further $<V\left(P_{n} \overline{P_{n}}\right)-D>=\left(\left\lfloor\frac{n}{3}\right\rfloor-1\right) K_{2} \cup 2 K_{1} \cup \overline{P_{n-1}}$, hence every independent set in $V\left(P_{n} \overline{P_{n}}\right)-D$ contains at most $\left\lfloor\frac{n}{3}\right\rfloor+3$ vertices. Thus, if $n>7$

$$
\left\lfloor\frac{n}{3}\right\rfloor+3<\left\lceil\frac{n-3}{2}\right\rceil+3=\left|\beta\left(P_{n} \overline{P_{n}}\right)\right|
$$

holds and $\gamma_{i t}\left(P_{n} \overline{P_{n}}\right)=\gamma\left(P_{n} \overline{P_{n}}\right)$.
If $n \equiv 2(\bmod 3)$ then the set $D=\left\{\bar{v}_{1}\right\} \cup\left\{v_{3 i-1}: 1 \leq i \leq\left\lceil\frac{n}{3}\right\rceil\right\}$ is $\gamma$-set and $<V\left(P_{n} \overline{P_{n}}\right)-D>=\left\lfloor\frac{n}{3}\right\rfloor K_{2} \cup K_{1} \cup \overline{P_{n-1}}$ hence every independent set in $V\left(P_{n} \overline{P_{n}}\right)-D$ contains at most $\left\lfloor\frac{n}{3}\right\rfloor+3$ vertices. Thus, if $n>7$

$$
\left\lfloor\frac{n}{3}\right\rfloor+3<\left\lceil\frac{n-3}{2}\right\rceil+3=\left|\beta\left(P_{n} \overline{P_{n}}\right)\right|
$$

holds and $\gamma_{i t}\left(P_{n} \overline{P_{n}}\right)=\gamma\left(P_{n} \overline{P_{n}}\right)$.

Theorem 3.2. Let $G \cong C_{n}$ be the cycle graph with $n$ vertices. Then we have the equality $\gamma_{i t}\left(C_{n} \overline{C_{n}}\right)=\gamma\left(C_{n} \overline{C_{n}}\right)$ if one of the following cases hold,

$$
\begin{gathered}
n>6 \text { and } n \equiv 0(\bmod 3) \\
\text { or } \\
n>10 \text { and } n \equiv 1(\bmod 3) \\
\text { or } \\
n>8 \text { and } n \equiv 2(\bmod 3) .
\end{gathered}
$$

Proof. Let $V=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$ and $\bar{V}=\left\{\bar{v}_{1}, \bar{v}_{2}, \bar{v}_{3}, \ldots, \bar{v}_{n}\right\}$. Let $n \equiv$ $0(\bmod 3)$. The set $D=\left\{\bar{v}_{n-1}, \bar{v}_{n}\right\} \cup\left\{v_{3 i+1}: 0 \leq i \leq \frac{n-3}{3}\right\}$ is $\gamma$-set. Further $<$ $V\left(C_{n} \overline{C_{n}}\right)-D>=\frac{n}{3} K_{2} \cup \overline{C_{n-2}}$ and hence every independent set in $V\left(C_{n} \overline{C_{n}}\right)-D$ contains at most $\frac{n}{3}+2$ vertices. If $n>6$, then we have

$$
\frac{n}{3}+2<\left\lceil\frac{n}{2}\right\rceil+1=\left|\beta\left(C_{n} \overline{C_{n}}\right)\right| .
$$

It follows that $V\left(C_{n} \overline{C_{n}}\right)-D$ contains no $\beta$-set and hence $\gamma_{i t}\left(C_{n} \overline{C_{n}}\right)=\gamma\left(C_{n} \overline{C_{n}}\right)$.
Now consider $n \equiv 1(\bmod 3)$. The set $D=\left\{\bar{v}_{1}, v_{n}\right\} \cup\left\{v_{3 i-1}: 0 \leq i \leq\left\lfloor\frac{n}{3}\right\rfloor\right\}$ is $\gamma$-set. Further $<V\left(C_{n} \overline{C_{n}}\right)-D>=\left(\left\lfloor\frac{n}{3}\right\rfloor-1\right) K_{2} \cup 2 K_{2} \cup \overline{P_{n-1}}$ and hence every independent set in $V\left(C_{n} \overline{C_{n}}\right)-D$ contains at most $\left\lfloor\frac{n}{3}\right\rfloor+3$ vertices. Thus, if $n>10$, then

$$
\left\lfloor\frac{n}{3}\right\rfloor+3<\left|\beta\left(C_{n} \overline{C_{n}}\right)\right|
$$

holds and $\gamma_{i t}\left(C_{n} \overline{C_{n}}\right)=\gamma\left(C_{n} \overline{C_{n}}\right)$.
If $n \equiv 2(\bmod 3)$ then the set $D=\left\{\bar{v}_{1}\right\} \cup\left\{v_{3 i-1}: 0 \leq i \leq \frac{n+1}{3}\right\}$ is $\gamma$-set and $<V\left(C_{n} \overline{C_{n}}\right)-D>=\left(\frac{n}{3}\right) K_{2} \cup K_{1} \cup \overline{P_{n-1}}$ and hence every independent set in $V\left(C_{n} \overline{C_{n}}\right)-D$ contains at most $\left\lfloor\frac{n}{3}\right\rfloor+3$ vertices. Thus, if $n>8$, then we have

$$
\left\lfloor\frac{n}{3}\right\rfloor+3<\left\lceil\frac{n-4}{2}\right\rceil+3=\left|\beta\left(C_{n} \overline{C_{n}}\right)\right|
$$

and $\gamma_{i t}\left(C_{n} \overline{C_{n}}\right)=\gamma\left(C_{n} \overline{C_{n}}\right)$.

Theorem 3.3. Let $G \cong K_{n}$ be the complete graph with $n$ vertices. Then $\gamma_{i t}\left(K_{n} \overline{K_{n}}\right)=n$.

Proof. Let $V\left(K_{n}\right)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$ and $V\left(\overline{K_{n}}\right)=\left\{\bar{v}_{1}, \bar{v}_{2}, \bar{v}_{3}, \ldots, \bar{v}_{n}\right\}$. Since the degree of all vertices in $V\left(\overline{K_{n}}\right)$ is 1 and none of them are adjacent, we have $\gamma\left(K_{n} \overline{K_{n}}\right) \geq n$. The set $V\left(\overline{K_{n}}\right)$ is also a dominating set. Thus $\gamma\left(K_{n} \overline{K_{n}}\right)=n$. Which concludes $\gamma_{i t}\left(K_{n} \overline{K_{n}}\right) \geq n . V\left(\overline{K_{n}}\right)$ is also an -ITD set. So $\gamma_{i t}\left(K_{n} \overline{K_{n}}\right)=n$.

Theorem 3.4. Let $G \cong S_{n}$ be the star graph with $n$ vertices. Then $\gamma_{i t}\left(S_{n} \overline{S_{n}}\right)=3$

Proof. Let $V\left(S_{n}\right)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n}\right\}$ and $\bar{V}=\left\{\bar{v}_{1}, \bar{v}_{2}, \bar{v}_{3}, \ldots, \bar{v}_{n}\right\}$ where $v_{1}$ is the center of $S_{n}$ and $\bar{v}_{1}$ is the corresponding vertex in $\bar{V}$. We know that for any $\bar{v}_{i} \in\left(\bar{V}-\left\{\bar{v}_{1}\right\}\right)$, the set $D=\left\{v_{1}, \bar{v}_{i}\right\}$ is dominating set. For any independent set $\beta, \bar{v}_{1} \in \beta$. So $D \cup\left\{\bar{v}_{1}\right\}$ is ITD-set. $\gamma_{i t}\left(S_{n} \overline{S_{n}}\right) \leq 3$. Now consider the independent set $\beta_{0}=\left\{v_{2}, v_{3}, v_{4}, \ldots, \bar{v}_{1}\right\}$. For any dominating set $D, \beta_{0} \cap D=\emptyset$ which concludes $\gamma_{i t}\left(S_{n} \overline{S_{n}}\right)>2$. Hence $\gamma_{i t}\left(S_{n} \overline{S_{n}}\right)=3$.

Theorem 3.5. Let $G \cong W_{n}$ be the wheel graph with $n$ vertices. Then $\gamma_{i t}\left(W_{n} \overline{W_{n}}\right)=4$.

Proof. Let $V\left(W_{n}\right)=\left\{v_{1}, v_{2}, v_{3}, \ldots, v_{n-1}, c\right\}$ and $V\left(\overline{W_{n}}\right)=\left\{v_{1}^{\prime}, v_{2}^{\prime}, v_{3}^{\prime}\right.$,
$\left.\ldots, v_{n-1}^{\prime}, c^{\prime}\right\}$ where $c$ is the center of $W_{n}$ and $c^{\prime}$ is the corresponding vertex in $V\left(\overline{W_{n}}\right)$. Any dominating set of $W_{n} \overline{W_{n}}$ can be defined as $D=\left\{c, v_{i}^{\prime}, v_{k}^{\prime}\right.$ : $v_{i}^{\prime}, v_{k}^{\prime} \in V\left(\overline{W_{n}}\right)$ and $\left.d\left(v_{i}, v_{k}\right) \neq 2\right\}$ where $d\left(v_{i}, v_{k}\right)$ is the distance between $v_{i}$ and $v_{k}$. Some of the independent sets of $W_{n} \overline{W_{n}}$ include only one vertex from $V\left(\overline{W_{n}}\right)$ and none of them includes $c$. Thus we can find an independent set $\beta_{0}$ which does not include $c, v_{i}^{\prime}, v_{k}^{\prime}$. Hence $\gamma_{i t}\left(W_{n} \overline{W_{n}}\right)>\gamma\left(W_{n} \overline{W_{n}}\right)$. Since all independent sets include $c^{\prime}$, for any dominating set $D,\left\{c^{\prime}\right\} \cup D$ is ITD-set and $\gamma_{i t}\left(W_{n} \overline{W_{n}}\right) \leq \gamma\left(W_{n} \overline{W_{n}}\right)+1$. Thus $\gamma_{i t}\left(W_{n} \overline{W_{n}}\right)=\gamma\left(W_{n} \overline{W_{n}}\right)+1=4$.

Theorem 3.6. Let $G \cong K_{n, m}$ be a complete bipartite graph with two partite sets, $A$ and $B$ having $n$ and $m$ vertices respectively where $n \geq m$. If $n>m, \gamma_{i t}\left(K_{n, m} \overline{K_{n, m}}\right)=\gamma\left(K_{n, m} \overline{K_{n, m}}\right)+1$. If $n=m$, then $\gamma_{i t}\left(K_{n, m} \overline{K_{n, m}}\right)=$ $\gamma\left(K_{n, m} \overline{K_{n, m}}\right)+2$.

Proof. Now assume $n>m$. Let $V(G \bar{G})=A \cup B \cup V(\bar{G})$ where $A \cup B=$ $V(G)$. It is clear that the complement part of $G \bar{G}$ is $K_{n}+K_{m}$. Now $D=$ $\left\{v_{i}, v_{j}, v_{k}, v_{t}\right\}$ where $v_{i} \in A, v_{j} \in B, v_{k} \in K_{n}, v_{t} \in K_{m}$ is minimal dominating set and $\gamma_{i t}(G \bar{G})>\gamma(G \bar{G})$. For any maximal independent set $\beta$, either $\beta=$ $A \cup\left\{v_{i}\right\}$ where $v_{i} \in K_{m}$ or $\beta=A-\left\{v_{j}\right\} \cup\left\{v_{i}\right\} \cup\left\{v_{k}\right\}$ where $v_{i} \in K_{m}, v_{j} \in A$, $v_{k}=\overline{v_{j}}$. Thus adding two vertices to $D$ from $A$ guarantees that $D$ intersects every $\beta$. Since $D$ already has a vertex from $A, D \cup\left\{v_{s}\right\}$ is $I T D$-set where $v_{s} \in A$ and $v_{s} \neq v_{i}$. Hence $\gamma_{i t}\left(K_{n, m} \overline{K_{n, m}}\right)=\gamma\left(K_{n, m} \overline{K_{n, m}}\right)+1$. Let $n=m$. Now we can also construct $\beta$ set by replacing the set $A$ with set $B$. Thus we have to add one more vertex to $D$ from B.Thus, $\gamma_{i t}\left(K_{n, m} \overline{K_{n, m}}\right)=\gamma\left(K_{n, m} \overline{K_{n, m}}\right)+2$.

Theorem 3.7. Let $G \cong t K_{2}$ where $t$ is positive integer. Then $\gamma_{i t}\left(t K_{2} \overline{t K_{2}}\right)=$ $t+3$.

Proof. We known that $\gamma\left(t K_{2} \overline{t K_{2}}\right)=t+1$ by Proposition 2.6. Now consider one of the $K_{2} \overline{K_{2}}$ blocks of $t K_{2} \overline{t K_{2}}$. Every $\beta$ set has at least one, at most two vertices from this block. Each minimal dominating set $D$ has two vertices where this two vertices are in the same $K_{2} \overline{K_{2}}$ block. Thus adding the remaining vertices from this block to $D$ guarantees the intersection of $D$ with any maximal $\beta$ sets.

Theorem 3.8. Let $G \cong K_{n} \circ K_{1}$ be the graph obtained by adding an endpoint to all vertices of $K_{n}$. Then $\gamma_{i t}(G \bar{G})=\gamma(G \bar{G})$.

Proof. Since there is only one maximal independent set, the equality is obvious.

Theorem 3.9. Let $G$ be the graph with the maximum degree of $n-1$. If $G$ has an endpoint then $\gamma_{i t}(G \bar{G})=3$.

Proof. Let $V(G)=\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}, V(\bar{G})=\left\{\bar{v}_{1}, \bar{v}_{2}, \ldots, \bar{v}_{n}\right\}, \operatorname{deg}_{G}\left(v_{n}\right)=n-1$ and $\operatorname{deg}_{G}\left(v_{1}\right)=1$. The set $D=\left\{v_{n}, \bar{v}_{1}\right\}$ is the only dominating set. We can
find at least one maximum independent set $\beta$ such that $v_{n}, \bar{v}_{1} \notin \beta$. Thus $\gamma_{i t}>\gamma$. Since $\bar{v}_{n} \in \beta$ for all maximum independent sets, $D \cup\left\{\bar{v}_{n}\right\}$ is an $I T D$-set $\gamma_{i t}(G \bar{G})=3$.

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