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Monthly Hanwoo supply and forecasting models

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Abstract

As the number of scaled-up ranches increased and agile responses to market changes became possible, decision-making by Hanwoo cattle farms also began to affect short-term shipments. Considering the changing environment of the Hanwoo supply market and the response speed of producers, it is necessary quickly to grasp the forecast ahead of time and to respond accordingly in an effort to stabilize supply and demand in the Hanwoo market. In this study, short-term forecasting model centered on the supply of Hanwoo was established. The analysis conducted here indicates that the slaughter of Hanwoo males increases by 0.248 as the number of beef cattle raised over 29 months of age in the previous month increases by one, and 0.764 Hanwoo females were slaughtered under average conditions for every Hanwoo male slaughtered. With regard to time, the slaughtering of Hanwoo was higher in January and August, which are months known for holiday food preparation activities for the New Year and Chuseok in Korea, respectively. Simulations indicated that errors were within 10% in all simulations performed through the Hanwoo supply model. Accordingly, it is considered that the estimation results from the supply model devised in this study are reliable and that the model has good structural stability.

Keywords: forecasting model, Hanwoo, market supply, price analysis, SUR (seemingly unrelated regression)

Introduction

Hanwoo (a native Korean breed of cattle) beef has been recognized as the preferred protein source by consumers (Jeong et al., 2020) and has become an indispensable food in their diet. The Hanwoo industry has achieved quantitative and qualitative growth through advanced and differentiated strategies after the opening of imports, and in this regard, related studies that contribute to the improvement of productivity, such as the breeding and rearing of Korean cattle, have developed together. One of the most notable achievements in the realm of growth of the Korean beef industry and expansion of research is the growth of the outlook program.



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the Creative Commons Attribution Non-Commercial License (http: //creativecommons.org/licenses/bync/4.0/) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited. Outlook plays a role in providing relevant information to consumers and producers of Hanwoo on industry trends, supplydemand analysis, price forecasts, and so on, and it ultimately contributes to the stable maintenance of farms' household income. As more precise data, called the Hanwoo traceability system, was available, the outlook program became adept at providing the necessary information, such as the number of animals raised and slaughtered, leading to the availability of the basic data needed for the study.

Along with these changes attributable to the outlook program, several attempts have been made to predict supply and price through the construction of a supply and demand model for Hanwoo, and the forecasting system related to the supply and demand response of Hanwoo has been further developed. Hanwoo has a relatively long production cycle compared to other livestock products; it takes more than 30 months for stock to be shipped to market.

According to these characteristics, the outlook for Hanwoo has been focused on mid- to long-term forecasting, and the development of a forecasting system based on model setting has laid the foundation on the side of mid- to long-term to some extent. However, unlike in the past when the proportion of farmers who raise one or two cows was absolutely a lot, currently, cattle herds owned by scaled-up ranches account for two-thirds of the total. As the number of scaled-up ranches increased and agile responses to market changes became possible, the decision-making of Hanwoo ranchers also began to affect short-term shipments.

However, Hanwoo supply and demand models are still insufficient for short-term forecasting. Considering the changing environment of the Hanwoo supply market and the response speed of producers, it is necessary to quickly grasp the forecast ahead of time and respond accordingly in order to stabilize supply and demand in the Hanwoo market. In this study, using the Hanwoo production cycle, a short-term forecasting model was established centered on the supply of Korean beef, and the model carried out simulations and provided verification data as part of providing a plan for preemptive supply and demand response.

Literature review

The Hanwoo forecasting model provides information related to supply, demand, and pricing to achieve a stable level of supply and demand and performs the function of predicting Hanwoo supply, demand, and pricing in advance through model estimation. The methodological basis of forecasting models relating to outlook can be found as early as the early 20th century, and as statistical figures on the supply and demand of agricultural products were released, an outlook-related forecasting model has been developed, enabling comparative analysis between items (Warren and Pearson, 1928). Since then, as economic theories have developed, various methodologies have been developed to derive empirical analysis results that fit economic principles (Waugh, 1964; McKillop, 1967; Deaton and Muellbauer, 1980; Chambers, 1988). In modern times, performing econometric analysis of the system of simultaneous equations of supply and demand has been established as a general methodology for empirical analysis.

In particular, in the field of beef industry research, the movement to increase the realistic explanatory power of the model in consideration of the biological characteristics of livestock and cattle's life cycle has become a key issue (Rosen, 1987; Rosen et al., 1994). Since then, the precision of outlook has begun to improve as techniques, such as time series analysis, dynamic analysis, and simulation analysis, have been used for forecasting and projecting further into the future (Leuthold, 1974; Martin and Garcia, 1981; Chudleigh and Cezar, 1982; Huffinan and Evenson, 1989; Sarmiento and Allen, 2000). Currently, it is considered the most appropriate method to operate variably an optimized model according to the purpose of the study under a system of simultaneous equations of supply and demand.

There have been several attempts to analyze the price of Hanwoo in academia in Korea, and it is true that meaningful research results have been derived. A representative research method is forecasting model analysis, which mainly consists of constructing and operating a structural model for livestock outlook and projections (Seo et al., 2020).

These models can dynamically reflect changes in internal and external conditions related to the livestock industry and have the advantage of being able to effectively simulate the supply and demand situation of the Korean beef market according to policy or market changes (Seo et al., 2017; 2020). Since the time unit is established annually, there are limits in terms of the precision of supply/demand forecast delicacy, and they have the disadvantage of being difficult to estimate due to their complicated step-by-step structure.

A few short-term model studies have been conducted that reflect the biological characteristics and seasonality of livestock ranching and show superiority in terms of precision (Jeong et al., 2005; Cho et al., 2014; 2017; Seo et al., 2017), but since analysis results are represented in quarterly figures, they still have limitations in that they are insufficient to provide information on smaller units of time, such as monthly forecasts.

In this study, a supply model with strong predictive power that can reflect Hanwoo's short-term supply and demand situation was constructed using the highly accurate data from the beef traceability system, and the model used in this paper has the distinction of being precise in units and the inability of dynamic analysis on a monthly basis compared to existing research on cattle.

Considering that the short-term forecasting model constructed in this study is a monthly model developed to enable a one-month advance outlook, it has the advantage of enabling producer and consumer to respond in a timely manner when an irregular supply and demand situation occurs, and also has policy significance as it can preemptively utilize the derived analysis results within a relatively short period.

Materials and Methods

Data

To construct a dataset for model estimation, the beef traceability system information from the Korea Institute for Animal Products Quality Evaluation (KAPE) was used, and the summary of data used for the analysis is presented in Table 1. Traceability system data are monthly, and the period used for estimating the supply model is from January 2015 to October 2020. To estimate the Hanwoo male supply, the number of slaughtered Hanwoo males was used as a dependent variable, and as an explanatory variable, the number of Hanwoo beef males raised over 29 months of age before one month was used, focusing on the fact that the slaughter age of a male Hanwoo is around 30 months on average.

To estimate the female Hanwoo supply, the number of slaughtered Hanwoo females was used as the dependent variable, and the ratio of Hanwoo females slaughtered to Hanwoo males slaughtered, derived using the number of slaughtered Hanwoo males in the current month, was used as an explanatory variable. In addition, month dummies and structural dummies that can affect the Hanwoo supply model were used as control variables. D1 is a dummy variable for August 2016, September 2017, January, August, October, November, December 2019, and September 2020, and D2 is a dummy variable for June to September 2016 and February 19, and is a dummy at which the supply shock occurs as a supply shifter. SD1 indicates a structural change that controls the impact effect caused by the outbreak of COVID-19, and SD2 indicates an implementation of the Hanwoo-related policy from February 2016 to August 2017.

Variable	Mean	Std Dev	Max	Min	Freq	Pct
SLm	34,787	8,968.27	59,345	20,155	-	-
Bm	94,622	24,532.43	144,662	51,740	-	-
SLf	29,598	6,972.47	47,898	13,973	-	-
D1	-	-	-	-	8	11.43
D2	-	-	-	-	5	7.14
SD1	-	-	-	-	19	27.14
SD2	-	-	-	-	6	8.57

Table 1. Data summary.

Std Dev, standard deviation; Freq, frequency; Pct, percentage.

Methods

In general, the supply of livestock products is determined by the number of slaughtered cattle, imports, and exports. The relation of Hanwoo supply (S) as a function of the number of slaughtered cattle (SL), imports (IM), and exports (EX) can be represented as:

$$S = f(SL, IM, EX) \tag{1}$$

As Hanwoo refers to a native breed of cattle raised in Korea, it can be assumed that there is no importing of Hanwoo beef under general circumstances, so the estimation of Hanwoo supply is limited to the number of slaughtered cattle. To estimate the Hanwoo supply model, a function for the number of slaughtered cattle was established, and a regression model was constructed centered on the variables affecting the number of slaughtered cattle.

In this study, equations for estimating a Hanwoo male and a Hanwoo female were set up to construct a Hanwoo supply model, considering that factors that affect the number of slaughtered cattle may vary depending on the sex of the Hanwoo.

In the case of Hanwoo males, the number of beef cattle decreases rapidly after 30 months, and the ecological environment shipped to market before that month is created. This relationship can be expressed as:

$$SLm = \beta_0 + \beta_1 Bm_{t-1} + \rho \sum_{j}^{11} M_{t,j} + \delta_k \sum_{k}^{2} D_{t,k} + \phi_1 \sum_{k}^{2} SD_{t,l} + \varepsilon_t$$
(2)

where SLm is the number of slaughtered cattle (Hanwoo males), Bm is the number of beef cattle 29 months old in the previous month (Hanwoo males), M is the dummy month variable, D is the dummy variable representing the point when supply shock occurs as the supply shifter effect, and SD is the dummy representing the point of structural change based on policy implementation, and ε , is the error term.

The number of beef cattle, which is a predetermined variable, is free from discussion of endogeneity since the variable is not determined by other systems such as simultaneous equations and has the advantage that one-step-ahead forecasting and dynamic forecasting are easy when information is given up to time *t* in relation to the variable.

The relationship between the number of slaughtered Hanwoo females (SLf) and males can be represented as:

$$SLf = \gamma_0 + \gamma_1 SLm_t + \mu_i \sum_{i}^{11} M_{ti} + u_t$$
(3)

where SLm is the number of slaughtered cattle (Hanwoo males), M is the dummy month variable, and u is the error. Equations (2) and (3) construct a system of equation model terms with the figures estimated from regression, simultaneously calculating the number of slaughtered Hanwoo males and females. In this study, because there might be a contemporaneous correlation between the Hanwoo male model and the Hanwoo female model, it was considered more appropriate to use a joint estimation method rather than a separate independent estimation method to estimate the model, and a seemingly unrelated regression (SUR) was used to estimate it.

SUR refers to a system of regression equations that appear unrelated but are actually related (Zellner, 1962). If a change or event at a specific time affects Hanwoo males and Hanwoo females, and the effects are not observed or represented by the independent variables, effects are inherent in the error term.

SUR follows the assumption that there is no correlation between error and observations for each equation, but there is a correlation between the two error terms in the equation. This relationship can be expressed as:

$$E(\varepsilon_{t},u_{t}|Bm,M,D,SD,SLm) = \sigma_{male, female} \text{ for } t = s \text{ ; otherwise, } E(\bullet) = 0$$
(4)

In this study, a performance evaluation for simulation was carried out to examine how well the numbers of slaughtered cattle derived from the model forecast the actual values. This process is a kind of post-test analysis that evaluates the proximity between the actually observed values and the simulated values calculated from estimated coefficients and predetermined variables through the constructed model in the study.

The performance criterion for simulation applied in the analysis is the root mean square percent error (RMSPE) approach, which can be calculated as:

$$\text{RMSPE} = \sqrt{\left(\frac{1}{T}\right)\sum_{t=1}^{T} \left[(\widehat{SL}_t - (\widetilde{SL}_t)/\widetilde{SL}_t\right]^2} \times 100$$
(5)

where SL_t is the simulated value and SL_t is actual value. The RMSPE method is similar to the root mean square error (RMSE) method commonly used to determine the accuracy of a model (Joshi et al., 2019), but it converts the root mean square error into a percentage unit so that the error does not change, even if the measurement unit changes. It is a devised method and has advantages in terms of mutual comparison.

Another criterion to determine the model performance is the mean absolute percentage error (MAPE) approach, which can be calculated as:

$$MAPE = \left(\frac{1}{T}\right) \sum_{t=1}^{T} |(\widehat{SL}_t - (\widetilde{SL}_t)/\widetilde{SL}_t)| \times 100$$
(6)

Through the analysis described above, it is possible to complement the performance criteria centered on individual equations led by explanatory power or significance judgment, and the verification process of whether the forecasting model is working effectively can be carried out systematically, applying the model performance criteria considering the equation system as a whole.

Results and Discussion

The Hanwoo supply model analysis

In order to examine Hanwoo supply, the SUR model was estimated. The result of the estimation of the Hanwoo supply model is shown in Table 2. As a result of estimating the two models using the SUR model, the explanatory power of the Hanwoo male supply model was 0.9416, and that of the Hanwoo female supply model was 0.9074. Examining the Hanwoo male supply model using the number of beef cattle raised over 29 months of age as an explanatory variable, it was shown that the slaughter of Hanwoo males increases by 0.248 as the number of beef cattle raised over 29 months of age in the

previous month increases by one. In the case of the month dummy, it was estimated that the slaughtering of Korean cattle was relatively higher in January and August, when holiday food preparations for the New Year and Chuseok (the Korean Thanks giving day) take place, compared to other months. In the case of April, it was estimated that 7,000 more Hanwoos were slaughtered compared to December due to the influence of May's Family Month and holiday season demand. It was shown that the Hanwoo male supply increased further after the outbreak of Covid-19, and it is assumed that the current number of slaughtered Hanwoo males has increased due to the influence of and increase in household consumption along with the increase in the number of beef cattle.

Hanwoo male					Hanwoo female			
Variable	Estimate	SE	Z	Variable	Estimate	SE	Z	
Intercept	7,478.01**	2,816.82	2.65	Intercept	-21.00	2,296.14	-0.01	
Bm_{t-1}	0.25***	0.021	11.71	SLm _t	0.76***	0.052	14.45	
M01	16,920.30***	1,328.39	12.74	M01	284.69	1,513.57	0.19	
M02	-1229.05	1434.49	-0.86	M02	626.64	1,402.72	0.45	
M03	2,295.62	1,507.97	1.52	M03	2,082.84	1,348.26	1.54	
M04	7,007.38***	1,621.41	4.32	M04	4,318.74***	1,307.08	3.30	
M05	4,959.96***	1,725.96	2.87	M05	5,850.42***	1,375.30	4.25	
M06	5,906.28***	1,772.61	3.33	M06	6,184.20***	1,390.06	4.45	
M07	5,558.39***	1,735.64	3.20	M07	6,448.95***	1,383.54	4.66	
M08	7,748.66***	1,458.48	5.31	M08	4,023.84***	1,257.45	3.20	
M09	5,736.33***	1,311.89	4.37	M09	5,921.65***	1,264.50	4.68	
M10	-5,442.48***	1,269.24	-4.29	M10	-593.74	1,312.51	-0.45	
M11	-7,654.34***	1,308.42	-5.85	M11	690.53	1,373.95	0.50	
D1	8,828.64***	861.56	10.25					
D2	-5,001.68***	1,087.36	-4.60					
SD1	3,840.63***	1,004.43	3.82					
SD2	-3,532.29***	621.838	-5.68					
R^2		0.9416		R^2		0.9074		

Table 2. The results of the model estimation.

SE, standard error.

****p<0.01, ***p<0.05.

In the case of Hanwoo females, unlike Hanwoo males whose breeding cycle is carried out according to the guidelines, the breeding intentions on farms differ according to the price of calves and cattle in the corresponding year, so the amount of slaughter varies from year to year. Taking this into consideration, the amount of slaughtered Hanwoo females was estimated by the ratio of the slaughter rate of Hanwoo males to Hanwoo females. As a result of analysis of the Hanwoo female supply model, it was estimated that 0.764 Hanwoo females were slaughtered under average conditions for every Hanwoo male slaughtered.

Simulation analysis and performance evaluation

The model established in this study is basically a step-by-step prediction, and the supply amount after the period used for estimation in the model can also be predicted by dynamic calculation. For this purpose, the stability and accuracy of the model must be premised-the higher the predictive power of the model, the more persuasive the stability and fit of the model will be. To evaluate the forecast accuracy of the models, simulation analysis was performed based on the estimation formula constructed in the previous section, and for the analysis, the actual number of Korean cattle slaughtered in the last three years (January 2018 to December 2020) was applied to the model to compare the actual supply figure and a hypothetical trend line; RMSPE and MAPE statistics were calculated to assess the models' predictive ability.

Fig. 1 and Fig. 2 show the simulated values estimated from the Hanwoo male model and Hanwoo female model, respectively, and actual observations. As shown in the figures, the accuracy of the forecast can be reviewed by comparing the size, trend direction, and turning point of simulated and actual observed values. As a result of the analysis, both models showed that the predicted values were performed properly, as the simulation values' actual observations were roughly consistent in terms of size, trend direction, and turning points. Looking at each model, it was observed that the Hanwoo male model follows the flow of overestimation–underestimation–overestimation over time and that the Hanwoo female model keeps the flow of underestimation as a whole.

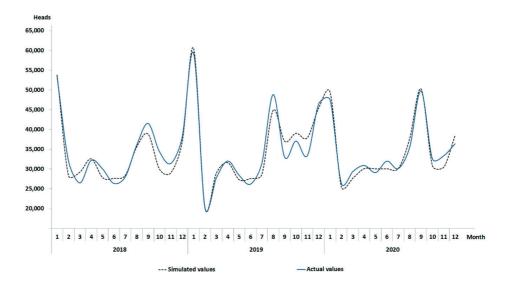


Fig. 1. Result of simulation and actual observations of slaughtered Hanwoo male.

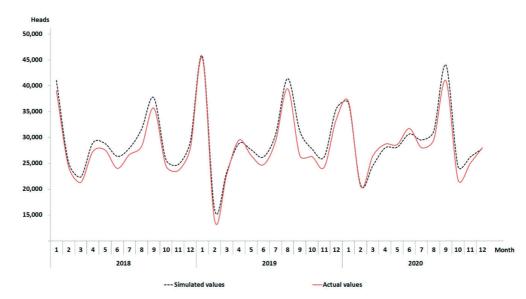


Fig. 2. Result of simulation and actual observations of slaughtered Hanwoo female.

The fact that directions of error of the Hanwoo male model and the Hanwoo female model do not match implies the effect of reducing errors by offsetting the positive and negative errors when performing simulation analysis of the entire Hanwoo herds, resulting in more accurate results in that procedure than when performing individual Hanwoo analysis. This is because the simultaneous estimation technique was carried out in the process of estimating two models, and as a result, it is considered that the accuracy of the model tends to be higher when analyzing total Korean cattle.

The results for the calculations of RMSPE and MAPE are shown in Table 3. Errors were represented within 10% in all simulations performed through the Hanwoo supply model, and the accuracy was higher in 2020 than in past periods. As a result of the simulation for the entire analysis period, RMSPE and MAPE were shown as 4.85% and 3.82%, respectively. Given that the predictive power of the model is regarded as excellent in general when the level of error is less than 5%, it is considered that the estimation results of the supply model constructed in the study are reliable and the model has structural stability.

Quarter	RMSPE			MAPE			
	2018	2019	2020	2018	2019	2020	
Q1	5.64	4.24	3.98	5.23	3.70	3.30	
Q2	4.66	3.45	3.14	4.07	2.54	2.67	
Q3	3.09	8.67	4.28	2.74	6.50	4.06	
Q4	3.47	7.23	2.56	2.62	5.93	2.46	
Annual	4.33	6.27	3.56	3.67	4.67	3.12	
Total		4.85			3.82		

Table 3. Results of RMSPE and MAPE from simulation analysis by period.

RMSPE, root mean square percent error; MAPE, mean absolute percentage error.

Conclusion

In this study, to improve the supply and demand situation of Hanwoo, the Hanwoo male and female models were established in the short-term, and the forecasting of Hanwoo supply was performed through the established equations and estimated coefficients of the model. Existing models tend to indicate the structure of Hanwoo well through step-by-step techniques that reflect the cohort characteristics of Hanwoo, but are limited to mid-term forecasts and long-term projections. Short-term analysis has to be conducted as well as medium- and long-term analyses in terms of balance, considering that the most important purpose of the outlook model is to forecast the supply and demand situation. In this respect, the short-term model that was carried out in this study is expected to be utilized as an important framework for supply-demand analysis of the Korean Hanwoo industry along with existing models.

As a result of the simulation analysis based on figures estimated from the study's supply model, the error between the actual result line and the virtual trend line was less than 5%, indicating that the short-term predictive power was significant. The simulation results for the Hanwoo supply model reveal great predictive power of the study's model, considering that the sensitivity of the short-term analysis is greater than that of the long-term analysis, and the stability of the model is relatively low in general. Also, considering that the estimation results for individual equations are theoretically consistent and have statistical fitness, it is concluded that the analysis results of the study have great significance in the practical sense.

In this paper, a Hanwoo supply model was established based on information on the number of slaughtered Hanwoo included in the beef traceability system, but the model used in the study is a supply-side model and does not reflect the demand-side structure related to Hanwoo, leaving a future research task of constructing a demand model.

Nevertheless, dealing with the monthly supply, which has not been dealt with in depth in previous studies, the analysis derives short-term forward forecasts on a monthly basis, and it provides a framework in which the analysis model can be utilized in industry policy by enabling simulation analysis. In future studies, it is expected that studies on the short-term price structure of Hanwoo will be discussed in more depth through continuous monitoring of the supply status of Hanwoo and additional research on demand.

Conflict of Interests

No potential conflict of interest relevant to this article was reported.

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