

# A Four Pole, Double Plane, Permanent Magnet Biased Homopolar Magnetic Bearing with Fault-Tolerant Capability

Uhn-Joo Na<sup>1\*</sup>

## 〈Abstract〉

This paper develops the theory for a novel fault-tolerant, permanent magnet biased, 4-active-pole, double plane, homopolar magnetic bearing. The Lagrange Multiplier optimization with equality constraints is utilized to calculate the optimal distribution matrices for the failed bearing. If any of the 4 coils fail, the remaining three coil currents change via a novel distribution matrix such that the same opposing pole, C-core type, control fluxes as those of the un-failed bearing are produced. Magnetic flux coupling in the magnetic bearing core and the optimal current distribution helps to produce the same c-core fluxes as those of unfailed bearing even if one coil suddenly fails. Thus the magnetic forces and the load capacity of the bearing remain invariant throughout the failure event. It is shown that the control fluxes to each active pole planes are successfully isolated. A numerical example is provided to illustrate the new theory.

*Keywords : Magnetic Bearing, Active Vibration Control, Hybrid Magnetic Bearing, Fault Tolerance, Permanent Magnet Device*

---

<sup>1\*</sup> Corresponding Author, School of Mechanical Engineering, Professor  
E-mail: uhnjoona@kyungnam.ac.kr

## 1. Introduce

A magnetic bearing system is a mechatronics device consisting of a magnetic force actuator (an active magnetic bearing, or AMB), motion sensors, power amplifiers, and a feedback controller (DSP), that suspends the spinning rotor magnetically without physical contact, and suppresses vibrations. Magnetic bearings find greater use in high speed, high performance, applications such as gas turbines, energy storage flywheels, and turbo-molecular pumps since they have many advantages over conventional fluid film or rolling element bearings, such as lower friction losses, lubrication free, temperature extremes, no wear, quiet, high speed operations, actively adjustable stiffness and damping, and dynamic force isolation.

Magnetic bearings have been investigated extensively for the last decades. Unlike heteropolar bearings[1-4], homopolar magnetic bearings have a unique biasing scheme that directs the bias flux flow into the active pole plane where it energizes the working air gaps, and then returns through the dead pole plane and the shaft sleeve. Use of rare earth permanent magnets such as Samarium-Cobalt (Sm-Co) and Neodymium-Iron-Boron (Nd-Fe-B) yields a very high efficiency when the permanent magnets are used as the source of bias flux to energize the air gaps and electromagnets are used to supply control fluxes in the active plane.

Some of the results on modeling, design, and control of homopolar magnetic bearings are

shown in literature. Meeks utilized a permanent magnet biased homopolar magnetic bearing to provide smaller, lighter, and power-efficient operation. Sortore et al.[5] and Allaire et al.[6] also presented design methods and experimental verifications for permanent magnet biased magnetic bearings. Similarly Maslen et al.[7] showed analytical and experimental results on the design and construction of permanent magnet biased magnetic bearings. Lee et al.[8,9] provides extensive discussion of design, testing and performance limits of the permanent magnet biased magnetic bearings. Fan et al.[10] also suggested systematic design procedures for permanent magnet biased magnetic bearings. Fukada and Yutani[11] work studied the frequency response of permanent magnet biased magnetic bearings.

Fault-tolerance of the magnetic bearing system is of great concern for highly critical applications of turbomachinery since a failure of any one control components may lead to the complete system failure. Much research has been devoted to fault-tolerant heteropolar magnetic bearings. Maslen and Meeker[12] introduced a fault-tolerant 8-pole magnetic bearing actuator with independently controlled currents and experimentally verified it in[13]. Flux coupling in a heteropolar magnetic bearing allows the remaining coils to produce force resultants identical to the unfailed bearing, if the remaining coil currents are properly redistributed. Na and Palazzolo [14-15] also investigated the optimized

realization of fault-tolerant magnetic bearing actuators, so that fault-tolerant control can be realized for an 8-pole bearing for up to 5 coils failed. Na and Palazzolo[16] suggested the fault-tolerant control scheme utilizing the grouping of currents to reduce the required number of controller outputs and to remove decoupling chokes.

Stricter constraints are required for the permanent magnet biased homopolar magnetic bearings to generate the equivalent fault-tolerance. The goal of the present work is to develop a fault-tolerant 4-active-pole, double plane, permanent magnet biased magnetic bearing such that the bearing can preserve the same decoupled magnetic forces identical to the unfailed bearing even after any one coil out of 4 coils fails.

## 2. The Fault Tolerant Bearing Model

The schematic drawing of a 4-active pole, double plane, permanent magnet biased homopolar magnetic bearing is shown in Fig. 1. Assuming that eddy current effects and material path reluctances are neglected, Maxwell's equations are reduced to the equivalent magnetic circuit for the homopolar magnetic bearing as shown in Fig. 2

The reluctance in air gap  $j$  of the active pole plane is;

$$R_j = \frac{g_j}{\mu_0 a_0} \tag{1}$$

$$g_j = g_0 - x \cos \theta_j - y \sin \theta_j \tag{2}$$

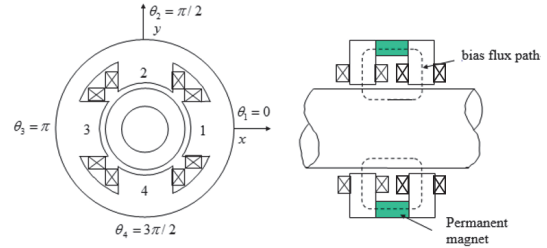


Fig. 1. Homopolar Magnetic Bearing

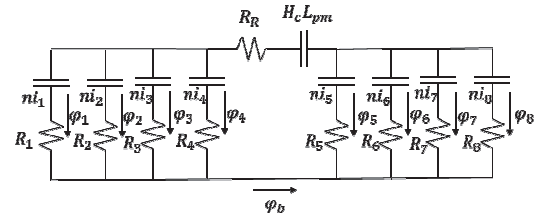


Fig. 2. Equivalent Magnetic Circuit for Double Plane Homopolar Magnetic Bearing

The parameters  $\mu_0$ ,  $a_0$ ,  $x$ ,  $y$ , and  $g_0$  represent the permeability of air, the pole face area of the active pole, the rotor positions, and nominal air gap, respectively. The permanent magnets are modeled as a source,  $H_c L_{pm}$ , and the return path reluctance  $R_R$ . Applying Ampere's loop law to the magnetic circuit results in 4 independent equations.

$$R_j \phi_j - R_{j+1} \phi_{j+1} = n i_j - n i_{j+1}, j=1,2,3 \tag{3}$$

$$R_j \phi_j + R_R \phi_b = H_c L_{pm} + n i_j, j=4 \tag{4}$$

The parameters  $\phi_j$ ,  $\phi_b$  and  $i_j$  represent the control flux through  $j$ -th pole, the return path flux, and currents through the  $j$ -th pole, respectively. Applying flux conservation law to

the magnetic circuit results in one more equation.

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 = \phi_b \tag{5}$$

Rearranging Eqs.(3)-(5) leads to a matrix equation.

$$\begin{bmatrix} R_1 - R_2 & 0 & 0 & 0 \\ 0 & R_2 - R_3 & 0 & 0 \\ 0 & 0 & R_3 & -R_4 \\ 1 & 1 & 1 & 1 + R_4/R_R \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{H_c L_{pm}}{R_R} \end{bmatrix} + \begin{bmatrix} n - n & 0 & 0 & 0 \\ 0 & n & -n & 0 \\ 0 & 0 & n & -n \\ 0 & 0 & 0 & 1 + n/R_R \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} \tag{6}$$

or

$$R\Phi = H + NI \tag{7}$$

The magnetic circuit equation (7) for a homopolar magnetic bearing has a permanent magnet bias forcing term  $H$  that energize the working air gaps. The coil current vector  $I$  then provides only AC control currents. The fault tolerant, 4-active pole, double plane, homopolar magnetic bearing utilizes 4 independent coils each driven by its power amplifiers. The coil wound clockwise in pole 1 of one plane is wound counterclockwise in pole 1 of another plane. Control fluxes through poles are then fully coupled with independent currents. The currents distributed to the 4-active-pole bearing are generally expressed as a  $4 \times 2$  distribution matrix  $T$  and control voltage vector  $v_c$ . The current vector is:

$$I = Tv_c \tag{8}$$

$$T = \begin{bmatrix} T_x & T_y \end{bmatrix}, v_c = \begin{bmatrix} v_{cx} \\ v_{cy} \end{bmatrix}$$

The parameters  $v_{cx}$  and  $v_{cy}$  represent the x and y control voltages. A typical current distribution scheme (coil winding scheme) for a homopolar magnetic bearing is;

$$\tilde{T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{9}$$

The feedback control voltages  $v_{cx}$  and  $v_{cy}$ , determined with any type of control law and measured rotor motions, are distributed to each pole via  $\tilde{T}$  in normal operation, and create effective stiffness and damping of the bearing to suspend the rotor around the bearing center position. If one of the 4 coils fails, the full  $(4 \times 1)$  current vector is related to the reduced current vector by introducing a failure map matrix  $W$ .

$$I = WI \tag{10}$$

For example the matrix for the 4<sup>th</sup> coil failed bearing is described as;

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The reduced distribution matrix is defined as;

$$\hat{I} = \hat{T}v_c \tag{11}$$

$$\hat{T} = \begin{bmatrix} \hat{T}_x & \hat{T}_y \end{bmatrix} \tag{12}$$

$$\hat{T}_x = [t_1 \ t_2 \ t_3]^T, \hat{T}_y = [t_4 \ t_5 \ t_6]^T$$

The reduced distribution matrix is to be

determined such that the magnetic forces should remain very much invariant before and after failure. The flux density vector can be determined from the magnetic circuit equation in Eq. (7). Leakage and fringing effects can be empirically determined and are simply derated by the fringing factor  $\zeta$ . The flux density vector in the air gaps is described as;

$$B = \zeta A^{-1} R^{-1} (H + NI) \quad (13)$$

where the pole face area matrix is  $A = \text{diag}([a_0, a_0, a_0, a_0])$ . The flux density vector is then reformulated as;

$$B = Gv \quad (14)$$

$$G = [G_b H G_c \hat{T}_x G_c \hat{T}_y], v = [1 v_{cx} v_{cy}]^T \quad (15)$$

$$G_b = \zeta A^{-1} R^{-1}, G_c = \zeta A^{-1} R^{-1} N W$$

Magnetic forces developed in the active pole plane are then described as;

$$f_x = v^T M_x v \quad (16)$$

$$f_y = v^T M_y v \quad (17)$$

$$M_x(\hat{T}) = -G^T \frac{\partial D}{\partial x} G, M_y(\hat{T}) = -G^T \frac{\partial D}{\partial y} G \quad (18)$$

and where the air gap energy matrix is;

$$D = \text{diag}([g, a_0 / (2\mu_0)]) \quad (19)$$

Employing an optimal current distribution matrix  $T$  may decouple the linearized forces of the failed bearing, and even maintain the same decoupled magnetic forces as those of an unfailed magnetic bearing. Maslen and Meeker

[12] introduced a linearization method which effectively decouple the control forces for a failed bearing by choosing a proper distribution matrix. The necessary conditions to yield the same decoupled magnetic control forces are;

$$M_x = k_v \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, M_y = k_v \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix} \quad (20)$$

If the distribution matrix  $\hat{T}$  is determined such that Eq. (18) satisfies Eq. (20), the nonlinear magnetic forces in Eqs. (16) and (17) become linearized at bearing center positions.

$$f_x = k_v v_{cx}, f_y = k_v v_{cy}, \quad (21)$$

Equations (18) and (20) can be written in 18 scalar forms, and then boils down to 10 algebraic equations if redundant terms are eliminated. The equality constraints to yield the same control forces before and after failure are;

$$\begin{aligned} h_1(\hat{T}) &= \hat{T}_x^T Q_{x0} \hat{T}_x = 0 \\ h_2(\hat{T}) &= \hat{T}_y^T Q_{x0} \hat{T}_y = 0 \\ h_3(\hat{T}) &= H^T Q_{bx0} \hat{T}_y = 0 \\ h_4(\hat{T}) &= \hat{T}_x^T Q_{x0} \hat{T}_y = 0 \\ h_5(\hat{T}) &= H^T Q_{bx0} \hat{T}_y = k_v / 2 \\ h_6(\hat{T}) &= \hat{T}_x^T Q_{y0} \hat{T}_x = 0 \\ h_7(\hat{T}) &= \hat{T}_y^T Q_{y0} \hat{T}_y = 0 \\ h_8(\hat{T}) &= H^T Q_{by0} \hat{T}_x = 0 \\ h_9(\hat{T}) &= \hat{T}_x^T Q_{y0} \hat{T}_y = 0 \end{aligned} \quad (22)$$

$$h_{10}(\hat{T}) = H^T Q_{\phi y0} \hat{T}_y = k_v/2$$

where

$$Q_{\phi b0} = -G_b \frac{\partial D}{\partial \phi} G_b \Big|_{\omega=0}, Q_{\phi c0} = -G_c \frac{\partial D}{\partial \phi} G_c \Big|_{\omega=0}$$

The criterion for choosing the best candidate is the one that will yield the maximum load capacity prior to any saturation. To accomplish this a distribution matrix  $\hat{T}$  can be determined by using the Lagrange Multiplier method to minimize the Euclidean norm of the flux density vector  $B$ . The cost function is defined as;

$$J = B(\hat{T})^T P B(\hat{T}) \tag{23}$$

where the diagonal weighting matrix  $P$  is also selected to maximize the load capacity. The Lagrange Multiplier method is then used to solve for  $\hat{T}$  that satisfies Eq. (23). Define:

$$L(\hat{T}) = B(\hat{T})^T P B(\hat{T}) + \sum_{j=1}^{10} \lambda_j h_j(\hat{T}) \tag{24}$$

Partial differentiation of Eq. (24) with respect to  $t_i$  and  $\lambda_j$  leads to 16 nonlinear algebraic equations to solve for  $t_i$  and  $\lambda_j$ .

$$\Psi = \begin{bmatrix} \Psi_1(t, \lambda) \\ \Psi_2(t, \lambda) \\ \vdots \\ \Psi_{15}(t, \lambda) \\ \Psi_{16}(t, \lambda) \end{bmatrix} = 0 \tag{25}$$

$$\Psi_i = \frac{\partial L}{\partial t_i} = 0, i = 1, 2, \dots, 6 \tag{26}$$

$$\Psi_{6+j} = h_j(\hat{T}) = 0, j = 1, 2, \dots, 10 \tag{27}$$

Various initial guesses of and are tested to

find the solution of Eq.(25) for the 4th coil failed bearing. The distribution matrix solutions for the 4 cases of failed bearing are determined as;

$$T_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \\ -2 & 0 \\ -1 & -1 \end{bmatrix}, T_2 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ -1 & -1 \\ 0 & -2 \end{bmatrix}, T_3 = \begin{bmatrix} 2 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}, T_4 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} \tag{28}$$

### 3. Numerical Analysis

The normal unfailed operation with the current distribution matrix  $\tilde{T}$  and the fault tolerant operation with an optimal distribution matrix  $T_4$  are simulated with the 3-D finite element model. The designed magnetic bearing has parameters,  $a_0$  (0.00024 m<sup>2</sup>),  $g_0$  (0.0005 m), and  $n$  (100 turns), respectively. The permanent magnet (SmCo) are also designed to maintain the air gap bias flux density of the active poles at nominal gaps to be 0.6 tesla. A commercial magnetic field software (MAXWELL3D) is used for the 3-D field and force calculation. The 3D finite element model of the magnetic bearing is shown in Fig. 3. The designed magnetic bearing has two active core plane separated with permanent magnets.

It is assumed that the final feedback voltages,  $v_{cx}$  and  $v_{cy}$ , determined with the control law and the measured rotor motions are simply two sinusoidal functions with 90° in phase for the steady state whirling. Two sinusoidal voltage signals are redistributed with the matrices,  $\tilde{T}$  and  $T_4$ , for the unfailed bearing and the 4th coil failed bearing,

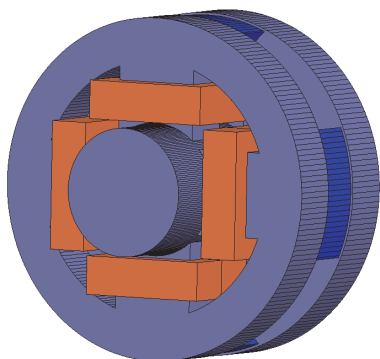


Fig. 3. The 4 Pole, Double plane, Homopolar magnetic bearing

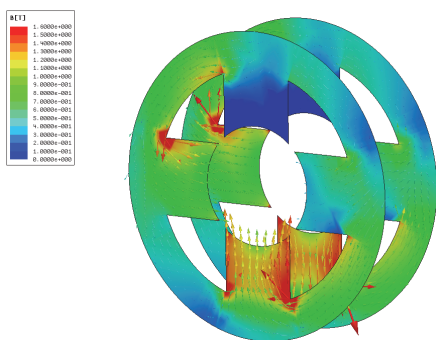


Fig. 4. Flux distribution Calculated with 3D Finite Element Model by Current Set  $I_2|_{t=5\pi/(8\Omega)}$

respectively. Two cases of currents,  $I_1 = \tilde{T}v_c$  and  $I_2 = T_4 v_c$ , with  $v_{cx} = 2.0\cos(\Omega t)$ ,  $v_{cy} = 2.0\sin(\Omega t)$ , and  $t \in [0, 2\pi/\Omega]$  are applied on the 3D magnetic model.

Firstly, the currents  $I_1$  and  $I_2$  at  $t = 5\pi/(8\Omega)$  are applied on the finite element model. Flux distribution in the active pole plane driven with the current set  $I_2|_{t=5\pi/(8\Omega)}$  is shown in Fig. 4.

It is also verified that the flux distribution driven with  $I_1|_{t=5\pi/(8\Omega)}$  is nearly identical to that with  $I_2|_{t=\pi/(6\Omega)}$ . Secondly, 16 current sets

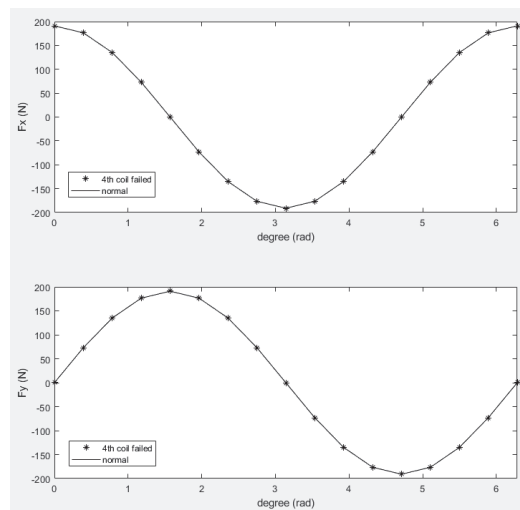


Fig. 5. Magnetic Forces calculated with the 3D finite element model by  $I_1$  (solid lines), and  $I_2$  (asterisks)

of  $I_1$  and  $I_2$  at  $t = [\pi/(8\Omega), 2\pi/(8\Omega), \dots, 12\pi/(8\Omega)]$  are applied on the finite element model such that the corresponding magnetic forces should be calculated at each current sets. The magnetic forces with and calculated by using 3-D finite element model are shown in Fig. 5.

Fig. 5 shows that the magnetic forces with 4-th coil failed bearing are almost identical to the normal unfailed bearing. The 3D magnetic forces distributed with the optimal current distribution  $T_4$  for the 4th coil failed operation are perfectly sinusoidal same as those with the normal current distribution  $\tilde{T}$  for the normal unfailed operation.

#### 4. Conclusion

A fault-tolerant control scheme is developed

for the permanent magnet biased, 4-active-pole, double plane, homopolar magnetic bearings. The remaining three coil currents can be redistributed in a way such that the same C-core type, control fluxes as those of an un-failed bearing are preserved in the active-pole plane even if any one coil out of four coils suddenly fail. The bias voltage linearization can be realized for the failed bearing in a manner such that the magnetic forces are decoupled by using a modified current distribution matrix after a coil failure. The Lagrange Multiplier optimization with equality constraints is utilized to calculate the optimal distribution matrices for the failed bearing.

Three dimensional finite element magnetic bearing model is constructed to determine the fault tolerance of the magnetic bearing with failed coil. The magnetic forces with 4-th coil failed bearing are almost identical to the normal unfailed bearing.

The fault tolerance of homopolar magnetic bearings is achieved at the expense of additional hardware requirements such as independent coils (power amplifiers), fault detection system, additional DSP controller channels. The fault-tolerant control scheme presented in this paper allows any one coil out of four coils to fail freely without sacrificing the load capacity, position stiffness or current stiffness. The energy efficient homopolar magnetic bearings having fault tolerant capability without sacrificing the bearing load capacity may find great use in some

applications of the high speed, high performance rotating machinery.

## Acknowledgement

This paper was supported by Kyungnam University research fund

## References

- [1] J.G. Bitterly, S.E. Bitterly, "Flywheel Based Energy Storage System," U.S. Patent No. 5,614,777 (1997).
- [2] P. E. Allaire, D. W. Lewis, J. D. Knight, "Active Vibration Control of a Single Mass Rotor on Flexible Supports," *Journal of the Franklin Institute* vol. 315 pp. 211-222, (1983).
- [3] J. Salm, G. Schweitzer, "Modeling and Control of a Flexible Rotor with Magnetic Bearing," *Proceedings of the Third International Conference on Vibrations in Rotating Machinery* pp. 553-561, (1984).
- [4] F. Matsumura, T. Yoshimoto, "System Modeling and Control of a Horizontal-Shaft Magnetic-Bearing System," *IEEE Transactions on Magnetics* vol. 22, pp. 197-206, (1986).
- [5] C.K. Sortore, P.E. Allaire, E.H. Maslen, R.R. Humphris, P.A. Studer, "Permanent Magnet Biased Magnetic Bearings – Design, Construction and Testing," *Proceedings of the Second International Symposium on Magnetic Bearings*, pp. 175-182, (1990)
- [6] P.E. Allaire, E.H. Maslen, R.R. Humphris, C.K. Sortore, P.A. Studer, "Low Power Magnetic Bearing Design for High Speed Rotating Machinery," *Proceedings of the NASA International Symposium on Magnetic*



- Suspension Technology, pp. 317-329, (1992).
- [7] E.H. Maslen, P.E. Allaire, M.D. Noh, C.K. Sortore, "Magnetic Bearing Design for Reduced Power Consumption." ASME Journal of Tribology, vol. 118, pp. 839-846, (1996).
- [8] A.C. Lee, F.Z. Hsiao, D. Ko, "Analysis and Testing of a Magnetic Bearing with Permanent Magnets for Bias." JSME International Journal, Series C vol. 37, pp. 774-782, (1994).
- [9] A.C. Lee, F.Z. Hsiao, and D. Ko, "Performance Limits of Permanent-Magnet-Biased Magnetic Bearings." JSME International Journal, Series C vol. 37, pp. 783-794, (1994).
- [10] Y. Fan, A. Lee, F. Hsiao, "Design of a Permanent/Electromagnetic Magnetic Bearing-Controlled Rotor System." Journal of the Franklin Institute, vol. 334B, pp. 337-356, (1997).
- [11] S. Fukata, K. Yutani, "Characteristics of Electromagnetic Systems of Magnetic Bearings Biased with Permanent Magnets," Proceedings of the Sixth International Symposium on Magnetic Bearings, pp. 234-243, (1998).
- [12] E. H. Maslen, D. C. Meeker, "Fault Tolerance of Magnetic Bearings by Generalized Bias Current Linearization." IEEE Transactions on Magnetics, vol. 31, pp. 2304-2314, (1995).
- [13] E. H. Maslen, C. K. Sortore, G. T. Gillies, R. D. Williams, S. J. Fedigan, R. J. Aimone, "A Fault Tolerant Magnetic Bearings." ASME Journal of Engineering for Gas Turbines and Power, vol. 121, pp. 504-508, (1999).
- [14] U. J. Na, A. B. Palazzolo, "Fault Tolerance of Magnetic Bearings with Material Path Reluctances and Fringing Factors," IEEE Transactions on Magnetics, vol. 36, pp. 3939-3946, (2001).
- [15] U. J., Na, A. B., Palazzolo, and A., Provenza, "Test and Theory Correlation Study for a Flexible Rotor on Fault-Tolerant Magnetic Bearings." ASME Journal of Vibration and Acoustics, vol. 124, pp. 359-366, (2002).
- [16] U.J. Na and A. B. Palazzolo, "The Fault-Tolerant Control of Magnetic Bearings With Reduced Controller Outputs." ASME Journal of Dynamic Systems, Measurement, and Control, vol. 123, pp. 219-224, (2001).

---

(Manuscript received November 01, 2021;

revised November 28, 2021; accepted December 01, 2021)