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Laminate Tensile Failure Strength Prediction using Stress Failure Criteria

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Abstract

This paper presents a method that uses the stress failure criteria to predict the tensile failure strength of open-hole laminates with stress concentrations. The composite material used in this study corresponds to a 177 °C cured, carbon/epoxy unidirectional tape prepreg. The results obtained by testing ten different laminates were compared and analyzed to verify the tensile strength of the open-hole laminates predicted using the proposed stress failure criteria. The findings of this study confirm that the tensile strength predictions performed using the proposed method are generally accurate, except in cases involving highly soft laminates (10% of 0° ply).

Key Words : advanced composite material, average-stress failure criterion, point-stress failure criterion, open-hole laminate, tensile failure strength

1. Introduction

The quest to reduce the weight of the aircraft structure components has resulted in the increased use of composite materials. Consequently, many recently developed aircraft structures are designed using composite materials, which account for nearly 50% of the total structure weight [1]. Because composite materials, unlike metals, demonstrate different strength characteristics based on their composite constituents and their microstructural orientation, several methods have been proposed to predict the failure of composite materials. The representative failure theories include those pertaining to the maximum stress and maximum strain as well as those reported by Tsai–Hill [2], Tsai–Wu [3], Hashin–Rotem [4], and Hashin [5]. Moreover, several investigations concerning the prediction of progressive failures and strength degradation of composite materials via numerical analysis and advanced computing techniques are currently underway [6–15].

Whitney and Nuismer [16] applied the stress failure criteria to predict the failure strength of laminates containing stress concentrations. In this study, the stress failure criteria are applied to laminates with holes subjected to tensile loads, thereby facilitating failure-strength prediction. The prediction results obtained using the proposed method are validated via a comparative analysis between test results obtained for ten specimens.

The stress failure criteria can be classified as the average- and point-stress failure categories. In this study, the tensile failure strengths of open-hole laminates are predicted using these failure criteria.

2. Material and Methods

2.1 Composite Material

The composite material used in this study corresponds to unidirectional (UD) carbon–epoxy tape prepreg cured at 177 °C (350 °F). The mechanical properties of the cured lamina are listed in Table 1. The specimens used in this study include ten different unnotched and open-hole laminates stacked in the 0°, +45°, -45°, and 90° orientations. The specimens were prepared in accordance with the following process.

(1) The specimens were first stacked considering standard stacking angles of 0°, +45°, -45°, and 90°, and each stacking angle was used for at least 10% of the total stacked-laminate count.
(2) The specimen thicknesses lied in the 2.03–5.08 mm range.
(3) The specimen stacking angle of 0° did not exceed 60% of the total stacked-ply count.
(4) All specimens were balanced and stacked symmetrically.

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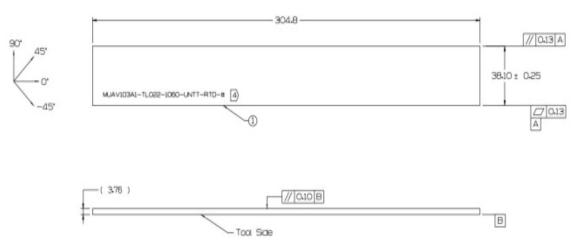
Table 1 Mechanical Properties of Cured Lamina

Item		Unit	Value (21 °C @ AMB)
Lamina Thickness		mm	0.1882
Elastic Modulus	E_1*	GPa	146.03
	E_2	GPa	9.03
	G_{12}	GPa	3.38
Poisson's Ratio		-	0.33

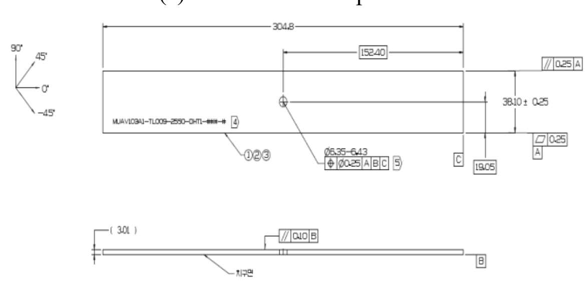
*The average of axial tensile and compressive elastic moduli

2.2 Test Methods

In this study, the unnotched and open-hole tensile tests (UNT and OHT, respectively) were performed in accordance with the ASTM-D-3039 and ASTM-D-5766 testing standards. Figure 1 depicts the UNT and OHT specimen shapes. The test matrix and stacking sequence of each specimen are described in Tables 2 and 3, respectively.



(a) UNT Laminate Specimen



(b) OHT Laminate Specimen

Fig. 1 UNT and OHT Specimen Configurations

Table 2 Test Matrix

Specimen* (0/±45/90% plies)	Specimen Count	Specimen (0/±45/90% plies)	Specimen Count
10/30/60	7	30/40/30	7
10/50/40	7	30/60/10	7
10/80/10	7	40/20/40	7
20/60/20	7	40/50/10	7
25/50/25	7	50/40/10	7

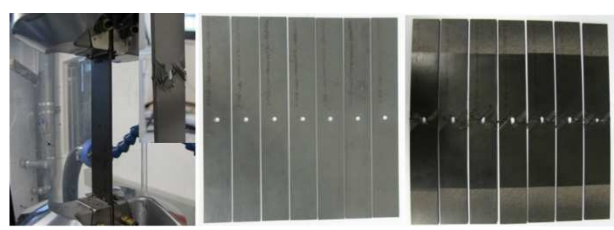
* Percentage of plies inclined at 0°, ±45°, and 90° with respect to the total stacked ply count.

Table 3 Stacking Sequence of Specimens

Specimen (0/±45/90% plies)	Ply Count	Stacking Sequence
10/30/60	20	[45/90/-45/90_2/0/90_3/45] s
10/50/40	20	[45/90/-45/90/45/0/-45/90_2/45] s
10/80/10	20	[45/90/-45/0/-45/45/-45/45/-45/45] s
20/60/20	20	[45/90/-45/0/-45/45/90/0/45/-45] s
25/50/25	16	[45/90/-45/0/90/0/-45/45] s
30/40/30	20	[45/90/-45/0/90/0/-45/90/45/0] s
30/60/10	20	[45/90/-45/0/-45/45/0/45/-45/0] s
40/20/40	20	[45/90/-45/0/90/0/90/0/90/0] s
40/50/10	20	[45/90/-45/0/45/0_2/-45/0/45] s
50/40/10	20	[45/90/-45/0_3/-45/0/45/0] s

(a) Test Setup (b) Before Test (c) After Test

Fig. 2 Test Setup and Failure Configuration for UNT Specimens



(a) Test Setup (b) Before Test (c) After Test

Fig. 3 Test Setup and Failure Configuration for OHT Specimens

In this study, all tests were performed at room temperature (21 °C) until the specimens failed. Figures 2 and 3 depict the test setup and specimen-failure shape. The tensile failure strength of the UNT and OHT specimens (F^t_{UNT} and F^t_{OHT}, respectively) were calculated as described in Eqs. (1) and (2).

$$F^t_{UNT} = \frac{P_{max}}{(w)(t_{calc})} \quad 1)$$

$$F^t_{OHT} = \frac{P_{max}}{(w)(t_{calc})} \quad 2)$$

where

F^t_{UNT} = tensile failure strength of UNT laminates (MPa)

F^t_{OHT} = tensile failure strength of OHT laminates (MPa)

P_{max} = maximum tensile load of specimen failure (N)

w = measured specimen width (mm)

t_{calc} = thickness calculated by multiplying the cured lamina thickness by stacked ply count (mm)

3. Stress Failure Criteria

When a uniform stress ($\bar{\sigma}$) is applied to an infinite orthotropic plate with a hole of radius R located parallel to the y-axis (transverse direction) as Fig. 4, the stress (σ_y) acting along the x-axis (longitudinal direction) can be evaluated as Eq. (3) [16].

$$\sigma_y(x,0) = \frac{\bar{\sigma}}{2}\{2 + \left(\frac{R}{x}\right)^2 + 2\left(\frac{R}{x}\right)^4 - (K_T^{\infty} - 3) \left[5\left(\frac{R}{x}\right)^6 - 7\left(\frac{R}{x}\right)^8\right]\}, \quad x > R \tag{3}$$

where K_T^{∞} denotes the orthotropic stress-concentration factor of the infinite-width plate. K_T^{∞} can be evaluated as Eq.(4) [17].

$$K_T^{\infty} = 1 + \sqrt{\frac{2}{A_{22}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}\right)} \tag{4}$$

where A_{ij} denotes the in-plane stiffness of the laminate in accordance with the laminate theory. The subscript i denotes the direction parallel to that of load application, and j denotes the direction of the laminate coordinate axis. Equation (4) can be expressed in terms of the effective elastic moduli of the laminate as Eq. (5).

$$K_T^{\infty} = 1 + \sqrt{2\left(\sqrt{\frac{E_x}{E_y}} - \gamma_{xy} + \frac{E_x}{G_{xy}}\right)} \tag{5}$$

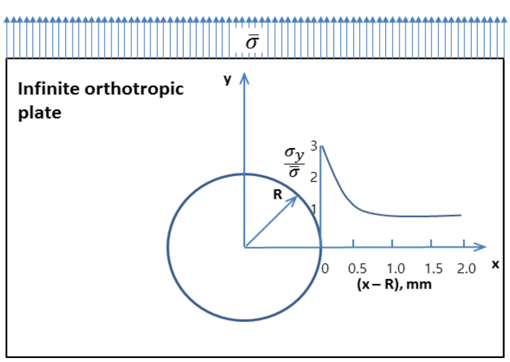


Fig. 4 Normal Stress (σ_y) Distribution for Circular Hole in Infinite Plate

3.1 Average-Stress Failure Criterion

The average-stress failure criterion assumes failure occurrence when the average stress σ_y at a specific distance a_0 from the hole equals the UNT laminate failure strength F_{UNT}^t (Fig. 5).

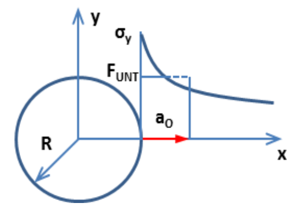


Fig. 5 Average-Stress Failure Criterion

The average-stress failure criterion [16] can be expressed as Eq. (6).

$$\frac{1}{a_0}\int_R^{R+a_0} \sigma_y(x,0)dx = F_{UNT}^t \tag{6}$$

The ratio of the strength of a notched laminate to that of an unnotched laminate can be evaluated using Eqs. (3) and (6).

$$\frac{\sigma_N^{\infty}}{F_{UNT}^t} = \frac{2(1-\emptyset_1)}{\{2 - \emptyset_1^2 - \emptyset_1^4 + (K_T^{\infty} - 3)(\emptyset_1^6 - \emptyset_1^8)\}} \tag{7}$$

where $\emptyset_1 = R/(R + a_0)$ and σ_N^{∞} denotes the notched strength of a laminate with infinite width under the application of stress $\bar{\sigma}$.

3.2 Point-Stress Failure Criterion

The point-stress failure criterion assumes failure occurrence when the point stress σ_y at a specific distance d_0 from the hole equals the UNT laminate failure strength F_{UNT}^t (Fig. 6). This criterion [16] can be expressed as Eq. (8).

$$\sigma_y(x,0)|_{x = R + d_0} = F_{UNT}^t \tag{8}$$

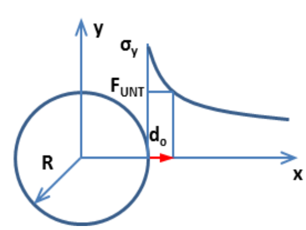


Fig. 6 Point-Stress Failure Criterion

By substituting Eq. (3) in Eq. (8), it follows that

$$\frac{\sigma_N^{\infty}}{F_{UNT}^t} = \frac{2}{\{2 + \emptyset_2^2 + 3\emptyset_2^4 - (K_T^{\infty} - 3)(5\emptyset_2^6 - 7\emptyset_2^8)\}} \tag{9}$$

where $\emptyset_2 = R/(R + d_0)$ and σ_N^{∞} denotes the notched strength of an infinite-width laminate subjected to stress $\bar{\sigma}$.

3.3 Failure-Stress Correction

As already stated, the failure strengths F_{UNT}^{t} and F_{OHT}^{t} can be calculated using Eqs. (1) and (2), respectively. They represent average values obtained from specimen tests. The stress-concentration factor of a finite-width notched plate [18] can be expressed as Eq. (10).

$$\frac{K_T}{K_T^{\infty}} = \frac{2+(1-\frac{2R}{w})^3}{3(1-\frac{2R}{w})} \tag{10}$$

where w denotes the specimen width. Equation (10) yields high-accuracy results in cases where $2R/w \leqq 1/2$.

The failure-strength correction for a finite-width laminate can be expressed as Eq. (11).

$$\sigma_N^{\infty} = F_{OHT}^{t}\frac{K_T}{K_T^{\infty}}. \tag{11}$$

4. Results and Discussion

In this study, the tensile tests were performed on the UNT and OHT laminate specimens, as described in Section 2. The tensile failure strengths of the laminates were calculated using Eqs. (1) and (2). Table 1 lists the average tensile failure strengths of the UNT and OHT laminates and the corresponding F_{OHT}^{t}/F_{UNT}^{t} ratios while Fig. 7 depicts the trends concerning F_{OHT}^{t} and F_{UNT}^{t} variations for different specimens.

Table 4 Test Results for Laminate Tensile Failure Strength

Specimen (0/±45/90% plies)	Strength (MPa)		F_{OHT}^{t}/F_{UNT}^{t} (%)
	F_{UNT}^{t}	F_{OHT}^{t}	
10/30/60	420.4	300.1	71.4
10/50/40	573.3	364.6	63.6
10/80/10	467.3	298.6	63.9
20/60/20	862.9	447.2	51.8
25/50/25	964.5	514.6	53.3
30/40/30	1048.5	565.6	53.9
30/60/10	1041.7	564.1	54.2
40/20/40	1212.7	675.1	55.7
40/50/10	1258.2	693.4	55.1
50/40/10	1559.2	853.0	54.7

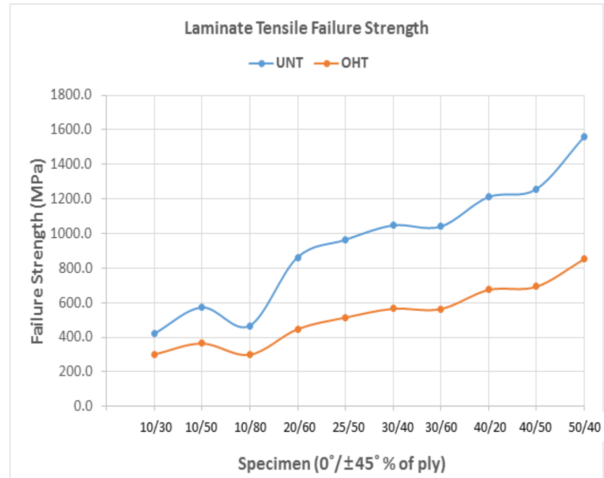


Fig. 7 Variations in F_{OHT}^{t} and F_{UNT}^{t} Values for Different Specimens

The tensile failure strength of laminates with a hole was predicted by applying the two stress failure criteria described in Section 3. The specific distances a_0 and d_0 equal 2.62 mm and 0.91 mm, respectively. The optimum values of a_0 and d_0 that yield the best results were deduced by comparing the F_{OHT}^{t} value obtained from the tests involving the ten laminate specimens. The value of σ_N^{∞} was obtained using Eq. (11). Table 5 lists the values of the failure strengths of laminates with a hole of radius R = 3.175 mm calculated using Eqs. (7) and (9). Figure 8 compares the trends concerning the variations in the average and point stresses for the different specimens.

Table 5 OHT Laminate Tensile Failure Strengths

Specimen (0/±45/90% plies)	$\sigma_N^{\infty}/F_{UNT}^{t}$ (%)	
	Average Stress	Point Stress
10/30/60	56.3	53.6
10/50/40	56.4	53.2
10/80/10	56.6	52.6
20/60/20	56.3	53.6
25/50/25	56.2	54.2
30/40/30	56.0	54.8
30/60/10	56.2	54.1
40/20/40	55.5	56.6
40/50/10	55.9	55.0
50/40/10	55.7	55.9

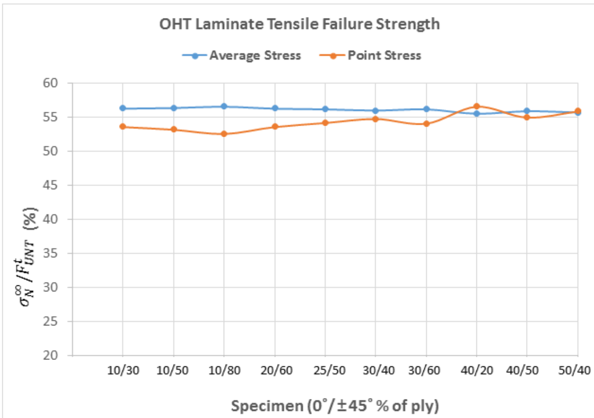


Fig. 8 Variations in Average- and Point-Stress-Criteria-Based OHT Laminate Tensile Failure Strengths for Different Specimens

The ratio of the tensile failure strengths of the notched laminates (with a hole) to that of their unnotched counterparts was evaluated by performing the tests and applying the stress failure criteria. The errors between the experimental and predicted results were subsequently compared. Table 6 and Fig. 9 describe the comparison between the experimental and predicted results with the latter obtained using the average-stress failure criterion. Furthermore, Table 7 and Fig. 10 describe the comparison between the experimental and predicted results based on the point-stress failure criterion.

These results reveal that the tensile failure strength of the notched laminates can be accurately predicted for most laminates except for those demonstrating high stress–strain nonlinearity as well as characterized by a 0° and ±45° ply-stacking ratios of 10% and 30–80%, respectively.

The large errors observed in the laminates demonstrating high stress–strain nonlinearity are likely caused by large errors in the values used to calculate the stress-concentration factor (Eqs. (4) or (5)) concerning these laminates.

Table 6 Comparison between Test Results and Those Predicted using Average-Stress Failure Criterion

Specimen (0/±45/90% plies)	F_{OHT}^t/F_{UNT}^t (%)		Error Rate % (test/aver.)
	Test	Ave. Stress	
10/30/60	71.4	56.3	-21.1
10/50/40	63.6	56.4	-11.3
10/80/10	63.9	56.6	-11.5
20/60/20	51.8	56.3	8.6
25/50/25	53.3	56.2	5.3
30/40/30	53.9	56.0	3.8
30/60/10	54.2	56.2	3.7
40/20/40	55.7	55.5	-0.2
40/50/10	55.1	55.9	1.5
50/40/10	54.7	55.7	1.8

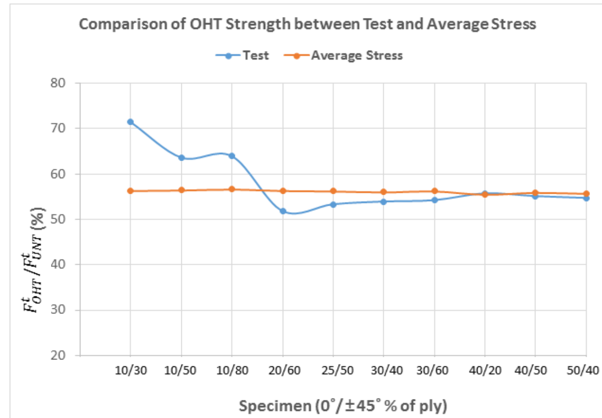


Fig. 9 Comparison between OHT Strength Values Obtained from Tests and Predicted using Average-Stress Failure Criterion

Table 7 Comparison between Test Results and Those Predicted using Point-Stress Failure Criterion

Specimen (0/±45/90% plies)	F_{OHT}^t/F_{UNT}^t (%)		Error Rate % (test/aver.)
	Test	Pt. Stress	
10/30/60	71.4	53.6	-24.8
10/50/40	63.6	53.2	-16.3
10/80/10	63.9	52.6	-17.6
20/60/20	51.8	53.6	3.5
25/50/25	53.3	54.2	1.5
30/40/30	53.9	54.8	1.5
30/60/10	54.2	54.1	-0.1
40/20/40	55.7	56.6	1.7
40/50/10	55.1	55.0	-0.3
50/40/10	54.7	55.9	2.3

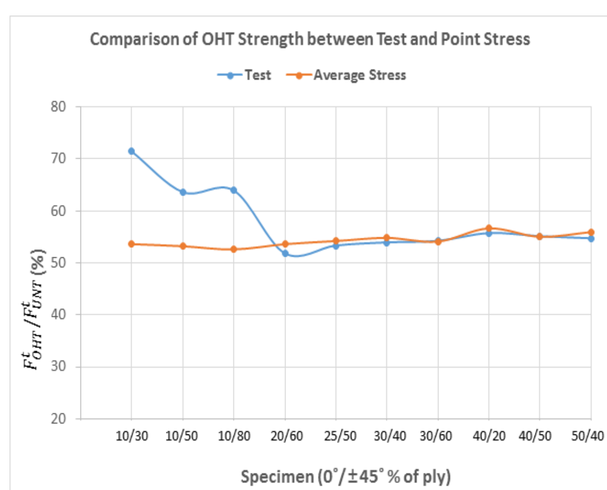


Fig. 10 Comparison between OHT Strength Values Obtained from Tests and Predicted using Point-Stress Failure Criterion

The stress failure criteria represent a technique for predicting the ratio of the failure strengths of the notched and unnotched laminates. If test results concerning the failure strength of notched laminates are not available owing to significant differences in the a_0 and d_0 values depending on the composite-material characteristics, the values predicted using the stress failure criteria cannot be considered reliable.

The values of a_0 and d_0 used in a previous study [16] equaled 3.81 and 1.01 mm, respectively. These are significantly different from the a_0 value of 2.62 mm and slightly higher than the d_0 value of 0.91 mm used in this study. A large value of a_0 for each composite material implies that the stress failure criteria predict different failure-strength values based on the composite material considered.

Although all-encompassing conclusions cannot be drawn based on the results obtained for the composite materials considered in this paper and [16] exclusively, it is undeniable that more accurate predictions can be realized by specifying d_0 because the difference in d_0 values corresponding to the two above-specified composite materials is small.

5. Conclusions

This paper presents the results obtained by applying the stress failure criteria to predict the failure strength of OHT laminates using the experimental results of the average failure strength of UNT-laminate specimens. The use of both the average- and point-stress failure criteria facilitates the accurate prediction of the tensile failure strength of the OHT laminates. The errors between the experimental and predicted results lie within ±5% for most laminates, except those that demonstrate high stress–strain nonlinearity as well as the ones with the 0° and ±45° ply-stacking ratios of 10% and 30–80%, respectively.

The values of a_0 = 2.62 mm and d_0 = 1.01 mm deduced in this study were found to demonstrate errors of approximately 31% and 10%, respectively, compared to those reported in [2].

Although the composite materials considered in this paper and [16] are insufficient to draw all-encompassing conclusion, it is reasonable to infer that the small difference in d_0 implies that the mechanical properties of OHT laminates do not significantly influence the prediction of their corresponding tensile failure strength. Therefore, the point-stress failure criterion can be used to predict the tensile failure strength of OHT laminates using the corresponding strength of their UNT counterparts. The findings of this study reveal that in cases where the tensile failure strengths of unnotched laminates are available, the application of the point-stress failure criterion yields accurate predictions of the tensile failure strength of laminates with holes.

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