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Development of Elementary Teachers' Mathematical Beliefs Scale: A Validity and Reliability Study

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The purpose of this study was to develop and validate a scale of Korean elementary teachers' mathematics beliefs. We examined 299 elementary teachers' mathematical beliefs using 30 items, out of which 12 items covered beliefs about the nature of mathematics and 18 items covered beliefs about mathematics teaching and learning. In the first stage, we performed exploratory factor analysis using 149 survey data to examine the factor structure. In the second stage, we performed confirmatory factor analysis using 150 survey data. Building upon previous studies, we examined the construct validity of three different models to find the best factor structure. The study results indicate that the four-factor model with 14 items provides the best fit for the data: transmissive view of mathematics, constructivist view of mathematics, transmissive view of teaching and learning, and constructivist view of teaching and learning. The findings of the study reveal that each factor has adequate internal consistency and reliability. These results confirm that the beliefs scale is a reliable and valid measurement tool to measure Korean elementary teachers' mathematical beliefs. The implications of the study are discussed.

Key words: elementary school teacher, factor analysis, scale development, teachers' mathematical beliefs

Ⅰ. Introduction

Todays' societies are changing rapidly, and individuals are likely to confront difficulties that they have rarely experienced and learned at school (Organization for Economic Cooperation and Development [OECD], 2005, 2019). Owing to the development of technologies and globalization, individuals are interconnected across the world and expected to work and compete with diverse people of different cultures, races, languages, and nationalities. Moreover, a large amount of information is rapidly produced and distributed every day, and previous knowledge acquired from school tends to become

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outdated information. In these environments, individuals should have various competencies beyond the acquisition of basic skills and information.

In response to social change, schools are expected to improve students' competencies to prepare them for the challenges of life. These competencies include not only knowledge and skills but also the ability to mobilize various resources to solve problems in a certain context (OECD, 2005, 2019). The competencies that students should acquire through school learning include using tools (e.g., knowledge, technology, and language) interactively, interacting in heterogeneous groups, and acting autonomously in a complex world (OECD, 2005). In line with this social change, the 2015 revised Korean mathematics curriculum highlighted the importance of improving six student competencies, including problem-solving, reasoning, and communication through mathematics classrooms (Ministry of Education, 2015). To construct this mathematics classroom environment, teachers are asked to transform their instructional practices from traditional teacher-centered practices into innovative student-centered practices. They are expected to work as facilitators, not transmitters, and allow active student engagement, discussion, and authority. Moreover, teachers are required to improve student reasoning and problem-solving abilities and view students as coconstructors of knowledge (Ministry of Education, 2015; National Council of Teachers of Mathematics [NCTM], 2014).

Researchers have suggested that teachers should first change their mathematical beliefs to shift their traditional instructional practices because teachers' mathematical beliefs play a critical role in their instructional practices (Buehl & Beck, 2015; Charalambous, 2015; Cross, 2009; Hwang, 2018; NCTM, 2014; Philipp, 2007; Skott, 2015; Yang & Lee, 2019). Teachers who believe that students learn mathematics best through rote memorization may introduce mathematical procedures directly and ask students to use them to solve similar problems. Conversely, teachers who believe that students should be free to explore ideas may choose to invoke personal investigation and group discussion to teach mathematics. That is, their mathematical beliefs often justify teachers' decision to implement traditional teacher-centered practices or innovative student-centered practices (Buehl & Beck, 2015; Hwang, 2018; Philipp, 2007; Skott, 2015). Moreover, researchers have found that teachers' mathematical beliefs affect students' mathematical beliefs (Philipp, 2007) and their achievement (Boonen, Van Damme, & Onghena, 2014). More specifically, teachers' constructivist views of teaching and learning are positively associated with student learning outcomes, whereas teachers' transmission views of teaching and learning are negatively associated with them. Therefore, it is important to measure teachers' mathematical beliefs accurately to understand and transform their instructional practices.

Previous Korean researchers have developed several scales to measure elementary teachers' mathematical beliefs (e.g., Kim & Ryu, 2019; Lee, 2013; Rim, Choo, & Kim, 2010). However, these studies have the following two limitations: having small sample size or implementing improper statistical analysis. Researchers have suggested that the desired ratio between the number of items and participants is approximately 1:5 (Hair, Black, Babin, & Anderson, 2014). However, some researchers have developed a beliefs scale without considering this principle.

Among the several methods to develop a scale, factor analysis has been most widely used by Korean researchers. However, none have used both exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) to develop a scale for elementary teachers' mathematical beliefs. Consequently, previous scales could not accurately measure these beliefs in Korean teachers. Therefore, the purpose of this study was to develop a scale measuring elementary teachers' mathematical beliefs with sufficient sample size and appropriate statistical methods. The findings of this study help researchers measure elementary teachers' mathematical beliefs more accurately. Moreover, the scale would be used to understand the mathematical beliefs of current elementary school teachers and provide additional suggestions for these educators.

Ⅱ. Literature Review

1. Teachers' Mathematical Beliefs

Beliefs represent individuals' mental constructs, which are subjectively true for them (Richardson, 2003). Pajares (1992) conceptualized beliefs as an "individual's judgment of the truth or falsity of a proposition" (p. 316). Similarly, Philipp (2007) defined beliefs as follows: "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 257). Hence, beliefs include cognitive and affective aspects and are more stable and less likely to change than other affective domains, such as attitudes and emotions. Moreover, beliefs can serve as an instrument to define behavior and organize knowledge and information (Buehl & Beck, 2015; Skott, 2015). Hence, teachers interpret information and make decisions as to what mathematics to teach and how according to their beliefs. From this perspective, mathematics teachers' beliefs are seen as mediators in the creation of mathematics classroom norms by which expectations, roles, and authorities of students and teachers are defined (Cross, 2009; Yackel & Rasmussen, 2002).

Beliefs are complex structures (Green, 1971; Pajares, 1992; Skott, 2015). Teachers can have different beliefs simultaneously because some beliefs are peripheral and relatively easy to change based on the contextual environment, while others are not. Green (1971) explained that the relationship between individual beliefs is quasi-logical and that the psychological strength of the relationship varies based on the individual. Hence, the relationship between individual beliefs is not always aligned. For example, teachers may have contradictory beliefs about mathematics teaching and learning because their mathematical beliefs are influenced by not only their educational philosophy but also their school contexts (Buehl, & Beck, 2015; Francis, 2015; Skott, 2015; Warfield, Wood, & Lehman, 2005). In a case study of seven elementary teachers, Warfield et al. (2005) found that while some teachers admitted the value of constructivist orientation, they espoused drill and practices in the mathematics classroom because they believed that their students could not understand mathematical concepts through investigation.

Teachers' mathematical beliefs are also influenced by their cultural context. As a result, East Asian mathematics teachers are likely to have different mathematical beliefs from their Western counterparts (Cai & Wang, 2010; Leung, 2001; Park & Leung, 2006). Western culture stresses independence and individualism, while East Asian culture emphasizes social orientation and collectivism. Because East Asian mathematics teachers tend to believe that teachers should control student learning activities, they explain mathematical concepts directly and are unlikely to sanction an individual investigation. Moreover, they stress mathematical content over mathematical process because they believe that memorizing content helps students' mathematical understanding; memorization comes before understanding (Cai & Wang, 2010). Therefore, Korean mathematics teachers with a constructivist view of mathematics can also have the transmissive view of teaching and learning by thinking that memorization is best for mathematical learning.

2. Dimension of Teachers' Mathematical Beliefs

Researchers have divided teachers' mathematical beliefs into beliefs about the nature of mathematics, mathematics teaching, and mathematics learning, following Ernest's (1989) categorization (Cross, 2009; Skott, 2015; Voss, Kleickmann, Kunter, & Hachfeld, 2013). Table 1 shows the dimension of teachers' mathematical beliefs. The beliefs about the nature of mathematics refer to an individual's belief about the nature of mathematical knowledge and knowing. Hence, the beliefs comprise beliefs about the simplicity of knowledge and the certainty of knowledge (Hofer & Pintrich, 1997). The simplicity of knowledge concerns whether knowledge is an accumulation of isolated rules or interrelated concepts. The certainty of knowledge concerns whether knowledge is absolute or changeable truths derived from human activities. Ernest (1989) proposed three different ways of how teachers typically think about the nature of mathematics: the instrumentalist view, the Platonist view, and the problem-solving view. The first view considered mathematics to be a collection of unrelated facts and rules; the second view assumed that mathematics is a static and unified body of knowledge; while the last view considered mathematics to be a process of investigation and invention of a steadily developing field. However, empirical studies have found that some mathematics teachers have both instrumentalist and Platonist views or both Platonist and problem-solving views (e.g., Shilling-Traina, & Stylianides, 2013), which makes it challenging to examine teachers' beliefs about the nature of mathematics. Therefore, other researchers have used a two-stage hierarchical model, including traditional and nontraditional beliefs (Raymond, 1997), view of rules and procedures and view of the inquiry process (Tatto et al., 2012), and transmissive and constructivist orientations (Voss et al., 2013).

Although Ernest (1989) divided teachers' beliefs about mathematics teaching and learning into two different constructs, other researchers have integrated them into a single construct. For example, Kuhs and Ball (1986) focused on teachers' beliefs about mathematics teaching and suggested the following three levels: (a) learner focused, believing learning of mathematics to be an active mathematics knowledge construction process in learning communities, (b) content focused, with an emphasis on learners' conceptual understanding, and (c) content focused, with an emphasis on mathematical rules and procedures. Similarly, Raymond (1997) proposed traditional and nontraditional beliefs, and Voss et al. (2013) and Chan (2011) used a transmissive and constructivist view of teaching and learning. Moreover, other researchers used different terms to describe teachers' constructivist orientation, such as student-centered beliefs (Remillard & Bryans, 2004), active learning (Tatto et al., 2012), and productive beliefs (NCTM, 2014).

<Table 1> Dimension of Teachers' Mathematical Beliefs

Dimension	Sub-dimension	Description
Beliefs about the nature of mathematics	The transmissive view of mathematics	Mathematical knowledge is - absolute and unquestionable - formalized language without error - immutable truths - a unified body with interconnecting truths
	The constructivist view of mathematics	Mathematical knowledge is - constructed from human experience - fallible, uncertain, and changeable An individual has a unique understanding of mathematics.
Beliefs about mathematics teaching and learning	The transmissive view of teaching and learning	Teachers are the source of knowledge, so students learn mathematics best by observing teachers' demonstration and memorizing procedures. Students should practice a certain type of problem until it becomes automatic. Students are passive recipients of knowledge. They are expected to reproduce knowledge in textbooks.
	The constructivist view of teaching and learning	Teachers should focus on students' reasoning and problem solving and increase student discourses to make mathematical learning communities. Students are believed to construct their mathematical understanding using prior experiences and knowledge. The roles of students and teachers are as active investigators and facilitators, respectively.

3. Developing Teachers' Mathematical Beliefs Scale

Researchers have developed teachers' mathematical beliefs scale in two different ways. The first group of researchers has divided beliefs about the nature of mathematics and mathematics teaching and learning into two or three constructs. They have then examined the validity and reliability of each construct separately (e.g., Purnomo, 2017; Tatto et al., 2012). The second group of researchers has integrated the beliefs about the nature of mathematics and mathematics teaching and learning into one construct. Then, they have implemented a factor analysis to examine the validity and reliability of a scale (Andrews & Hatch, 1999; Barkatsas & Malone, 2005; Beswick, 2005; Perry, Tracey, & Howard, 1999; Voss et al., 2013). It is important to emphasize that most scholars have used this method. These studies revealed that teachers' beliefs about the nature of mathematics and about mathematics teaching and learning are significantly aligned because they are deeply embedded in the philosophy and worldview of teachers. For example, Barkatsas and Malone (2005) examined 465 Greek secondary teachers' mathematical beliefs using 34 items, consisting of beliefs about the nature of mathematics, mathematics teaching, and learning. They implemented factor analysis with all items and found two major constructs: (a) constructivist orientation consisting of socio-constructivist orientation, problem-driven orientation, and cooperative orientation and (b) transmission orientation consisting of mechanistic-transmission and static-transmission orientation. Similar findings have been found for mathematics teachers from Austria (Perry et al., 1999) and Germany (Voss et al., 2013).

Most Korean scholars have developed beliefs scales following the first group of researchers (e.g., Lee, 2013; Rim et al., 2010). They have developed items in different constructs and examined construct validity separately. Moreover, most of them do not implement CFA. As a result, the exploration of teachers' mathematical beliefs from integrated perspectives is highly limited in Korea. However, teachers' beliefs about the nature of mathematics and mathematics teaching and learning tend to be aligned in certain patterns (Barkatsas & Malone, 2005). Because teachers' beliefs about the nature of mathematics are reflected in their beliefs about mathematics teaching and learning, they are intertwined and interconnected. Therefore, in this study we implemented a factor analysis of elementary teachers' mathematical beliefs without dividing them into different constructs.

Ⅲ. Methods

1. Participants

We recruited the respondents through email and SMS and asked them to share the survey with their colleagues. The study participants completed the questionnaire and sent the scanned copy of the questionnaire to the researchers. Moreover, we posted a survey announcement in the online community for Korean elementary school teachers (www.indischool.com) to recruit more diverse participants. These teachers completed the survey using the online survey system Qualtrics. Although 335 teachers (112 online and 223 offline) participated in the survey, we excluded 36 teachers' data because they did not complete more than half of the questionnaire. We used a total of 299 teachers' data (male 57, female 238, and no response 4) for data analysis. Most teachers had a bachelor's (n = 174) or master's degree (n = 118), and only two had a Ph.D. Years of teaching were evenly distributed across the five groups; teachers were asked to select one of the five options. The percentage of each group was approximately 20%. Table 2 shows more detailed information about participants. This demographic information is similar to that of the target group. According to the Korean Education Statistics Service (2019), of the complete group of South Korean elementary teachers, female and male teachers accounted for 77.1% and 22.9%, respectively. Moreover, their years of experience were 19.9%, 23.0%, 17.7%, 16.2%, and 23.2% for 0–4, 5–9, 10–14, 15–19, and more than 20 years, respectively.

<Table 2> Demographic Information of the Participants

Demographic variables	n	%
Gender		
Male	*57*	*19.1*
Female	*238*	*79.6*
No response	*4*	*1.3*
Years of teaching		
0 - 5	*64*	*21.4*
6 - 10	*76*	*25.4*
11 - 15	*48*	*16.1*
16 - 20	*56*	*18.7*
More than 21	*51*	*17.1*
No response	*1.3*	*1.3*
The highest degree		
Bachelor	*174*	*58.2*
Master	*118*	*39.5*
Ph.D.	*2*	*0.6*
No response	*5*	*1.7*
Total	*299*	*100*

2. Scale Development

We developed a mathematics teachers' beliefs questionnaire following scale development literature (DeVellis, 2011; Hair et al., 2014; Netemeyer, Bearden, & Sharma, 2003). We first created an item pool based on a literature review of mathematics teachers' beliefs (Andrews & Hatch, 1999; Barkatsas & Malone, 2005; Beswick, 2005; NCTM, 2014; Perry et al., 1999; Philipp, 2007; Skott, 2015; Tatto et al., 2012; Voss et al., 2013). The first item pool comprised a total of 42 items. The scale consisted of 15 items covering beliefs about the nature of mathematics and 27 items covering beliefs about mathematics teaching and learning. Second, we determined the measurement format. A 5-point Likert scale was adopted with the following options: strongly disagree (1), disagree (2), neutral (3), agree (4), and strongly agree (5). This was because the Likert scale is one of the most commonly used tools to measure respondents' beliefs, and more than seven points could decrease the validity and reliability (DeVellis, 2011). Moreover, providing an even number of scale points might force participants to have an opinion, resulting in a "weak commitment" (Netemeyer et al., 2003, p.101).

Third, two professors reviewed the initial item pool, along with three doctoral students specializing in mathematics education. We asked the experts to evaluate each item with one of the following three options: keep, revise, and delete (DeVellis, 2011). We asked them to choose "delete" if an item was not related to teachers' mathematical beliefs. Moreover, when they chose to revise, they were asked to write down how to revise an item. Based on reviewers' comments, we decided to remove 12 items and revised the wording of the remaining 30 items, resulting in 12 items for beliefs about the nature of mathematics and 18 items for beliefs about mathematics teaching and learning. Then, we administered the scale to three elementary school teachers with at least 10 years of experience. They were invited to an individual phone-call meeting and asked to read and evaluate each item whether the individual was stated to understand. We have made minor revisions based on their responses, but most items have not changed.

3. Data Analysis

We sequentially implemented EFA, CFA, and reliability analysis to examine the validity and reliability of elementary teachers' mathematical beliefs scale. We randomly divided the total sample into two subsamples. We used the first subsample data for the EFA, which we conducted in SPSS 21.0 with the 30 items to examine the factor structure. The ratio between the number of items and participants was approximately 1:5, which meets the threshold recommended by Hair et al. (2014). We used principal components analysis and Oblimin rotation. Kaiser criterion and screening plot were used to determine the number of factors to be extracted. Moreover, we examined the extracted items to interpret the characteristics of each factor. Items with less than .50 factor loading and .40 communalities and with cross-loading above .40 were removed from the scale (Hair et al., 2014). In addition, we used Kaiser-Meyer Olkin (KMO) and Bartlett sphericity test: the former to examine whether the sample size was appropriate for factor analysis and the latter to examine whether the assumption of sphericity was met. When KMO values are higher than .6 and Bartlett sphericity test is significant, factor analysis can be performed (Hair et al., 2014).

Then, we performed CFA in Mplus 8.2 to examine the construct developed from the first dataset. In particular, we compared three models to find the best factor structure. The second subsample data was used for CFA. We used a chi-square test, comparative fit index (CFI), Tucker-Lewis index (TLI), and root mean square error of approximation (RMSEA) to determine the adequacy of the model. The chi-square test is sensitive to sample size, so we focused more on other fit indices. Generally, CFI and TLI higher than .90 and RMSEA less than .08 are considered a threshold level to say that the model is reasonably fit (Kline, 2015). Last, we performed a reliability analysis to examine the internal consistency of each factor using Cronbach's alpha. A coefficient value greater than .6 is regarded as an acceptable level of reliability (Clark & Watson, 1995).

Ⅳ. Results

1. Preliminary Analysis

The first subsample used for EFA consisted of 149 teachers, including 18% male and 80.7% female teachers with a no response rate of 1.3%. The second subsample used for CFA consisted of 150 teachers, including 20.2% male and 78.5% female teachers with a no response rate of 1.3%. We implemented a t-test and chi-square test to ensure comparability between the two subsamples regarding background characteristics of the participants. The chi-square test results indicated that no significant differences were found for years of experience x^2 ([4, n = 295] = 3.251, p = .517) and for the highest educational degree ([3, n = 294] = 2.610, p = .456). Moreover, the distribution of gender between the two groups was not statistically different (t(293) = 0.469, p = .639). Therefore, we concluded that both subsamples were not statistically different in terms of teaching year, gender, and highest educational degree.

2. Exploratory Factor Analysis

Initial EFA revealed that the KMO measure of sampling adequacy was .736, and Bartlett's test of sphericity was significant at the .001 level, which indicated that the factor analysis could proceed. Based on the iterative factor analysis process, we removed 16 items from the pool and obtained 14 items. The final EFA produced four factors. The four factors explained 64.164% of the total variance. Each item had significant factor loading and communality scores (Hair et al., 2014). Factor loadings of 14 items ranged from .542 to .941. Additionally, their communalities ranged from .514 to .884. The first factor was associated with a transmissive view of mathematics (four items), the second factor was associated with a constructivist view of mathematics (three items), the third factor was associated with a transmissive view of teaching and learning (three items), and the fourth factor was associated with a constructivist view of teaching and learning (four items). Table 3 shows the items and factor loadings of each factor.

3. Confirmatory Factor Analysis and Reliability Analysis

We conducted CFA with the second subsample (n = 150) to confirm the factor structure. Based on the previous literature, we examined the construct validity of the three models to find the best factor structure. Table 4 depicts the factorial structure of the three models. The first two models tested whether the four factors produced by the EFA could be represented as two structural factors. The results of the first model, consisting of transmissive and constructivist orientations (e.g., Voss et al., 2013), indicated a poor fit based on the following model fit indices: x^2 (73, n = 150) = 135.698, $p < .001$, CFI = .877, TLI = .847, and RMSEA = 0.076.

<Table 3> The Results of the EFA

Item	Item description	Factor 1	Factor 2	Factor 3	Factor 4
1	People need to know the correct procedure to solve mathematics problems	*.822*			
2	Doing mathematics requires the correct application of certain steps	*.694*			
3	Mathematics is a collection of rules	*.800*			
5	To do mathematics requires remembering formulas	*.542*			
10	Mathematical knowledge develops over time		*.733*		
11	Mathematics is useful for solving everyday problems		*.945*		
12	Mathematics is strongly related to real-life situations		*.941*		
13	Teachers should teach exact procedures for solving problems			*.578*	
25	Memorizing formulae is the best way to do well in mathematics			*.684*	
27	Students learn mathematics best by attending to the teacher' s explanations			*.813*	
19	Teachers should provide opportunities for students to figure out their own ways to solve problems				*.738*
20	The teacher should allocate time to discuss mathematics problems				*.759*
21	Teachers should encourage students to find their problem-solving strategies				*.743*
23	It is highly productive for students to work together during mathematics class				*.728*

<Table 4> Structure and Fitness Indices for the Three Models

Model	Item numbers	x^2 (df)	CFI TLI RMSEA
Two factors		*139.194*	*.873*
-Transmissive orientation	1, 2, 3, 5, 13, 25, 27	*(74)*	*.843*
-Constructive orientation	10, 11, 12, 19, 20, 21, 23		*.077*
Two factors		*254.500*	*.647*
-Beliefs about the nature of mathematics	1, 2, 3, 5, 10, 11, 12	*(74)*	*.566*
-Beliefs about mathematics teaching and learning	13, 25, 27, 19, 20, 21, 23		*.128*
Four factors			
-The transmissive view of mathematics	1, 2, 3, 5	*95.742*	*.946*
-The constructivist view of mathematics	10, 11, 12	*(68)*	*.928*
-The transmissive view of teaching and learning	13, 25, 27		*.052*
-The constructivist view of teaching and learning	19, 20, 21, 23		

Moreover, the second model, consisting of beliefs about the nature of mathematics and mathematics teaching and learning (e.g., Tatto et al., 2012), revealed a poor fit with the data with the following model fit indices: x^2 (73, n = 150) = 250.156, p < .001, CFI = .654, TLI = .568, and RMSEA = .127. The results of the third model, consisting of four factors, indicated an adequate fit between the hypothesized model and the observed data with the following model fit indices: x^2 (68, n = 150) = 95.742, p = .015, CFI = .946, TLI = .928, and RMSEA = 0.052. Based on the comparison among the three models, we concluded that the four-factor model provides a better fit for the data than does a model hypothesizing two factors. Table 5 shows the unstandardized and standardized estimates between factors and each item of the third model. To examine the internal consistency of the four factors, we performed a reliability analysis using the total dataset (n = 299). The four factors all had acceptable reliability reaching .728-.848, which is above the suggested criteria of .60 (Clark & Watson, 1995).

<Table 5> Final Factor Structure of Teachers' Mathematical Beliefs

Factor	Item	B	SE of B	β	Cronbach's alpha
The transmissive view of mathematics	1	*1fixed****	-	*.671****	*.738*
	2	*1.243****	*.266*	*.824****	
	3	*.599****	*.148*	*.458****	
	5	*.731****	*.194*	*.509****	
The constructivist view of mathematics	10	*1fixed****	-	*.795****	*.848*
	11	*.883****	*.14*	*.709****	
	12	*.578****	*.135*	*.451****	
The transmissive view of teaching and learning	13	*1fixed****	-	*.650****	*.728*
	25	*.712****	*.296*	*.449****	
	27	*.708****	*.265*	*.456****	
The constructivist view of teaching and learning	19	*1fixed****	-	*.658****	*.794*
	20	*1.511****	*.232*	*.708****	
	21	*1.315****	*.175*	*.775****	
	23	*1.252****	*.213*	*.600****	

Note. B and β refer to unstandardized and standardized coefficients, respectively, ***p < .001 (two-tailed tests).

V. Conclusion and Discussion

1. Conclusion

The purpose of this study was to develop a new scale measuring elementary teachers' mathematical beliefs. To ensure the construct validity of the scale, we performed the EFA and CFA using 299 survey data. After the EFA of the beliefs scale, we found a four-factor structure with 14 items, accounting for 64.164% of the variance explained. The first factor (transmissive view of mathematics, four items) examined whether the teachers believed that mathematical knowledge is absolute and immutable truth without error. In contrast, the second factor (constructivist view of mathematics, three items) examined whether the teachers believed that mathematics knowledge is fallible and constructed from human experience.

Moreover, the third factor (transmissive view of teaching and learning, three items) examined whether participants believed that teachers are a source of knowledge and whether students are passive recipients of knowledge and learn best through memorization, whereas the fourth factor (constructivist view of teaching and learning, four items) examined whether the respondents believed that students could construct their mathematical understanding using previous experiences and knowledge and that the roles of teachers are as facilitators.

Based on previous studies, we examined whether the four factors could be integrated into the following two factors: (a) transmissive orientation and constructive orientation (Voss et al., 2013) and (b) beliefs about the nature of mathematics and beliefs about mathematics teaching and learning (Tatto et al., 2012). The comparison of the three competing models using fit indices showed that the four-factor model was the best-fit model to the data. Moreover, we tested the internal consistency and reliability of a belief scale using Cronbach's alpha coefficients. The results confirmed that each factor had an acceptable internal consistency and reliability.

However, the findings of the study contradicted the results of the literature findings in Western countries (e.g., Barkatsas & Malone, 2005; Perry et al., 1999; Voss et al., 2013). These scholars have reported that teachers' beliefs about the nature of mathematics and mathematics teaching and learning are intertwined in a way that can be integrated as one construct: transmissive or constructivist orientation. Meanwhile, the factor analysis results indicated that teachers' beliefs about the nature of mathematics and mathematics teaching and learning comprise distinct factors. That is, teachers could have different beliefs about the nature of mathematics and about mathematics teaching and learning. For example, teachers who believe mathematics knowledge is fallible and constructed from human experience may believe that memorization would be the best method for learning mathematics. One possible explanation for this contradictory finding pertains to the effect of cultural factors. Compared to Western teachers, East Asian teachers, including Koreans, are concerned about collectivism and mathematical contents (Cai & Wang, 2010; Leung, 2001; Park & Leung, 2006). As a result, some teachers are likely to believe that direct explanation and memorization might be the best methods to manage students' engagement and improve their achievement, while they have constructivist view of mathematics. However, this argument requires further evidence.

2. Implications and Limitations of the Study

The 2015 revised Korean mathematics curriculum asks teachers to change their instructional practices to enhance student competencies, including problem-solving, reasoning, and communication (Ministry of Education, 2015). To achieve this goal, teachers should open their mind to the change and try to transform their practices from traditional teacher-centered to innovative student-centered practices. In this process, mathematics teachers' beliefs play a critical role. Therefore, teacher educators, researchers, and school administrators need to assess teachers' mathematical beliefs accurately.

For this purpose, we developed a scale of elementary teachers' mathematical beliefs using EFA, CFA, and reliability analysis. Although some items of the scale were similar to those of previous scales, the value of the current study was providing additional information on not only factor structures of Korean elementary teachers' mathematical beliefs but also items of each factor. Educational stockholders may acquire in-depth insight into elementary teachers' mathematical beliefs using the scale. Moreover, the beliefs scale can be used to examine mathematical beliefs of different groups of teachers regarding gender, educational degree, and years of teaching. The beliefs scale can also be used to examine the effect of professional development and teacher education programs on changing teachers' beliefs. The results of this study provide a better understanding of mathematical beliefs of Korean elementary teachers.

This study has the following limitations: First, participants of this study could not represent all Korean elementary teachers. Because we used conventional sampling methods and did not consider teachers' working places, it is possible that participants of the survey may have been over-selected from a few cities. Thus, the findings of the study could not represent all Korean elementary teachers. Further research should be conducted with different samples and contexts. The second limitation was the validity of online participants. Some participants were recruited through the intra-email service of the National Education Information System and online community for elementary teachers, which is called Indischool. Indischool is a nonprofit online community for elementary school teachers, and members of the website should submit their job verification document every year to sustain their membership. Most importantly, the first question of the survey was "Are you currently working as an elementary school teacher?", which caused participants to self-check their job status. This recurring process helped us ensure that the participants of the survey were elementary teachers. However, we could not fully guarantee that all participants were elementary teachers. Third, some factors contained only three items. Although three items per factor are enough to identify a factor model (Costello & Osborne, 2005; Hair et al., 2018; Yong & Pearce, 2013), more items per factor could ensure more accurate factor identifications. Thus, further studies should be conducted with more items. Fourth, we collected the survey data from teachers' self-reported responses. Because they completed the survey anonymously, some teachers may have deliberately rated their beliefs to make a good impression. Researchers should interpret the findings of the study carefully.

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〈국문초록〉

초등학교 교사의 수학적 신념 측정도구 개발: 타당성 및 신뢰성 분석

황성환[2]

본 연구의 목적은 초등학교 교사들의 수학적 신념 측정도구를 개발하는 것이다. 초등학교 교사 299명의 수학적 신념을 30문항을 통해 분석하였다: 수학에 대한 신념 12 문항, 수학 교수에 관한 신념 18 문항. 탐색적 요인분석을 통해 (n = 149) 수학적 신념의 요인 구조를 탐색하였다. 또한 확인적 요인분석을 통해 (n = 150) 서로 다른 세 모델의 요인 구조를 비교 분석하였다. 분석 결과 14개 문항으로 구성된 네 개의 요인 구조가 데이타를 가장 잘 설명하는 것으로 나타났다: 전달적 수학관(transmissive view of mathematics), 구성주의적 수학관(constructivist view of mathematics), 전달적 수학 교수관(transmissive view of teaching and learning), 구성주의적 수학 교수관(constructivist view of teaching and learning). 본 연구의 결과를 바탕으로 개발된 측정도구가 초등학교 교사의 수학적 신념을 측정하기 위한 타당성과 신뢰성을 갖춘 것으로 나타났다. 이를 토대로 본 연구의 시사점을 논의하였다.

주제어: 초등학교 교사, 요인분석, 측정도구 개발, 교사의 수학적 신념

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Appendix. Korean Version of Elementary Teachers' Mathematical Beliefs Scale (The final with 14 items)

문항 번호	설문 문항
1	수학적 문제를 해결하기 위해서는 정확한 절차와 규칙을 알아야 한다.
2	수학을 한다는 것은 특정한 절차를 문제에 정확하게 적용하는 것을 의미한다.
3	수학은 규칙과 절차의 집합체이다.
5	수학 문제를 해결하기 위해서는 형식적 절차를 정확하게 기억하고 있어야 한다.
10	수학적 지식은 시간이 지남에 따라 변화하고 발전한다.
11	수학은 일상생활의 문제를 해결하는데 유용하다.
12	수학은 일상생활과 밀접하게 관련되어 있다.
13	교사는 문제를 해결하기 위한 정확한 절차를 학생들에게 가르쳐야 한다.
25	수학을 잘하는 가장 좋은 방법은 공식을 암기하는 것이다.
27	학생들이 수학을 잘하는 가장 좋은 방법은 교사의 설명에 집중하는 것이다.
19	교사는 학생들이 스스로 문제해결 방법을 찾을 수 있도록 충분한 시간과 기회를 주어야 한다.
20	교사는 학생들이 다양한 방법으로 문제를 해결할 수 있도록 토의 시간을 제공해야 한다.
21	교사는 학생들이 스스로 문제 해결 방법을 찾도록 격려해야 한다.
23	학생들간의 협동학습은 학생들의 수학 학습에 도움이 된다.