한국기계가공학회지, 제19권, 제7호, pp.98～105(2020.07) ISSN 1598-6721(Print)
Journal of the Korean Society of Manufacturing Process Engineers, Vol. 19, No. 7, pp.98~105(2020.07) ISSN 2288-0771(Online)

https://doi.org/10.14775/ksmpe.2020.19.07.098

Reduction of Vibration for an Elastic Structure by means of a Relocation of Part

Giman Kim*, Seongdae Choi*,#

*Department of Mechanical System Engineering, Kumoh National Institute of Technology

구조 재배치를 이용한 탄성체 진동 저감

김기만*, 최성대*,#

*국립금오공과대학교 기계시스템공학과

(Received 14 May 2020; received in revised form 18 June 2020; accepted 19 June 2020)

ABSTRACT

This study deals with the passive control of the dynamic characteristics of a theoretical model which is a string with fixed ends and loaded by two point masses - a main mass (M_o) and a secondary mass (M_s). It has been controlled passively by means of a relocation of a secondary mass. A main mass placed on the string is considered as a vibrating receiver which be forced to vibrate by a vibrating source being positioned on the string. By analyzing the motion of a string, the equation of motion for a string was derived by using a method of variation of parameters. To define the optimal conditions for the vibration reduction, the governing equation, which denotes the dynamic response of a string was derived in the closed form and then evaluated numerically. The possibility of reduction of an amplitude and a power being transmitted to a main mass were found to depend on the location and the magnitude of a secondary mass as well as the range of a forcing frequency.

Keywords : Point Mass(점질량), Method of Variation of Parameters(맥계변수 변환법), Dynamic Response(동응답), Power(시간당 에너지)

1. Introduction

In the industrial fields, vibration phenomena of machines and structures had been serious problems to be solved for the good environmental conditions. Until nowadays a lot of studies have been performed to analyze and control the vibration phenomena induced by machining operations in the past decades. These days the fast manufacturing time and the accurate level of the machining tools are known to be the important factors for the development of the industry. In case of the needs of the high speed operation, the vibration phenomenon should be one of the overcoming troubles for the stability of the machine structure.

For the simplification of the complex structures, each part of the machines is considered as a discrete system (a rigid body or a particle) or a continuous system such as string, beam, plate or shell and then in a theoretical model. Hence, the vibration characteristics of a system in a theoretical

Corresponding Author : sdchoi@kumoh.ac.kr
Tel: +82-54-478-7396, Fax: +82-54-478-7319

model had been studied by lots of persons. Recently for the investigation of the vibration characteristics, the vibration energy flow and the dynamic response of a rigid body or some compound system have been analyzed. By using a model of an elastic structure, the vibration energy and control technologies had been presented[1-5]. The structure analysis and the control of the vibrations of the machine tools had been introduced [6-9].

In this study a vibrating system in a theoretical model which consists of two point masses loaded string with a primary source is employed to analyze and improve the dynamic response of the original system which is a one point mass loaded string by relocating a secondary point mass. The edges conditions of a string are fixed. Based on the wave equation, the vibration of a string will be discussed. To define the dynamic response of a vibrating system, the closed form of a displacement of a vibrating system is derived by using a method of variation of parameters. For the reduction of the dynamic response of the main mass, the location and magnitude of a secondary mass is evaluated numerically until the optimal conditions be found. Fig. 1 shows the theoretical model of a uniformly stretched string which is loaded by two point masses and is subjected to a harmonically excited force F(t).

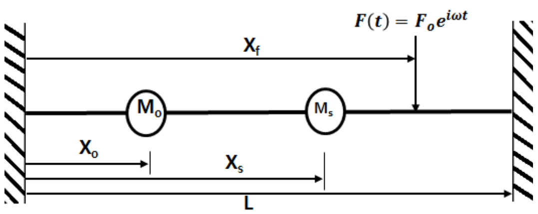


Fig. 1 A theoretical model of a vibrating system F(t)-[Fo(amplitude), ω (forcing frequency)], Distances away from the left end-[X_o(to the main mass), X_s(to the secondary mass), X_f(to the external force)], Mass items-[Mo(a main mass), Ms(a secondary mass)]

Here the total mass of the system is kept constant but the magnitude and the location of a secondary point mass are considered as variables.

2. The Governing Equations

The governing equation for the motion of a string which is loaded by two point masses, can be written as,

$$T\partial_{XX}u(X,t) = \rho\partial_{tt}u(X,t) + f(X,t) + M(X,t) \qquad (1)$$

where $\partial_{XX} = \dfrac{\partial^2}{\partial X^2}$, $\partial_{tt} = \dfrac{\partial^2}{\partial t^2}$,

$f(X,t) = -F_o e^{i\omega t}\delta(X-X_f)$,

$M(X,t) = \{M_o\delta(X-X_o) + M_s\delta(X-X_s)\}\partial_{tt}u(X,t)$

and u(X,t) represents the displacements of the string, T stands for the uniform tension of the string, ρ for the mass density of the string per unit length and δ is the Dirac delta function.

Assuming that the motion of the string is sinusodial in time, the displacement u(X,t) becomes

$$u(X,t) = y(X)e^{i\omega t} \qquad (2)$$

where y(X) represents the amplitude of the string and depends only on X. By inserting Eq.(2) into Eq.(1) and cancelling the term $e^{i\omega t}$ from both sides of equation, the governing equation of motion is written as a function of X,

$$\partial_{XX}y(X) + k^2 y(X) = F(X) + m(X) \qquad (3)$$

where $k^2 = \dfrac{\rho\omega^2}{T}$, $F(X) = -\dfrac{F_o}{T}\delta(X-X_f)$ and

$$m(X) = -\frac{M_o\omega^2}{T}\delta(X-X_o)y(X_o) - \frac{M_s\omega^2}{T}\delta(X-X_s)y(X_s)$$

Equation (3) satisfies the following boundary conditions,

$$y(0)=y(L)=0 \tag{4}$$

The complete solution of Eq. (3) can be expressed into the sum of a homogeneous solution (y_h) and a particular solution (y_p) as

$$y(X)=y_h(X)+y_p(X) \tag{5}$$

The homogeneous solution is determined by letting the right side of Eq.(3) be zero. Then the solution becomes as

$$y_h(X)=A\sin(kX)+B\cos(kX) \tag{6}$$

where A and B are the constant parameters. The particular solution is assumed to be of the form as

$$y_p(X)=V_1(X)\sin(kX)+V_2(X)\cos(kX) \tag{7}$$

Here $V_1(X)$ and $V_2(X)$ can be determined by means of the method of variation of parameters and then defined as follows

$$V_1(X)=\frac{1}{k}\int^{X}\cos(k\zeta)f(\zeta)\,d\zeta \tag{8}$$

$$V_2(X)=-\frac{1}{k}\int^{X}\sin(k\zeta)f(\zeta)\,d\zeta \tag{9}$$

where $f(\zeta)=F(\zeta)+m(\zeta)$.

By substituting Eqs. (8) and (9) in Eq. (7), the particular solution becomes as

$$y_p(X)=F(X-X_f)+m_1(X)y(X_o)+m_2(X)y(X_s) \tag{10}$$

where $F(X-X_f)=-\frac{F_o}{kT}\sin(k(X-X_o))\,H(X-X_o)$,

$m_1(X)=-\frac{M_o\omega^2}{kT}\sin(k(X-X_o))\,H(X-X_o)$ and

$m_2(X)=-\frac{M_s\omega^2}{kT}\sin(k(X-X_s))\,H(X-X_s)$.

Here $H(X-X_I)$ in Eq. (10) represents the Heaviside unit step function. Hence By evaluating the quantities, $y(X_o)$ and $y(X_s)$ in Eq. (10) and then the boundary conditions, Eq. (4), the complete solution of Eq. (3) becomes as

$$y(X)=\frac{A_1}{A_2}[\sin(kX)+m_1(X)G_{11}+m_2(X)G_{12}] + F(X-X_f)-m_1(X)G_{21}-m_2(X)G_{22} \tag{11}$$

where $G_{11}=\sin(kX_o)+m_2(X_o)\sin(kX_s)$,

$G_{12}=\sin(kX_s)+m_1(X_{so})\sin(kX_o)$,

$G_{21}=\sin(k(X_o-X_f))H(X_o-X_f)+m_2(X_o)\sin(k(X_s-X_f))H(X_s-X_f)$,

$G_{22}=\sin(k(X_s-X_f))H(X_s-X_f)+m_1(X_s)\sin(k(X_o-X_f))H(X_o-X_f)$,

and $A_1=\frac{F_o}{kT}[\sin(k(L-X_f))+m_1(L)G_{21}+m_2(L)G_{22}]$,

$A_2=\sin(kL)+m_1(L)G_{11}+m_2(L)G_{12}$.

Equation (11) which is the displacement of a string is used to define the characteristics of a vibrating system. All values obtained in this study have been expressed into the dimensionless forms which are divided by a certain quantity. The total mass of the system is expressed into M (= M_o + M_s + ρL) and kept constant. Also the magnitude and location of the main mass are kept constant. In Table 1, the dimensionless properties are introduced as

Table 1 Dimensionless properties

Items	Expression form
masses	$m_o = M_o/M = 0.3$ $m_s = M_s/M = 0.1,\ 0.2$ $\mu^2=\rho L/M$ $=1-m_o-m_s$
displacement	$Y(x) = y(X)/F_o/kT$
locations	$x_o=X_o/L = 0.3$ $x_f=X_f/L = 0.7$ $x_s=X_s/L = 0 \sim 1$
power	$P = \text{Power}/\sqrt{T^3/\rho}$
frequency	$\alpha = \omega/\sqrt{T/ML}$

3. Resonant Frequency Coefficients (α)

In Eq. (11), the denominator part A2 vanishes for certain specific values of the wave number k. The roots of denominator which are expressed in ω_r by $k^2=\rho\omega^2/T$ are called the resonant frequencies for the vibrating system. So the resonant frequency equation becomes as,

$$R(k) = \sin(kL) + m_1(L)G_{11} + m_2(L)G_{12} = 0 \qquad (12)$$

Here Eq. (12) can be rewritten in terms of the dimensionless form as,

$$R(\alpha) = \sin(\alpha\mu) + \overline{m_1}(1)\overline{G_{11}} + \overline{m_2}(1)\overline{G_{12}} = 0 \qquad (13)$$

where $\overline{m_1}(1) = -m_o\alpha\sin(\alpha\mu(1-x_o))/\mu$,

$\overline{m_2}(1) = -m_s\alpha\sin(\alpha\mu(1-x_s))/\mu$,

$\overline{G_{11}} = \sin(\alpha\mu x_o) + \overline{m_2}(x_o)\sin(\alpha\mu x_s)$,

$\overline{G_{12}} = \sin(\alpha\mu x_s) + \overline{m_1}(x_s)\sin(\alpha\mu x_o)$,

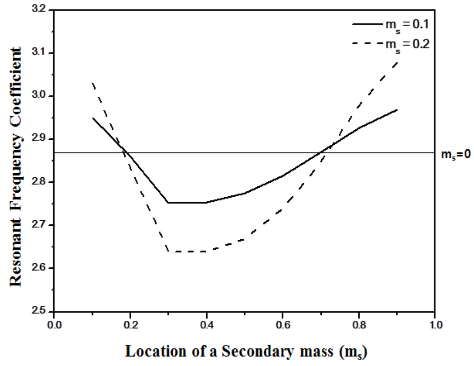
$\overline{m_1}(x_s) = -m_o\alpha\sin(\alpha\mu(x_s - x_o))H(x_s - x_o)/\mu$,

$\overline{m_2}(x_o) = -m_s\alpha\sin(\alpha\mu(x_o - x_s))H(x_o - x_s)/\mu$.

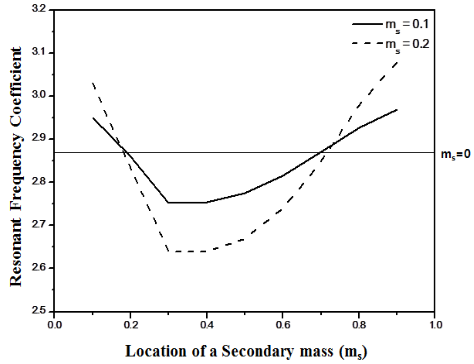
For two mass ratios (m_s = 0.1, 0.2), the variations of three resonant frequency coefficients (α_1, α_2, α_3) versus location of a secondary mass are plotted in Fig. 2 (a), (b) and (c), respectively. The emergence of secondary mass changes the resonant frequency of the system, but may also have the same resonant frequency as the original system being loaded by one point mass depending on the location of the secondary mass. Also it is observed that the larger the mass is, the greater be the variation in frequency.

4. Dynamic Characteristics

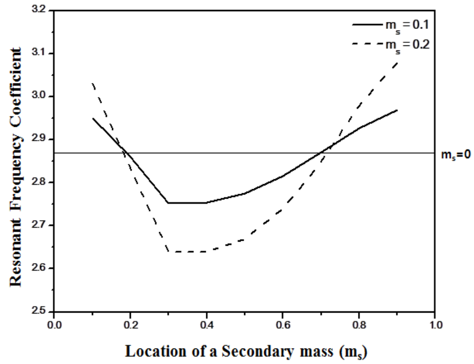
4.1 Reduction of Amplitudes



(a) a_1



(b) a_2



(c) a_3

Fig. 2 Resonant frequency coefficients vs x_s
m_o = 0.3, x_o = 0.3, x_f = 0.7
solid [m_s = 0.1], dot [m_s = 0.2]

In order to control passively the amplitude of a main mass, the dimensionless amplitude Y(x) in Eq. (11) is evaluated according to the secondary mass

(M_s) location, magnitude and the range of frequency. The loss factor is applied to avoid infinite amplification in resonance in all calculation processes. So the frequency coefficient is expressed in terms of a complex form as,

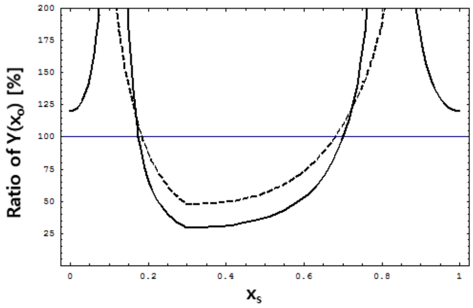
$$\alpha = \boldsymbol{\alpha}\left(1 - i\frac{\eta}{2}\right) \tag{14}$$

where η is the loss factor and typically given in the range of 10^{-1} to 10^{-3}. The reduction level of the amplitude (RA) is evaluated in comparison with two cases - an amplitude without the secondary mass and an amplitude with the secondary mass. The ratio of a Y(x) with M_s to one without M_s is expressed in terms of percentage[%] as follows;

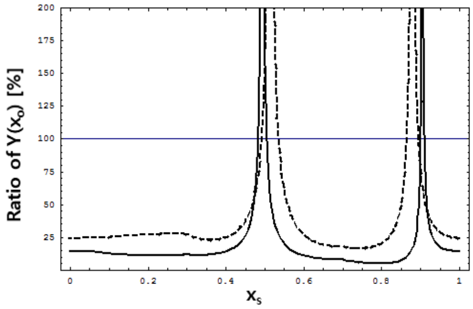
$$RA[x] = 100\,|\,Y(x)_{Ms}/Y(x)_{no\ Ms}| \tag{15}$$

The values below 100 in Eq. (15) verifies the reduction of amplitude. In Fig. 3 (a), (b) and (c), the reduction of amplitudes at the main mass (x_o) for two mass ratios (m_s = 0.1, 0.2), are plotted along location of a secondary mass for three frequency coefficients (α = 3, 6, 10), respectively. In Fig. 3 (a) for α = 3, the reduction of amplitude are observed to be done around the locations (x_s = 0.2~0.6). However in case of α = 6 and 10, the locations for the reduction are observed differently compared to the case of α = 3. Hence it is not easy to achieve reduction for wide frequency range by fixing the location of the secondary mass because the variation of reduction amplitude is so irregular depending on frequency.

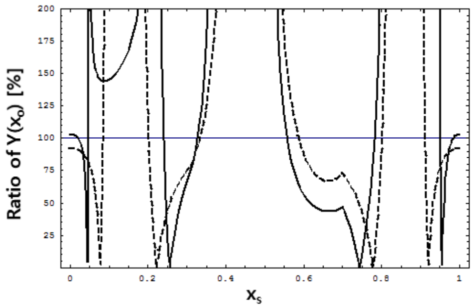
Especially, the secondary mass located near or at the main mass was found to satisfy the reduction in frequency zones in three cases (α = 3, 6, 10). In Fig. 4, the reduction of amplitudes for three frequency coefficients ratios (α = 3, 6, 10), are plotted along x for the secondary mass location (x_s = 0.29) which is very close to the main mass. As shown in Fig. 4, it is proved again that when the secondary mass is



(a) α = 3



(b) α = 6



(c) α = 10

Fig. 3 RA[x_o] vs x_s
m_o = 0.3, x_o = 0.3, x_f = 0.7
solid [m_s = 0.1], dot [m_s = 0.2]

located near or at the main mass, the amplitude of the main mass could be down to the level being less than the original system.

In Fig.5 (a), (b), (c) and (d), the reduction of amplitudes at the main mass for two mass ratios (m_s = 0.1, 0.2), are plotted along frequency coefficient for four locations of the secondary mass (x_s = 0.2, 0.3, 0.6, 0.7), respectively.

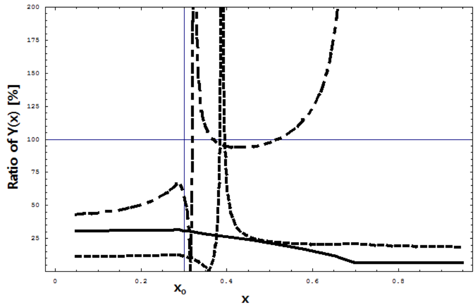


Fig. 4 RA[x] vs x
m_o=0.3, x_o=0.3, m_s=0.2, x_s=0.29, x_f=0.7
solid [α = 3], dot [α = 6], dash [α = 10]

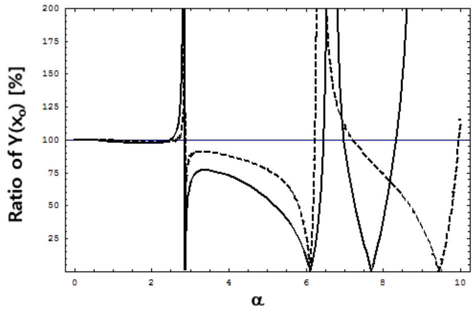
In Fig. 5 (a) and (b), it is confirmed that the reduction of an amplitude of the main mass agrees well with the results in Fig. 3 for α = 3 ~ 6 and the secondary mass being located near or at the location of the main mass. But as shown in Fig. 5 (c) and (d) which are the cases of the secondary mass being located away from the main mass, the amplitude of the main mass is found to be amplified for α = 3 ~ 6. It is surely stated that the reduction of an amplitude of the main mass is strongly depending on the location of the secondary mass and the range of the frequency.

4.2 Reduction of Transmitted Power

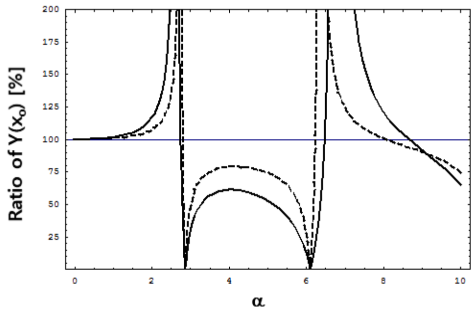
The Vibrational energy which is induced by the harmonic motion of the external force is known to transmit to the main and the neighbour systems. In practice, many structures and parts are known to be troubled by excessive energy which is transmitted to the main part and to the surrounding parts. Hence the power transmitted to the main mass which is the energy rated by time is evaluated and is expressed in dimensionless form as,

$$P[M_o] = Re[\frac{1}{2} i\alpha^3 \mu m_o Y(x_o) Y^*(x_o)] \tag{16}$$

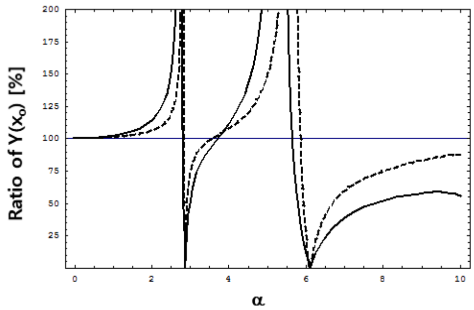
where ‘*’ means the conjugate complex form of



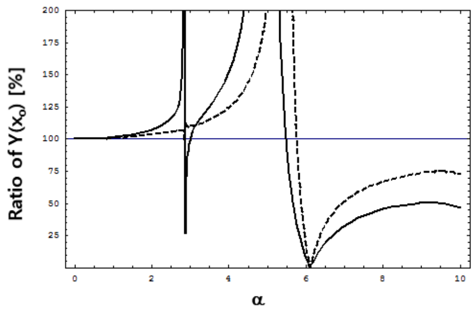
(a) x_s = 0.2



(b) x_s = 0.3

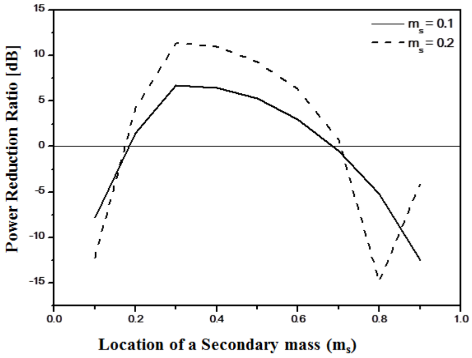


(c) x_s = 0.6

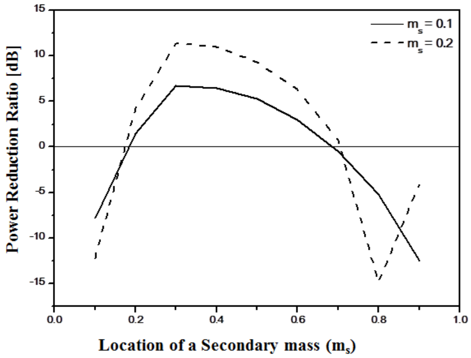


(d) x_s = 0.6

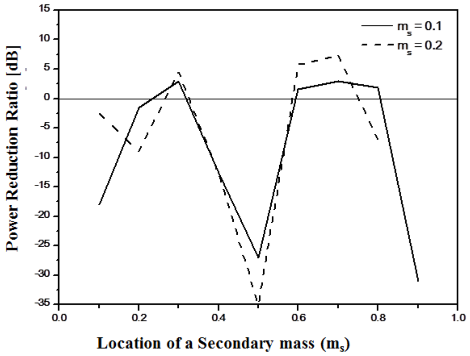
Fig. 5 RA[x_o] vs a
m_o = 0.3, x_o = 0.3, x_f = 0.7
solid [m_s = 0.1], dot [m_s = 0.2]



(a) α = 3



(b) α = 6



(c) α = 10

Fig. 6 PRR vs x_s
m_o = 0.3, x_o = 0.3, x_f = 0.7
solid [m_s = 0.1], dot [m_s = 0.2]

Y. The power reduction ratio (PRR) which is the ratio of a power with M_s to one without M_s is expressed in terms of decibel [dB] as;

$$PRR = 10^{*} Log_{10}\left(\frac{\left|P[\,]_{no\,M_s}\right|}{\left|P[\,]_{M_s}\right|}\right) \ [dB] \qquad (17)$$

In Fig.6 (a), (b) and (c), the reductions of the transmitted power to the main mass for two mass ratios (m_s = 0.1, 0.2) versus the location of the secondary mass are plotted for three frequency coefficient (α = 3, 6, 10), respectively.

As shown in Fig. 6, the reduction pattern of the power along the location of the secondary mass is found to be irregular with respect to the frequency coefficient. However, the reduction of power is observed to be down well when the secondary mass is placed at or near the main mass location.

5. Conclusion

In this paper, the dynamic characteristics of a vibrating string with two point masses is studied to determine the optimal design parameters of a secondary mass. On the bases of the analyses in this study, the conclusions are obtained as follows,

1. A secondary mass is observed to become the control tool for the reduction of the dynamic characteristics, which are the dynamic responses and the power be transmitted to a main mass placed on the string.
2. The location of a secondary mass should be positioned near or at the place of a main mass for the reduction of dynamic response of a system.
3. The variation of a secondary mass configuration might be a helpful tool to avoid the resonant phenomena by changing the range of the natural frequencies.

Acknowledgment

This paper was supported by Research Fund, Kumoh National Institute of Technology(2018-104-026).

References

1. Pan, J. and Hansen, C. H., “Active control of total vibratory power flow in a beam. I : Physical System Analysis,” The Journal of the Acoustical Society of America, Vol. 89, No. 1, pp. 200-209, 1991.
2. Scchwenk, A. E., Sommerfeldt, S. D., and Hayek, S. I., “Adaptive control of structural intensity associated with bending waves in a beam,” The Journal of the Acoustical Society of America, Vol. 96, pp. 2826, 1994.
3. Kim, G. M., “Active control of vibrational intensity at a reference point in an infinite elastic plate,” Transactions of the Korean Society for Noise and Vibration Engineering, Vol. 11. No. 4, pp. 22-30, 2001.
4. Kim, G. M. and Choi, S. D., “The Design Parameter analysis of a Dynamic Absorber for the Control of a Machine Body Vibration,” Journal of the KSMTE, Vol. 18, No. 1, pp. 1~8, 2019.
5. Kim, G. M. and Choi, S. D., “Active control of dynamic response for a discrete system in an elastic structure,” Advanced Materials Research, Vol. 505, pp. 512-516, 2012.
6. Son, I. S., Kim, C. H., Bae, S. H., Lee and J. Y., "Rescue Lift Development Using Structural Analysis", Journal of the Korean Society of Manufacturing Process Engineers, Vol. 14 No. 1, pp. 111-116, 2015.
7. Yi, C. S., Kang, D. S., Jeong E. I., Kim H. K., and Baek, I. C., “Structure Analysis on the Material Supply Device for Small Scale Machine Tool,” Proc. of the KSMPE Autumn Conference, pp. 126, 2018.
8. Kim, G. M. and Choi, S. D., “Analysis of the Reduction of the Dynamic Response for the CNC 5 Axles Machining Center,” The Korean Society of Manufacturing Process Engineers, Vol. 9, No. 5, pp. 83-89, 2010.
9. Moon, Y.-M. and Kota, S., “Design of Reconfigurable Machine Tools,” Journal of Manufacturing Science and Engineering. Vol. 124, No. 2, pp. 480-483, 2002.