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Theoretical observation of waves in cancellous bone

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Abseact Poroelasticity theory has been widely used for detecting cancellous bone deterioration because of the safe use for humans. The tortuosity itself is an important indicator for ultrasound detection for bone diseases. The transport properties of cancellous bone are also important in bone mechanotransduction. In this paper, two important factors, the wave velocity and attenuation are examined for permeability (or tortuosity). The theoretical calculation for the relationship between the wave velocity (and attenuation) and permeability (or tortuosity) for cancellous bone is shown in this study. It is found that the wave along the solid phase (trabecular struts) is influenced not by tortuosity, but the wave along the fluid wave (bone fluid phase) is affected by tortuosity significantly. However, the attenuation is different that the attenuation of a fast wave has less influence than that of a slow wave because the slow wave is observed by the relative motion between the solid and fluid phases.

Key Words : Ultrasound, Poroelasticity, Cancellous bone, Attenuation, Wave velocity

1. Introduction

Biot [1,2] published significant works on poroelasticity and these two papers play an important role in the wave propagation in porous media, which are well known in poroelastic societies. Cowin [3] published the survey paper in the application of poroelasticity theory or Biot theory to bone. As we know, the osteocyte, which a bone cell is residing in the lacunar space communicating with other osteocytes and, we believe, osteocytes receive signals from other osteocytes, osteoblasts (bone forming cells), and osteoclasts (bone resorbing cells). The communication among these cells are studied for many years, but the underlying mechanism is not clear at this moment. However, bone fluid flow is believed a primary stimulus in bone cell signaling. We call it as "mechanobiology" because cells are stimulated by mechanical loading, such as bone fluid flow. Then the permeability as well as tortuosity are important factors in bone fluid flow.

Williams [4] employed the Biot theory or so called poroelasticity theory to cancellous bone. Hosokawa and his colleague [5] observed two compressional waves in cancellous bone, which can be predicted by Biot theory. Haire and Langton [6] published the review paper in the application of Biot theory to cancellous bone. Yoon and his colleagues [7] published the article that elucidate the shape of trabeculae significantly affect the speed of wave propagation by using the Biot theory.

The wave velocity and attenuation are key parameters in bone loss diagnosis. We can

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simply measure the bone density. Well-known bar equations or bulk equations are used to find the bone deterioration. A term, BUA are widely used for detecting bone diseases, such as osteoporosis [8-14]. Aygun et al. observed the wave velocity and attenuation is affected by the orientation of pore structure and the anisotropy of cancellous bone [14].

The permeability is important to estimate the transport properties of cancellous bone. Experimentally Kohles et al. [15] measured the anisotropic permeability of cancellous bone, and Grimm and Williams [16] measured the anisotropic permeability of calcaneus. For cortical bone, Beno et al. [17] theoretically estimated anisotropic permeability of different species.

Structural anisotropy [18], or fabric tensor [19,20] determines the anisotropy of cancellous bone. It is assumed that the trabecular struts are isotropic and structural anisotropy determines elastic properties. We can simply the transport properties, such as permeability and tortuosity relate to structural anisotropy or fabric tensor. Thus, in this paper, we will examine the relationship between the wave velocity (and attenuation) and permeability (or tortuosity).

2. Method : Poroelasticity Theory

The governing equations for the poroelasticity theory are given by [21]

$$\rho_{11}\ddot{u}+\rho_{12}\ddot{U}-N_{i,jj}-(A+N)u_{j,ji}-QU_{j,ji}-b(\dot{u}_i-\dot{U}_i)=0 \tag{1}$$

$$\rho_{22}\ddot{U}_i+\rho_{12}\ddot{u}_i-(RU_{j,j}+Qu_{j,j})_{,i}-b(\dot{u}_i-\dot{U}_i)=0 \tag{2}$$

, where N, R, Q, and A are poroelastic parameters, u and U are displacements of fluid and solid constituents, and the variable b is defined by $\phi^2\mu/K$, where μ is the fluid viscosity, K is the permeability of bone fluid, ε is the permittivity, and the ϕ is the porosity. After we define $P\equiv A+2N$, the poroelastic parameters or, so called Biot parameters P, Q, and R are given by

$$P=\frac{\phi(K_s/K_f-1)K_b+\phi^2K_s+(1-2\phi)(K_s-K_b)}{(1-\phi-K_b/K_s+\phi K_s/K_f)}+\frac{4G}{3}\ , \tag{3}$$

$$Q=\frac{(1-\phi-K_b/K_s)\phi K_s}{1-\phi-K_b/K_s+\phi K_s/K_f}, \tag{4}$$

and

$$R=\frac{K_s\phi^2}{(1-\phi-K_b/K_s+\phi K_s/K_f)}. \tag{5}$$

By assuming that the displacements of fluid and solid, u and U, to be harmonic, i.e., $u_i=u_i^0\exp[i(k_ix_i-\omega t)]$ and $U_i=U_i^0\exp[i(k_ix_i-\omega t)]$, equations (1) and (2) are expressed by

$$[Pk^2-(\omega^2\rho_{11}+i\omega b)]u_1^0+[Qk^2-\rho_{12}-i\omega b)]U_1^0=0, \tag{6}$$

$$[Qk^2-(\omega^2\rho_{12}-iwb)]u_1^0+[Rk^2-(\omega^2\rho_{22}+iwb)]U_1^0=0. \tag{7}$$

In order to have non-trivial solutions, the determinant of equations (6) and (7) should be zero, i.e.,

$$\begin{vmatrix} Pk^2-(\omega^2\rho_{11}+i\omega b) & Qk^2-(\omega^2\rho_{12}-i\omega b) \\ Qk^2-(\omega^2\rho_{12}-i\omega b) & Rk^2-(\omega^2\rho_{22}+i\omega b) \end{vmatrix}=0. \tag{8}$$

Then the real part of the determinant, equation (8) is given by

$$-k^4Q^2+k^4PR-k^2R\rho_{11}\omega^2+2k^2P\rho_{22}\omega^2-\rho_{12}^2\omega^4+\rho_{11}\rho_{22}\omega^4, \tag{9}$$

and the imaginary part of the determinant, equation (8) is given by

$$-bk^2P\omega-2bk^2Q\omega-bk^2R\omega+b\rho_{11}\omega^3+2b\rho_{12}\omega^3. \tag{10}$$

Both real and imaginary parts of the determinant, or equations (9) and (10) should be zero. From the real part of the determinant, equation (9) to be zero, the wave velocity, i.e., $Re[\nu]=\frac{\omega}{k}$ is obtained,

$$Re[\nu]=\sqrt{\frac{R\rho_{11}-2Q\rho_{12}+P\rho_{22}\pm B}{2(-\rho_{12}^2+\rho_{11}\rho_{22})}}, \quad (11)$$

where

$$B=\sqrt{\begin{matrix}R^2\rho_{11}^2-4QR\rho_{11}\rho_{12}+4PR\rho_{12}^2+4Q^2\rho_{11}\rho_{12}\\-2PR\rho_{11}\rho_{12}-4PQ\rho_{12}\rho_{22}+P^2\rho_{22}^2\end{matrix}}. \quad (12)$$

From the definition of the wave number, k to be complex number, $k=Re[k]+i\alpha$, where α is the attenuation constant. Because the wave number k is expressed by

$$\begin{aligned}k&=\frac{\omega}{\nu}=\frac{\omega}{Re[\nu]+iIm[\nu]},\\&=\frac{\omega(Re[\nu]-iIm[\nu])}{|\nu|^2}=Re[k]+i\alpha\end{aligned} \quad (13)$$

then the attenuation α is obtained by

$$\alpha=-\frac{\omega Im[\nu]}{|\nu|^2}. \quad (14)$$

The imaginary part of the determinant, equation (10) to be zero, the imaginary part of velocity, $Im[v]$ is obtained by

$$Im[\nu]=\sqrt{\frac{P+2Q+R}{\rho_{11}+2\rho_{12}+\rho_{22}}}. \quad (15)$$

3. Numerical application to cancellous bone

To obtain the bulk moduli described in poroelastic parameters, P, Q, and R illustrated in equations (3), (4), and (5), the following relation is employed,

$$K_i=\frac{E_i}{3(1-2\nu_i)}, \quad (16)$$

where the subscript i can be replaced by b, s, and f for bone, solid bone material, and fluid (or water), respectively. The elastic modulus and Poisson's ratio of 141 human cancellous bones are theoretically estimated for the porosity ϕ [7]

$$E_b=822.367E_t\phi^{1.95},\ G_b=345.540E_t\phi^{1.99},$$

and (17)

$$\nu_b=0.198\phi^{-0.16}.$$

The dimension of elastic and shear moduli in equation (17) is GPa. To obtain the unknown variable, E_t, we set $E_b=E_s$ as the porosity ϕ is zero. The variable, E_t is obtained as 0.023 as the elastic modulus of solid bone material is 18.9 GPa. Then we can estimate the shear modulus of solid bone material is 7.94 GPa, and poroelastic parameters, P. Q, and R are numerically calculated with the bulk modulus of fluid (or water) to be 2.3 GPa. The poroelastic densities, ρ_{11}, ρ_{22}, and ρ_{12} are defined by

$$\rho_{11}+\rho_{12}=(1-\phi)\rho_s, \quad (18)$$

$$\rho_{22}+\rho_{12}=\phi\rho_f, \quad (19)$$

and

$$\rho_{12}=(1-\alpha)\phi\rho_f, \quad (20)$$

where ρ_s and ρ_f are densities of solid and fluid, respectively.

4. Discussion and conclusion

Figure 1 shows the relationship among the fast wave velocity, porosity, and tortuosity, and figure 2 shows the same relationship for the slow wave. The porosity changes both fast and slow wave velocities, but the tortuosity changes only the slow wave velocity for low tortuosity. Similar to figures 1 and 2, the attenuation of slow wave is affected by tortuosity, but that of fast wave is not affected by tortuosity significantly (Figures 3 and 4). However, for the low tortuosity we can see the influence of both wave velocity and attenuation.

Experimentally Kohles et al. [15] measure anisotropic permeability of cancellous bone.

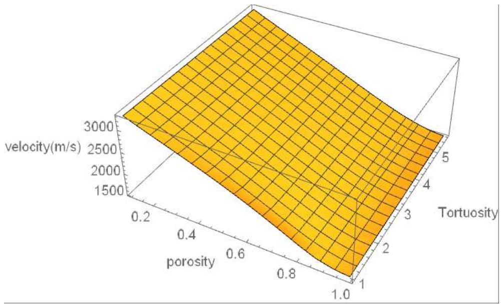

Fig. 1. The fast wave velocity is plotted against porosity and tortuosity

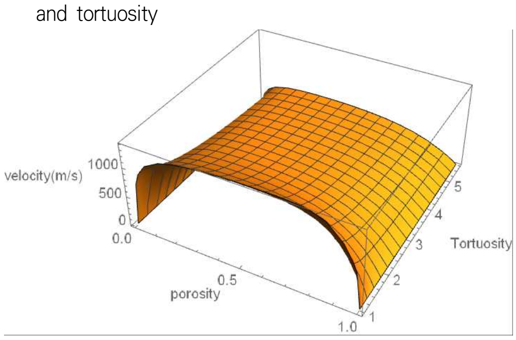

Fig. 2. The slow wave velocity is plotted against porosity and tortuosity

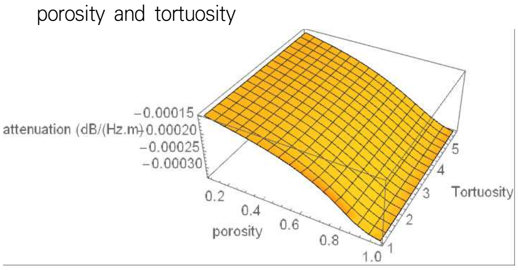

Fig. 3. The attenuation of fast wave is plotted against porosity and tortuosity

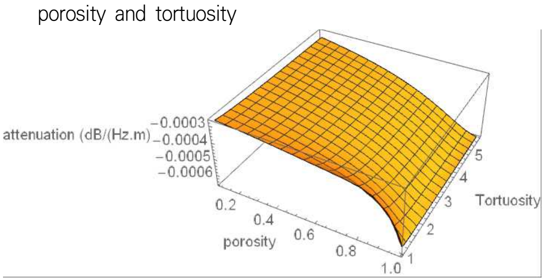

Fig. 4. The attenuation of slow wave is plotted against porosity and tortuosity

The permeability of the superior-inferior orientation is $4.65\times10^{-10}m^2$, that of the anterior-posterior orientation is $4.52\times10^{-10}m^2$, and that of medial-lateral orientation is $2.33\times10^{-10}m^2$, respectively.

However, the permeability doesn't affect both wave velocity and attenuation, when poroelastic theory (or Biot theory) is employed, but only velocity and attenuation of slow wave are affected by tortuosity. The fast wave is believed to penetrate through the trabecular struts (or solid phase) and it is believed that the slow wave goes along the fluid phase in cancellous bone.

From this observation, we can conclude that the tortuosity of trabecula does not affect the wave penetration along the trabecular struts or solid matrix of cancellous bone, but the wave along the fluid phase is influenced by tortuosity. It means that the wave along the solid phase is influenced not by tortuosity, but the wave along the fluid wave is affected by tortuosity significantly, especially for low tortuosity.

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