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Integrating Digital Technology into Elementary Mathematics: Three Theoretical Perspectives

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In this article, the author's intent is to begin a conversation centered on the question: How was the integration of digital technology into elementary mathematics classrooms framed? In the first part of the discussion, the author provides a historical perspective of the development of theoretical perspectives of the integration of digital technology in learning mathematics. Then, the author describes three theoretical perspectives of the role of digital technology in mathematics education: microworlds, instrumental genesis, and semiotic mediation. Last, based on three different theoretical perspectives, the author concludes the article by asking the reader to think differently.

Keywords: technology, microworlds, instrumental genesis, semiotic mediation.
MESC Classification: D22, U12
MSC2010 Classification:97C80, 97U70

I. INTRODUCTION

The growth of technology has been explosive in society. The technology changes not only how people access information but also how they think. This change also influences the identity of students by their experience within the technological environment (Dodge et al., 2008).

Many stakeholders have emphasized the incorporation of technology into mathematics education. National Council of Teachers of Mathematics (NCTM) suggested the integration of technology in the teaching and learning of mathematics by Technology Principle (NCTM, 2000): "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." Common Core State Standards for mathematics (Common Core State Standards Initiative, 2010) also pointed out a strategical use of appropriate tools including "a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software." (p. 7).

Digital technology has been increasingly integrated into the mathematics classroom, and this integration has piqued researchers' interests in understanding the role of technology in teaching, learning, and doing mathematics. Most studies show plenty of evidence that digital technology allows visualizing students' mathematical ideas (e.g., Moreno-Armella, Hegedus, & Kaput, 2008), to facilitate organizing and analyzing data (e.g., Konold, Harradine, & Kazak, 2007), and to support investigation in various content areas (e.g., Dick & Hollerbrands, 2011). Kaput (1992) argued that this is possible since "the new electronic media now afford a whole new class of dynamic, interactive notations of virtually any kind" (p. 522). Building on Kaput's foundation, Roschelle, Noss, Blikstein, and Jackiw (2017) asserted that such dynamism combined generic factors (e.g., digital, interactive, multimedia) helps students attain a clear mathematical relationship as a new path to conceptual understanding.

In this paper, I investigated how children make use of the technology when they are learning *with* it, not *from* it[1]. The focus of this paper is the incorporation of the technology to enhance mathematical learning rather than the technology itself or the pedagogy impacted by technology. To revisit the empirical studies on technology, I drew on three theoretical perspectives. The first perspective is the "microworld" that students construct and reconstruct elements of the environment based on a visible programming language. The second is the "instrumental" perspective that considers how a tool changes from an artifact to an instrument in the hands of a student. The third is the "semiotic" perspective that refers to how a tool mediates the children's mathematical understanding. These theoretical frameworks are also connected to relevant learning theories, which leverage the impact of technology (Kaput, 1992). In the next sections, I outline the theoretical framing for the paper and describe the sub-categories related to mathematical activities, providing detailed reviews to support each category with comparisons and contrasts of approaches, methods, and findings. Specifically, six empirical studies in the elementary school level were selected to reframe the role of digital technology in learning mathematics.

II. THEORETICAL FRAMING

The theoretical framework on technology in mathematics education has progressed along with the development of research paradigm from classifying tools to understanding the relationship among educational subjects. A few decades ago, with the advent of

[1] This expression was adapted from Howland, Jonassen, & Marra (2011, p. 5). The perspective of learning *from* technology is regarded as conveying academic knowledge to students via technology. On the other hand, the perspective of learning *with* technology focuses on supporting role for students' productive and meaningful thinking (pp. 5-7).

microcomputers, Taylor (1980) suggested "tutor, tool, and tutee" framework and categorized the fundamentally different roles of the computer: tutor role refers that students learn from computers, tool role refers that computers assist students to solve tasks, and tutee role refers that students learn through computer programing. A little later, Pea (1987) introduced cognitive technology to "transcend the limitations of the mind" (p. 910) and argued that reorganizing (two-way between technology and students) mathematical thinking with technology has more possibilities than the amplifying perspective (one-way from technology to students). More recently, Zbiek, Heid, Blume, and Dick (2007) accounted for the impact of technology by exploring its interactions with students, mathematical activities, mathematical representations, and mathematical content.

In this paper, Drijvers, Kieran, and Mariotti (2010) inspired me to develop a theoretical framework for integrating technological tools into mathematics education. They provided a historical overview which has contributed to bringing a new perspective to the field (e.g., microworld, white-black box). Then, they specifically focused on the theory of instrumental approach and the theory of semiotic mediation "to guide the design of teaching, to understand learning, and to improve mathematics education" (p. 90). Based on those perspectives, the three categories were selected as the theoretical framework: microworld, instrumental genesis, and semiotic mediation.

In the first category, "*microworlds*" (e.g., Logo) are defined by Papert (1980) as "a place, a province of Mathland, where certain kinds of mathematical thinking could hatch and grow with particular ease" (p. 125). The fundamental idea is based on the notion of constructionism, which refers learners reconstruct mental model for understanding (Papert & Harel, 1991). From this perspective, students learn through participation in programming-based learning where they make connections between different ideas and areas of knowledge (i.e., reconstructing). This process of learning is also promoted by teachers' guided facilitation rather than one-way lectures. Noss and Hoyles (1996) traced the origin of microworlds in mathematics education and reported novelty and pitfalls in using microworld programs. When teachers use the microworld programs in the mathematics classroom, those afford deep engagement and more opportunities to express the mathematical ideas on a monitor screen. Yet, a concern with the balance between guidance and investigation also emerged. Teachers can give enough time to manipulate and to play with the objects in the microworld environment, but students might have trouble to focus on mathematical ideas without reflecting on the relationship between the objects and the structure. With respect to the trend of studies on microworlds, early studies focused on investigating the symbolic notation in programming environments (Jones, 1998; Kynigos, Koutlis, & Hadzilacos, 1997; Sacristan, 2001). Then, researchers designed their own microworlds to understand how students make the relationship between symbolic and graphical representations (e.g, Mathsticks: Noss, Healy, & Hoyles, 1997; Turtle Mirros: Hoyles & Healy, 1997). The significance of the studies regarding microworlds has been transited from how to solve the given tasks in the microworld-program to how to design effective learning environments of microworlds.

In the second category, "*instrumental genesis*" refers to the process that an artifact becomes a part of an instrument (Artigue, 2002). In other words, an artifact influences the mental processes and impacts on the mathematical activity, and the artifact can be connected to the students' cognitive processes, then the artifact will become an instrument (artifact + scheme). This relationship between the artifact and the students is bidirectional. Here, it is important to distinguish between the artifact and the instrument (Verillon & Rabardel, 1995). The artifact has an apparent and a practical role, whereas the instrument is transformed from the artifact when the student shapes the tool for the purpose (*instrumentalization*) and the student's understanding is shaped by the tool (*instrumentation*). For example, a bucket is used to hold some materials (artifact). A student might estimate how much this bucket can store a specific material and recognize what material is not appropriate to hold with the bucket (instrumentation). The student also might change the form of the bucket such as attaching a string to hold it easily (instrumentalization). The gist of the instrumental approach is both utilizing the technology according to one's own purpose and understanding mathematics of the technology. Furthermore, in the classroom situation, each individual constructs their own instrumental genesis and collaborates with them. Trouche (2004) termed "*instrumental orchestration*", which means that the teachers organize the various artifacts on intention and with systematic ways to develop student's instrumental genesis in technology-based learning environments.

In the third category, "*semiotic mediation*" is the process of meaning-making from the signs. Vygotsky (1978) differentiated between tools and signs as follows: "The sign acts as an instrument of psychological activity in a manner analogous to the role of a tool in labor" (p. 52). The signs (e.g., visual mediators) are symbolic tools to be used in the psychological operation, as a practical tool in the labor. The signs generated by the use of technology are internalized and directed to other signs (e.g., words, drawings, and gestures) in the social activity. Bartolini and Mariotti (2008) described semiotic mediation as a process of meaning-making through internalizing the signs that are produced from an external, interpersonal activity. For example, Dynamic Geometry Software (DGS) produces many signs (Mariotti, 2009). An external, goal-oriented activity such as dragging and tracing in DGS can be internalized to construct individual meanings. The use of dragging and tracing tools also facilitates a whole class discussion to mediate the collaborative meaning-making process in the activities. Arzarello and Robutti (2008) termed the "semiotic bundle" as the collection of the signs and their reciprocal relationships. Therefore, the goal of teaching is to generate mathematical signs through the meaning-making process of the learners.

In order to revisit existing studies from the theoretical framework. When I was searching for the studies to analyze from the framework, I used some keywords to restrict the boundary of studies such as elementary school mathematics, technology, high-quality journals (Nivens & Otten, 2017), empirical study, and conceptual understanding. The searching process funneled down with six studies. The three categories had pertained with two studies each: microworlds (Clements & Battista, 1990; Thompson, 1992), instrumental perspective (Freiman, Polotskaia, & Savard, 2017; Soury-Lavergne & Maschietto, 2015), and semiotic perspective (Bartolini Bussi & Baccaglini-Frank, 2015; Ng & Sinclair, 2015). In the following sections, after briefly summarizing each study, I found clear evidence to compare the main characteristics of the theoretical categories and contrast research methodologies and major findings relevant to the framework.

III. THREE THEORETICAL PERSPECTIVES

1. MICROWORLDS

As the first theoretical category, microworlds focused on designing a computerized environment to promote mathematical ideas instead of merely giving programming statements. I selected two empirical studies (Clements & Battista, 1990; Thompson, 1992) targeted on fourth-grade mathematics classrooms. Clements and Battista (1990) investigated whether the Logo programming experience affected children's development of geometric concepts (e.g., angle, angle size), progress in geometric thinking (from visual to analytic level), and problem-solving skills. Thompson (1992) suggested that the involvement of Block Microworld (digital base-ten blocks) helped students engage in mathematical tasks and contributed to constructing the meaning of decimal numbers.

The main feature of microworlds is to bridge between a symbolic representation of the 'objects' and the 'operations' and the 'dynamic visual representation'. In Logo programming (Clements & Battista, 1990), for example, students commanded a 'turtle' to construct a square (dynamic visual representation), determining the exact length and the degree of rotation (objects and operations). In the same vein, *Block Microworld* (Thompson, 1992) allowed students to use the transformation (dynamic visual representations) of 1 'Long' to 10 'Singles' (objects) by using the 'Separate' tool (operations). Using these multiple representations, students can have opportunities to predict and act on mathematical objects from one modality to various forms of modality with animated feedback.

Regarding the research design of the implementation of the microworld programs, the systematic relationship between mathematical tasks and activities was essential to develop intended conceptual knowledge, since children may not be spontaneously “bumping into mathematical ideas, and indeed doing mathematics without even being aware of it” (Hoyles & Noss, 1996, p. 188). Clements and Battista (1990) implemented two 40-minute sessions per week for 40 sessions and each session focused on a variety of programming tasks in the Logo as a problem-solving environment. For example, when children conceptualized the angle concept with the Logo program, the first task was to construct angles with the ‘Angle’ tool which inputs two numerical values, the side length and the measurement of an angle. Students engaged in this task to construct diverse figures such as triangles, squares, and stars. Considering the connection from the former task, the second task was to estimate the measurement of the angle from the given random size angles on the screen. Then, they constructed the same size angle in the Logo programming. Thompson (1992) emphasized the extensive relationship between in-class instructions and homework. For example, when students learned various solutions for the addition problems, they also solved addition problems with many strategies as homework. In order to connect each day’s lesson, the homework problems could be started with the concrete actions previously taken by a student who used invented algorithm to solve one of the problems. To emphasize the need for multiple problem situations, Hoyles and Noss (1996) suggested that teaching with microworlds should provide several (or even many) kinds of situations related to all properties of a concept.

The role of the microworld programs is to enrich children’s mathematical conceptualization and the sophistication of their mathematical thinking. Regarding the development of conceptual understanding, both studies reported the students’ progress in the post-interview compared with the pre-interview. Clements and Battista (1990) articulated that the Logo programming contributed to developing children’s mathematically accurate and general conceptualizations of angles during the treatment. In the first and second interview, all children had a weak concept of the angles but the last interviews showed that the Logo group students had clearer and more accurate concept of the angles than the control group. This was because of the informal experience with rotating and constructing angles in the Logo context linked to the formal instructions about angle measurement. As a result, students could conceptualize the angles as the union of two lines or rays, not just tilted lines. Thompson’s (1992) study showed the evidence of the sophisticated mathematical thinking. He stated, “Students in the microworld group made repeatedly reference to actions on symbols as referring to actions on blocks (e.g., “Borrow a thousand so that we can break up a cube”) (p. 143). Students’ orientation toward the symbols might influence the operation of digital blocks. The experimental group transferred knowledge from acting on their notational activities to acting on digital blocks. The relationships between mathematical knowledge and digital objects in microworlds become more prominent through their experience.

2. INSTRUMENTAL GENESIS

The instrumental perspective focuses on the development of knowledge through the bidirectional nature of instrumental genesis: instrumentalization (a student's knowledge influences how to use the artifact) and instrumentation (a student's reasoning is guided by affordances and constraints of the artifact). Two studies (Freiman, Polotskaia, & Savard, 2017; Soury-Lavergne & Maschietto, 2015) are selected as examples of instrumental genesis in the second-grade mathematics classrooms. As the first case, Freiman, Polotskaia, and Savard (2017) investigated how computer-based story problems help students engage in analyzing mathematical structures and relationships with algebraic thinking rather than calculating results. As the second case, Soury-Lavergne and Maschietto (2015) studied the use of the digital graphical space as a bridge between the physical space and the geometrical space and the role of digital technology to construct geometrical knowledge built on spatial knowledge.

When students are given an artifact to solve a mathematical task, the process of externalizing (i.e. instrumentalization) and internalizing (i.e. instrumentation) is iterative between the student and the artifact. In the first case, Freiman, Polotskaia, and Savard (2017) believed students could solve story problem with generalized strategies through the use of the expression with letters. Within the computer environment, students solved a task, "Sarah had s apples. She ate f apples in d minutes. How many apples does she have now?" The goal of this task was to make a mathematical expression with dragging the letters (s, f, d) and the operational symbols (e.g., +, -); the solution of the task was (s - f). This computer-environment task situated students to construct an expression with algebraic representations (instrumentation), rather than to simply calculate the solution. Even though students have an option to replace the letters with numerical values, the environment allows them to keep focusing on the analysis of the problem and the general structure (part-part-whole). In the second case, Soury-Lavergne and Maschietto (2015) explored the use of the digital space designed by the 'Cabri Elem' (graphical space, see Figure 1) to connect the green carpet (physical space) and the concepts of lines and intersections (geometrical space), focusing on grids. The "*Cabri and the frog*" task, manipulating the frog through the grid to reach Cabri (a goat in French), was given to students. At first, the students recognized the grid as an object that belongs to the digital graphical space (instrumentalization). Then, the grid was considered as a geometrical instrument to solve a specific problem (instrumentation). The use of digital technology, here, can support the construction of connections between physical and geometrical areas by instrumental genesis.

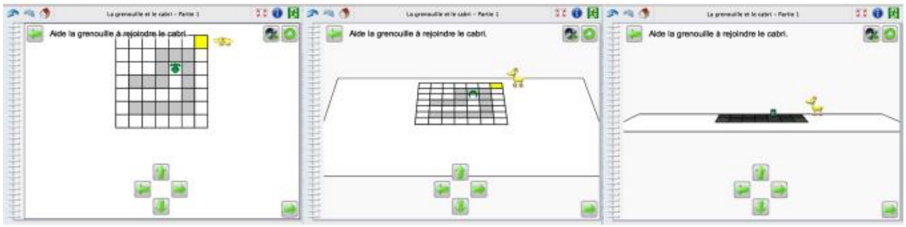

Figure1. The “Cabri and the frog” task (Soury-Lavergne & Maschietto, 2015, p. 440)

Interestingly, both studies had the various levels of the phase in the research design from offline learning to online learning. In the first case, Freiman, Polotskaia, and Savard (2017) designed two phases. The first phase was to solve addition story problems by using linear representations with pencil-and-paper, and the second phase was introduced by the computer-environment task with the story problems of the same addition structure. In the offline learning phase, the use of representations helped students develop mathematical connections and relational thinking. The online learning phase also supported the development of relational thinking toward algebraic thinking. In the second case, Soury-Lavergne and Maschietto (2015) designed three stages. The first stage was to solve a task on the carpet without the grid, the second was on the carpet with the grid, and the final stage was in the digital space. The grid in the three different spaces could be a tool to solve the locating tasks, but the construction of spatial knowledge was not enough with only one stage. The students did not recall the grid concept at the third stage, although they had the earlier experience in the physical space. These comparisons between offline and online learning apparently show how digital technology has an impact on other physical and abstract activities.

Compared with the pen-and-pencil environment, dynamic representations could be the most distinguishable feature in the use of digital technology. However, beyond the dynamism, one of the key affordances of digital technology is to provide continuous and instantaneous feedback on the computer-environment activities (Mackrell, Maschietto, & Soury-Lavergne, 2013). For example, children can keep checking whether their answer is valid in the computer-based learning task (Freiman, Polotskaia, & Savard, 2017). This type of feedback is called the “evaluation feedback” which refers to the systemic assessments of the students’ answer. As the other example, the “Cabri and the frog” task gave the feedback on whether or not the frog placed in the appropriate places on the grid

(Soury-Lavergne & Maschietto, 2015). These affordances of feedback are to provide students with the autonomy of mathematical activities in order to adapt the level of each child toward better understanding and learning. Feedback is also a key part to design learning environments which are different from the physical learning space.

3. SEMIOTIC MEDIATION

Mediation through dynamic mediators which are generated by a digital technology invokes mathematical relationships and properties to students. I chose two cases (Bartolini Bussi & Baccaglini-Frank, 2015; Ng & Sinclair, 2015) targeted on first-grade students as well as geometry. The first case is investigating the use of a programming robot (the bee-bot) for children's transition from a dynamic perception of paths to seeing paths as geometric figures, focused on squares and rectangles (Bartolini Bussi & Baccaglini-Frank, 2015). The second case is addressing the question of (a) how children's geometric conceptions of symmetry emerged and transformed through language, gestures, and the use of technology and (b) what role of dynamic digital technology has in the learning of symmetry from the semiotic perspective (Ng & Sinclair, 2015).

Extending the Vygotskian perception on signs as symbolic tools, Bartolini Bussi and Mariotti (2008) suggested the three categories of signs: artifact, mathematical, and pivot signs. *Artifact signs* are generated in the context of the use of the artifact and close to the artifacts itself. *Mathematical signs* are generated in the sophisticated mathematical context. *Pivot signs* refer to bridging between the artifact signs and the mathematical signs. Both cases described these three signs in their own contexts. For example, in the first case (Bartolini Bussi & Baccaglini-Frank, 2015), children used the arrow signs (↶,↑,↓,↷), which stemmed from the command-icons on the bee-bot's back, to represent paths as sequences of commands (e.g., a square: ↑↑↷↑↑↷↑↑↷↑↑↷). Here, the turning arrow (↷) was the pivot sign which connected the artifact signs (e.g., the command-icon on the bee-bot, the action of turning right in the path on the floor) and the corresponding mathematical signs (e.g., a right angle of the square). In the second case (Ng & Sinclair, 2015), children manipulated squares and/or a symmetry line in "The Symmetry Machine" activity, dragging squares and observing what happens to the corresponding symmetry points on the interactive whiteboard. The actions to move squares as the pivot sign produced both the artifact signs (the movement of a particular square) and the mathematical signs (the square as a mathematical object with the line of symmetry) as the following statement: "it will move like opposite, like this one will move to the windows, and this one will move to the wall" (p. 432).

To delve into semiotic mediation through digital technology, the studies (Bartolini

Bussi & Baccaglini-Frank, 2015; Ng & Sinclair, 2015) were designed not only to do teaching experiments as long-term projects but also to collect a comprehensive set of semiotic evidence. The researchers developed a series of activities based on mathematical microworlds (e.g., the bee-bot programming, Sketchpad) and used teaching experiments to "sow seeds" for the development of inclusive mathematical definitions (e.g., squares, symmetry). In keeping with the semiotic perspective, it was critical for children to make their own signs such as speaking, writing, drawing, gesturing, and acting on digital technology. Specifically, the data of the first case (Bartolini Bussi & Baccaglini-Frank, 2015) included the collection of protocols, photos, graphical productions, whereas the second case (Ng & Sinclair, 2015) collected the recorded videos which were also transcribed with languages, gestures, and drawings verbatim. The authors of both cases strove to collect language, gestures, and diagrams as a bundle in order to gain insights on how children learn mathematics in dynamic and multimodal ways.

In the process of semiotic mediation, visual representations, gestures and oral, written languages are obviously crucial mediators to facilitate the whole classroom discussions (Battista, 2008; Kaur, 2015). More importantly, the use of technology activates new types of potential mediators. In the first case, children could enhance their perception of a path with the periodic eye-blinking and beep sound of the bee-bot. Since the bee-bot produced the blinking and the sound in starting and ending points each, those visual and audible patterns made children reflect the action of the bee-bot related to geometrical meaning. In the second case, the dragging tool on the Sketchpad was utilized to mediates children with the dynamic action of the line of symmetry and the image squares. This tool allowed the children to attain the main features of symmetry: the shape of one side is the same as the other; a component is in the same distance as the counterpart; pre-image and image make a pair.

IV. CONCLUSIONS AND DISCUSIONS

The impact of digital technology on mathematics education has changed the nature of the relationship between mathematical thinking and tools. This change also brings the creation of new goals for what students can do with the deep understanding of tools to make progress on their technique and scheme in learning mathematics. In this study, I investigated the roles of digital technology in the elementary mathematics classroom. To revisit the existing studies, I used the framework based on the historical perspective of microworlds and the relevant theories in mathematics education with particular attention to instrumentation and semiotic mediation.

Each category focuses on the different roles of digital technology. The first category, microworlds, provides the domain of knowledge for students to explore through interactions between a set of objects, basic operations on the objects, and rules of the operations (Hoyles, 1993). These interactions triggered the reconstruction of students' knowledge structure. For example, Thompson (1992) suggested evident evidence of the assimilation from already developed structure into the newly investigating domain of knowledge. The Block microworlds group students assimilated decimal number system from whole-number and the arithmetic operations on the whole-numbers, when they operate with the digital base-ten block on the screen. The further use of the microworld would allow transforming the complex objects and operations into new objects and operations. This showed the development of students' knowledge structure along with the evolvement of the microworld reciprocally. The second category, instrument genesis, enriches children's techniques to use digital technology as well as mental schemes through the instrumentation. From the example of using the digital graphical space (Soury-Lavergne & Maschietto, 2015), the online grid enhances not only the geometrical reasoning in children's problem-solving processes but also the interchangeability between 2D and 3D. The third category, semiotic mediation, concentrates on children's own signs in using digital technology. The dynamic manipulation produces potential semiotic signs which trigger a different type of mediation. For example, children realized new mathematical properties (equidistant from the line of symmetry) by dragging a square and the line of symmetry dynamically (Ng & Sinclair, 2015).

Due to different theoretical backgrounds, the design of studies is varied. In earlier microworld studies, there was no description and analysis of the microworld programs themselves and how they worked. The main focus on microworlds was how children made progress between pretest and posttest through using such technology. However, this approach has been challenged by other theories. The instrumental and the semiotic perspective bridge between the use of technology and children's learning practice in mathematics classrooms such as whole group discussions, group collaboration, and problem-solving, rather than the laboratory environment. In addition, these perspectives appreciate both online and offline learning phases with the investigation of the similarity and connection between them. However, the way how to analyze the relationship of both phases is different from each other. The instrumental perspective emphasizes the comparison between two phases with the same role of tools (e.g., how is the authentic experience with online grid tool different from the corresponding of the offline carpet activities?). On the other hand, the semiotic perspective emphasizes to collect student's own signs in order to recognize the potential mediator in the learning process (e.g., how does the semiotic potential of the bee-bot influence the conceptualization of the definition of squares?).

No theoretical framework can explain all phenomena in the complex learning situation.

To make ‘the whole picture’, additional studies from alternative perspectives on the use of technology in mathematics classrooms are necessary. Across all selected studies in this paper, along with the recognition of the technology-based tools, the relationship between tools and students, and the activity structure, it is also important to understand how students are influenced by the design of the tools and tasks fundamentally. Since this design factor is a sensitive means for student’s interaction with tools, we need more investigations how the tasks and tools can facilitate students’ meaningful mathematics learning based on the interactive design principles such as visibility, feedback, constraints, consistency, and affordance (Preece, Rogers, & Sharp, 2015): how the instrumental genesis would be facilitated by the design of software? how can the design of tools make the pivotal signs bridge between artifact signs and mathematical signs effectively in the process of meaning-making from the semiotic perspective?

With respect to the instrumental genesis, there is a room for further studies. The main focus of this perspective is the relationship between an individual student and an artifact. However, the mathematics lessons in the classroom consist of a variety of the social interactions among other students and teachers. The artifacts themselves also have the social aspects since they are the products of social experience such as activities and contexts (Trouche, 2004). As a result, we can think about the following questions to understand the instrumental perspective comprehensively: what is the role of language and discourse in the instrumental genesis? What is the role of the teacher in the technology-integrated mathematics classroom? To what extent is the instrumental genesis influenced by the learning environment?

A new theoretical perspective enables us to ask refined questions about the role of technology. Consider a study to find the better way for children to learn fractions with a given technology. Instead of thinking whether children learn better and effectively, we can focus on the new technical-conceptual schemes that the students developed in using the technology from the instrumental perspective; on the mathematical signs that students generated with the technology. Technology in mathematics education is not an additional ‘artifact’ but the ‘infrastructure’ as technological environments.

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