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# AN APPLICATION OF BINARY SOFT MAPPINGS TO THE PROBLEM IN MEDICAL EXPERT SYSTEMS

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Abstract. We initiate and introduce the notion of binary soft mapping, which is defined on collection of binary soft sets named as binary soft class over two initial universes $U_1$ and $U_2$ with fixed set of parameters. We also define and study the properties of binary soft images and binary soft inverse images of binary soft sets. Examples and counter examples are also given in support of presented properties. Moreover, these concepts are applied to the problem of medical diagnosis in medical expert systems.

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## 1. Introduction

Binary soft sets and binary soft topology is found to be very important during the study towards possible applications in classical and non classical logic. Classical methods are not always successful because of various types of complexities of modelling uncertain data in engineering, medical science, economics, sociology and other fields of life. In [19], Zadeh presented new theory of fuzzy sets, which is considered as a most suitable framework to handle with uncertain data. To describe uncertainty, mathematical tools such as probability theory, rough set theory [18] etc., are also considered as useful approaches. In [15-16], Molodtsov et. al highlighted inherent difficulties in each of these theories and initiated a refined approach of soft sets theory for modelling vagueness and uncertainty. Soft set theory has potential applications in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory. In [13] and [14], Maji et. al introduced and discussed many basic notions of soft set theory and applied them in a multicriteria decision making problems. For detailed study

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of algebraic structures of soft sets and soft topological spaces as well as their applications, interested readers are refer to [1],[3] [5-6], [8-9], [11-12] and [17].

In [2], Ackgoz and Tas initiated binary soft set over two initial universal sets and a parameter set and studied its properties. In [5], Hussain introduced binary soft topological spaces, which are, in fact, generalization of soft topological spaces in broader sense and are defined over two initial universes $U_1$ and $U_2$ with fixed set of parameters. In [7], Hussain initiated and explored binary soft connectedness in binary soft topological spaces and presented application of binary soft sets in decision making problems as well.

In this paper, we initiate and introduce the notion of binary soft mapping, which is defined on collection of binary soft sets named as binary soft class over two initial universes $U_1$ and $U_2$ with fixed set of parameters. We also define and study the properties of binary soft images and binary soft inverse images of binary soft sets. Examples and counter examples are also given in support of presented properties. Moreover, these concepts are applied to the problem of medical diagnosis in medical expert systems.

## 2. Preliminaries

**Definition 2.1.** [16] Let $X$ be an initial universe and $E$ be a set of parameters. Also let $P(X)$ denotes the power set of $X$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \to P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $e \in A$, $F(e)$ may be considered as the set of $e$-approximate elements of the soft set $(F, A)$.

**Definition 2.2.** [8] Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is said to be a soft topology on $X$, if
(1) $\Phi$, $\tilde{X}$ belong to $\tau$.
(2) the union of any number of soft sets in $\tau$ belongs to $\tau$.
(3) the intersection of any two soft sets in $\tau$ belongs to $\tau$.
The triplet $(X, \tau, E)$ is called a soft topological space over $X$.

Consider $U_1$ and $U_2$ are two initial universal sets, E be a set of parameters and $P(U_1)$, $P(U_2)$ denote the power sets of $U_1$ and $U_2$ respectively. Also, let $A, B, C \subseteq E$.

**Definition 2.3.** [2] A pair $(F, A)$ is said to be a binary soft set over $U_1, U_2$, where $F : A \to P(U_1) \times P(U_2)$ is defined by, $F(e) = (X, Y)$, for each $e \in A$ such that $X \subseteq U_1, Y \subseteq U_2$.

**Definition 2.4.** [2] Let $(F, A)$ and $(G, B)$ are two binary soft sets over the universes $U_1, U_2$. $(F, A)$ is called a binary soft subset of $(G, B)$, if
(1) $A \subseteq B$,
(2) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ such that $F(e) = (X_1, Y_1)$, $G(e) = (X_2, Y_2)$, for each $e \in A$ such that $X_1, X_2 \subseteq U_1, Y_1, Y_2 \subseteq U_2$.

We denote it by $(F,A)\tilde{\tilde{\subseteq}}(G,B)$. $(F,A)$ is called a binary soft super set of $(G,B)$, if $(G,B)$ is a binary soft subset of $(F,A)$. We write $(F,A)\tilde{\tilde{\supseteq}}(G,B)$. $(F,A)$ is called binary soft equal to $(G,B)$, if $(F,A)$ is binary soft subset of $(G,B)$ and $(G,B)$ is binary soft subset of $(F,A)$. We denote it by $(F,A)\tilde{\tilde{=}}(G,B)$.

**Definition 2.5.** [2] The complement of binary soft set $(F,A)$ is denoted by $(F,A)^c$ and is defined $(F,A)^c = (F^c, \rceil A)$, where $F^c : \rceil A \to P(U_1) \times P(U_2)$ is a mapping given by $F^c(e) = (U_1 - X, U_2 - Y)$ such that $F(e) = (X,Y)$, for each $e \in A$ with $X \subseteq U_1, Y \subseteq U_2$. Clearly, $((F,A)^c)^c = (F,A)$.

**Definition 2.6.** [4] A binary soft set $(F,A)$ over $U_1, U_2$ is called binary absolute soft set denoted by $\tilde{\tilde{A}}$, if $F(e) = (U_1, U_2)$, for each $e \in A$.

**Definition 2.7.** [4]A binary soft set $(F,A)$ over $U_1, U_2$ is called binary null soft set denoted by $\tilde{\tilde{\Phi}}$, if $F(e) = (\Phi, \Phi)$, for each $e \in A$.

**Definition 2.8.** [2] Intersection of two binary soft sets $(F,A)$ and $(G,B)$ over the universes $U_1, U_2$ is the binary soft set $(H,C)$, where $C = A \cap B$, and $H(e) = (X_1 \cap X_2, Y_1 \cap Y_2)$, for each $e \in C$ such that $F(e) = (X_1, Y_1)$, for each $e \in A$ and $G(e) = (X_2, Y_2)$, for each $e \in B$ such that $X_1, X_2 \subseteq U_1, Y_1, Y_2 \subseteq U_2$. We denote it by, $(F,A)\tilde{\tilde{\cap}}(G,B)\tilde{\tilde{=}}(H,C)$.

**Definition 2.9.** [2] Union of two binary soft sets $(F,A)$ and $(G,B)$ over the universes $U_1$, $U_2$ is the binary soft set $(H,C)$, where $C = A \cup B$, and for each $e \in C$ such that $X_1, X_2 \subseteq U_1, Y_1, Y_2 \subseteq U_2$,

$$H(e) = \begin{cases} (X_1, Y_1), & \text{if } e \in A - B \\ (X_2, Y_2), & \text{if } e \in B - A \\ (X_1 \cup X_2, Y_1 \cup Y_2), & \text{if } e \in A \cap B \end{cases},$$

such that $F(e) = (X_1, Y_1)$, for each $e \in A$ and $G(e) = (X_2, Y_2)$, for each $e \in B$. We denote it by $(F,A)\tilde{\tilde{\cup}}(G,B)\tilde{\tilde{=}}(H,C)$.

**Definition 2.10.** [4] The binary soft set $(F,E)$ is called a binary soft point over $U_1, U_2$ denoted by $e_F$, if for the element $e \in A$, $F(e) \neq (\Phi, \Phi)$ and $F(e') = (\Phi, \Phi)$, for all $e' \in E - \{e\}$.

**Definition 2.11.** [4] Let $\tau$ be the collection of binary soft sets over $U_1, U_2$ and $E$ denotes the set of parameters. Then $\tau$ is said to be binary soft topology over $U_1, U_2$, if

(1)$\tilde{\tilde{\Phi}}, \tilde{\tilde{E}}$ belong to $\tau$.

(2) the union of any number of binary soft sets in $\tau$ belongs to $\tau$.

(3) the intersection of any two binary soft sets in $\tau$ belongs to $\tau$.

The $(U_1, U_2, \tau, E)$ is called a binary soft topological space over $U_1, U_2$ and the members of $\tau$ are said to be binary soft open sets over $U_1, U_2$. A binary soft set $(F,E)$ over $U_1, U_2$ is said to be a binary soft closed set over $U_1, U_2$, if its binary soft relative complement $(F,E)^c$ belongs to $\tau$.

## 3. Binary Soft Mappings on Binary Soft Classes

**Definition 3.1.** The binary soft class denotes as $(U_1, U_2, E)$ and is the collection of all binary soft sets over $U_1, U_2$ with parameters from $E$. Where $U_1, U_2$ be two universes and $E$ a set of parameters.

**Definition 3.2.** Suppose $(U_1, U_2, E)$ and $(V_1, V_2, E')$ are two binary soft classes. Let $u = m \times n : P(U_1) \times P(U_2) \to P(V_1) \times P(V_2)$; where $m : P(U_1) \to P(V_1)$, $n : P(U_2) \to P(V_2)$ and $P : E \to E'$ are mappings. Then binary soft mapping from binary soft class $(U_1, U_2, E)$ to binary soft class $(V_1, V_2, E')$ is denoted as $f : (U_1, U_2, E) \to (V_1, V_2, E')$ and is defined as: for a binary soft set $(F, A)$ in $(U_1, U_2, E)$, $(f(F, A), B)$; where $B = p(A) \subseteq E'$, is a binary soft set in $(V_1, V_2, E')$ given by

$$f(F, A)(\beta) = u\Big(\bigcup_{\alpha \in p^{-1}(\beta) \cap A} F(\alpha)\Big), \quad for \;\; \beta \in B \subseteq E'.$$

$(f(F, A), B)$ is called a binary soft image of a binary soft set $(F, A)$. If $B = E'$, then we shall write $(f(F, A), E')$ as $f(F, A)$.

**Definition 3.3.** Suppose that $f : (U_1, U_2, E) \to (V_1, V_2, E')$ be a binary soft mapping from a binary soft class $(U_1, U_2, E)$ to another binary soft class $(V_1, V_2, E')$. Let $(G, C)$ be a binary soft set in binary soft class $(V_1, V_2, E')$, where $C \subseteq E'$. Suppose $u : m \times n : P(U_1) \times P(U_2) \to P(V_1) \times P(V_2)$, $m : P(U_1) \to P(V_1)$, $n : P(U_2) \to P(V_2)$ and $p : E \to E'$ are mappings. Then $(f^{-1}(G, C), D)$, where $D = p^{-1}(C)$, is a binary soft set in the binary soft class $(U_1, U_2, E)$ defined as:

$$f^{-1}(G, C)(\alpha) = u^{-1}(G(p(\alpha))), \quad for \;\; \alpha \in D \subseteq E.$$

$(f^{-1}(G, C), D)$ is called a binary soft inverse image of $(G, C)$. Here after we shall write $(f^{-1}(G, C), E)$ as $f^{-1}(G, C)$.

**Example 3.4.** Suppose the following sets:

$$\begin{array}{rr} U_1 = \{t_1, t_2, t_3\}, & U_2 = \{b_1, b_2, b_3\}, \\ V_1 = \{x_1, y_1, z_1\}, & V_2 = \{x_2, y_2, z_2\}, \\ E = \{e_1, e_2, e_3, e_4\}, & E' = \{e'_1, e'_2, e'_3\} \end{array}$$

and $(U_1, U_2, E)$ , $(V_1, V_2, E')$ are binary soft classes. Define $P : E \to E'$, $m : P(U_1) \to P(V_1)$ and $n : P(U_2) \to P(V_2)$ as:

$$\begin{array}{llll} m(t_1) = y_1, & m(t_2) = z_1, & m(t_3) = y_1, & \\ n(b_1) = z_2, & n(b_2) = y_2 & n(b_3) = z_2, & \\ p(e_1) = e'_3, & p(e_2) = e'_3, & p(e_3) = e'_2, & p(e_4) = e'_3. \end{array}$$

Choose two binary soft sets over $U_1, U_2$ and $V_1, V_2$ respectively as:

$$\begin{array}{rcl} (F, A) & = & \{e_2 = \Phi, e_3 = (\{t_2, t_3\}, \{b_1, b_3\}), e_4 = (\{t_1, t_3\}, \{b_2, b_3\})\}, \\ (G, C) & = & \{e'_1 = (\{x_1, y_1\}, \{x_2, y_2\}), e'_2 = (\{y_1\}, \{y_2\})\}. \end{array}$$

Therefore, the binary soft mapping $f:(U_1,U_2,E)\ \rightarrow (V_1,V_2,E')$ is given as: for a binary soft set $(F,A)$ in $(U_1,U_2,E)$; $(f(F,A),B)$, where $B=p(A)=\{e'_2,e'_3\}$, is a binary soft set in $(V_1,V_2,E')$ obtained as:

$$\begin{aligned} f(F,A)e'_2 &= u(\bigcup F(\{e_3\})) \qquad ,(since\ p^{-1}(e'_2)\cap A=\{e_3\}),\\ &= u(\{t_2,t_3\},\{b_1,b_3\})=(\{z_1,y_1\},\{z_2\}) \end{aligned}$$

$$\begin{aligned} f(F,A)e'_3 &= u(\bigcup_{\alpha\in p^{-1}(e'_3)\cap A} F(\alpha)) \qquad (since\ p^{-1}(e'_3)\cap A=\{e_2,e_4\}).\\ &= u(\{F(e_2)\cup F(e_4)\})\\ &= u(\Phi\cup(\{t_1,t_3\},\{b_2,b_3\}))=(\{y_1\},\{y_2,z_2\}). \end{aligned}$$

Thus

$$(f(F,A),B)=\{e'_2=(\{z_1,y_1\},\{z_2\}),e'_3=(\{y_1\},\{y_2,z_2\})\}.$$

Moreover, for the binary soft inverse images, we get

$f^{-1}(G,C)e_3=u^{-1}(G(p(e_3)))=u^{-1}(G(e'_2))=u^{-1}(\{y_1\},\{y_2\})=(\{t_1,t_3\},\{b_2\})$,

where $D=p^{-1}(C)=\{e_3\}$ . Thus, we get

$$(f^{-1}(G,C),D)=\{e_3=(\{t_1,t_3\},\{b_2\})\}.$$

**Remark 3.1.** Note that the null (resp. absolute) binary soft set is not unique in a binary soft space $(U_1,U_2,E)$, rather it depends upon $A\subseteq E$ . Therefore, we denote it by $\tilde{\tilde{\Phi}}_A$(resp. $(\tilde{U}_1,\tilde{U}_2)_A$): If $A=E$, then we denote it simply by $\tilde{\tilde{\Phi}}$ (resp. $(\tilde{U}_1,\tilde{U}_2)$); which is unique null (resp. absolute) binary soft set, called full null (resp. full absolute) binary soft set.

**Theorem 3.5.** *Suppose $f:(U_1,U_2,E)\ \rightarrow (V_1,V_2,E')$ be a binary soft mapping and $u:m\times n:P(U_1)\times P(U_2)\rightarrow P(V_1)\times P(V_2)$; $m:P(U_1)\rightarrow P(V_1)$, $n:P(U_2)\rightarrow P(V_2)$ and $p:E\rightarrow E'$ are mappings. Then for binary soft sets $(F,A)$,$(G,B)$ and a family of binary soft sets $(F_i,A_i)$ in the binary soft class $(U_1,U_2,E)$ , we get*

*(1) $f(\tilde{\tilde{\Phi}})=\tilde{\tilde{\Phi}}$ and $f(\tilde{U}_1,\tilde{U}_2)\ \tilde{\tilde{\subseteq}}\ (\tilde{V}_1,\tilde{V}_2)$.*

*(2) If $(F,A)\ \tilde{\tilde{\subseteq}}\ (G,B)$, then $f(F,A)\ \tilde{\tilde{\subseteq}}\ f(G,B)$ .*

*(3) $f((F,A)\tilde{\tilde{\cap}}(G,B))\ \tilde{\tilde{\subseteq}}\ f(F,A)\tilde{\tilde{\cap}}f(G,B)$ and*
*$f((F,A)\ \tilde{\tilde{\cup}}\ (G,B))=f(F,A)\tilde{\tilde{\cup}}f(G,B)$. In general,*
*$f(\tilde{\tilde{\cap}}_i(F_i,A_i))\ \tilde{\tilde{\subseteq}}\ \tilde{\tilde{\cap}}_i f(F_i,A_i)$ and $f(\tilde{\tilde{\cup}}_i(F_i,A_i))=\tilde{\tilde{\cup}}_i f(F_i,A_i)$.*

*Proof.* (1)This is obvious.
(2) For $\beta\in E'$,

$$f(F,A)\beta=u(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha)).$$

Then

$$f(F,A)\beta = u(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha)) \subseteq u(\bigcup_{\alpha\in p^{-1}(\beta)\cap B} G(\alpha)) = f(G,B)\beta.$$

This proves (2).

(3) For $\beta \in E'$, first we prove that $f((F,A)\tilde{\tilde{\cap}}(G,B))\beta \tilde{\subseteq} (f(F,A)\tilde{\tilde{\cap}}f(G,B))\beta$. For this, consider

$$f((F,A)\tilde{\tilde{\cap}}(G,B))\beta = f(H, A\tilde{\tilde{\cap}}B)\beta = u(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} H(\alpha)),$$

where $H(\alpha) = F(\alpha)\cap G(\alpha)$. Thus

$$\begin{aligned} f(H, A\tilde{\tilde{\cap}}B)\beta &= u(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} H(\alpha)) \\ &= u(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} (F(\alpha)\cap G(\alpha))), \end{aligned}$$

or

$$f((F,A)\tilde{\tilde{\cap}}(G,B))\beta = u(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} (F(\alpha)\cap G(\alpha))).$$

On the other hand, we have

$$\begin{aligned} (f(F,A)\tilde{\tilde{\cap}}f(G,B))\beta &= f(F,A)\beta\tilde{\tilde{\cap}}f(G,B)\beta \\ &= u(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha))\tilde{\tilde{\cap}}u(\bigcup_{\alpha\in p^{-1}(\beta)\cap B} G(\alpha)). \end{aligned}$$

We get

$$\begin{aligned} (f(F,A)\tilde{\tilde{\cap}}f(G,B))\beta &= u(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha))\tilde{\tilde{\cap}}u(\bigcup_{\alpha\in p^{-1}(\beta)\cap B} G(\alpha)) \\ &\supseteq u(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cap B)} (F(\alpha)\tilde{\tilde{\cap}}G(\alpha))) \\ &= f((F,A)\tilde{\tilde{\cap}}(G,B))\beta. \end{aligned}$$

Now we prove the second part of (3). For $\beta \in E'$, we prove that $f((F,A)\tilde{\tilde{\cup}}(G,B))\beta = (f(F,A)\tilde{\tilde{\cup}}f(G,B))\beta$. For this consider

$$\begin{aligned} f((F,A)\tilde{\tilde{\cup}}(G,B))\beta &= f(H, A\tilde{\tilde{\cup}}B)\beta \qquad (say) \\ &= u(\bigcup_{\alpha\in p^{-1}(\beta)\cap(A\cup B)} H(\alpha)), \end{aligned}$$

where

$$H(\alpha) = \begin{cases} F(\alpha), & if \quad \alpha\in A-B\ ; \\ G(\alpha), & if \quad \alpha\in B-A\ ; \\ F(\alpha)\cup G(\alpha), & if \quad \alpha\in A\cap B, \end{cases}$$

such that $F(\alpha) = (X_1, Y_1)$, for each $\alpha \in A$, $G(\alpha) = (X_2, Y_2)$, for each $\alpha \in B$ and $F(\alpha) \cup G(\alpha) = (X_1 \cup X_2, Y_1 \cup Y_2)$. Then

$$f((F,A)\tilde{\tilde{\cup}}(G,B))\beta$$
$$= u(\bigcup \begin{cases} F(\alpha), & if \quad \alpha \in (A-B) \cap p^{-1}(\beta) \\ G(\alpha), & if \quad \alpha \in (B-A) \cap p^{-1}(\beta) \\ F(\alpha) \cup G(\alpha), & if \quad \alpha \in (A \cap B) \cap p^{-1}(\beta). \end{cases} ) \ldots \quad (*)$$

Further, for $\beta \in E'$, we get

$$(f(F,A)\tilde{\tilde{\cup}}f(G,B))\beta$$
$$= f(F,A)\beta\tilde{\tilde{\cup}}f(G,B)\beta$$
$$= u(\textstyle\bigcup_{\alpha \in p^{-1}(\beta)\cap A} F(\alpha))\tilde{\tilde{\cup}}u(\textstyle\bigcup_{\alpha \in p^{-1}(\beta)\cap B} G(\alpha))$$
$$= u(\textstyle\bigcup_{\alpha \in p^{-1}(\beta)\cap A} F(\alpha)\tilde{\tilde{\cup}}\textstyle\bigcup_{\alpha \in p^{-1}(\beta)\cap B} G(\alpha))$$
$$= u(\bigcup \begin{cases} F(\alpha), & if \quad \alpha \in (A-B) \cap p^{-1}(\beta) \\ G(\alpha), & if \quad \alpha \in (B-A) \cap p^{-1}(\beta) \\ F(\alpha) \cup G(\alpha), & if \quad \alpha \in (A \cap B) \cap p^{-1}(\beta). \end{cases} ) \ldots \quad (**)$$

From $(*)$ and $(**)$, we get the second part of (3) as required.
This completes the proof. □

In the following example, we show that equalities do not hold in second parts of (1) and first part of (3), in general, in above Theorem 3.5.

**Example 3.6.** Consider the binary soft mapping $f : (U_1, U_2, E) \to (V_1, V_2, E')$ as defined in Example 3.4. Then for a binary soft set $(\tilde{U}_1, \tilde{U}_2)$ in $(U_1, U_2, E)$; $(f(\tilde{U}_1, \tilde{U}_2), E')$ is a binary soft set in $(V_1, V_2, E')$ obtained as follows:

$$f(\tilde{U}_1, \tilde{U}_2)e'_1 = \Phi \qquad (since\ p^{-1}(e'_1) \cap (\tilde{U}_1, \tilde{U}_2) = \Phi),$$

$$\begin{aligned} f(\tilde{U}_1, \tilde{U}_2)e'_2 &= u(\bigcup F(\{e_3\})) \qquad (since\ p^{-1}(e'_2) \cap (\tilde{U}_1, \tilde{U}_2) = \{e_3\}) \\ &= u(\{t_1, t_2, t_3\}, \{b_1, b_2, b_3\}) \\ &= (\{y_1, z_1\}, \{y_2, z_2\}), \end{aligned}$$

$$\begin{aligned} f(\tilde{U}_1, \tilde{U}_2)e'_3 &= u(\bigcup_{\alpha \in p^{-1}(e'_3)\cap A} F(\alpha)) \quad (since\ p^{-1}(e'_3) \cap (\tilde{U}_1, \tilde{U}_2) = \{e_1, e_2, e_4\}) \\ &= u(\{F(e_1) \cup F(e_2) \cup F(e_4)\}) \\ &= u(\{t_1, t_2, t_3\}, \{b_1, b_2, b_3\})) \\ &= (\{y_1, z_1\}, \{y_2, z_2\}). \end{aligned}$$

Therefore, we get

$$(\tilde{V}_1, \tilde{V}_2)\tilde{\tilde{\nsubseteq}} f(\tilde{U}_1, \tilde{U}_2) = \{e'_1 = \Phi, e'_2 = (\{y_1, z_1\}, \{y_2, z_2\}), e'_3 = (\{y_1, z_1\}, \{y_2, z_2\})\}.$$

This proves that the inverse of second part of inequality (1) is not true.

To prove that the inverse of first part of inequality (3) does not hold, we choose binary soft sets in $(U_1, U_2, E)$ as:

$$\begin{aligned}(F,A) &= \{e_1 = (\{t_2\},\{b_2\}), e_2 = (\{t_2,t_3\},\{b_2,b_3\}), e_3 = (\{t_1\},\{b_2\})\},\\ (G,B) &= \left\{\begin{array}{c} e_1 = (\{t_1\},\{b_1\}), e_2 = (\{t_1,t_2\},\{b_1,b_2\}), e_3 = (\{t_3\},\{b_3\}),\\ e_4 = (\{t_1,t_3\},\{b_1,b_3\})\end{array}\right\}.\end{aligned}$$

Calculations show that

$$\begin{aligned}f(F,A) &= \{e_2' = (\{y_1\},\{y_2\}), e_3' = (\{z_1,y_1\},\{y_2,z_2\})\}.\\ f(G,B) &= \{e_2' = (\{y_1\},\{z_2\}), e_3' = (\{z_1,y_1\},\{y_2,z_2\})\}.\end{aligned}$$

Therefore,

$$f(F,A)\tilde{\tilde{\cap}}f(G,B) = \{e_2' = (\{y_1\},\Phi), e_3' = (\{y_1,z_1\},\{y_2,z_2\})\}... \quad (A^*)$$

Next, for binary soft set $(H,D) = (F,A)\tilde{\tilde{\cap}}(G,B) = \{e_1 = \Phi, e_2 = (\{t_2\},\{b_2\}), e_3 = \Phi\}$ in $(U_1,U_2,E)$; $(f(H,D),W)$, where $W = p(D) = \{e_3'\}$ is a binary soft set in $(V_1,V_2,E')$ obtained as follows:

$$f(H,D) = f((F,A)\tilde{\tilde{\cap}}(G,B)) = \{e_3' = (\{z_1\},\{y_2\})\}.... \quad (A^{**})$$

By $(A^*)$ and $(A^{**})$, we get

$$\begin{aligned}f(F,A)\tilde{\tilde{\cap}}f(G,B) &= \{e_2' = \Phi, e_3' = (\{z_1,x_1\},\{y_2,x_2\})\}\\ \tilde{\tilde{\nsubseteq}}\{e_3' &= (\{z_1\},\{y_2\})\} = f((F,A)\tilde{\tilde{\cap}}(G,B)).\end{aligned}$$

**Theorem 3.7.** *Suppose $f : (U_1,U_2,E) \to (V_1,V_2,E')$ be a binary soft mapping and $u : m \times n : P(U_1) \times P(U_2) \to P(V_1) \times P(V_2)$; $m : P(U_1) \to P(V_1)$, $n : P(U_2) \to P(V_2)$ and $p : E \to E'$ are mappings. Then for binary soft sets $(F,A)$, $(G,B)$ and a family of binary soft sets $(F_i,A_i)$ in the binary soft class $(V_1,V_2,E')$, we get*
*(1) $f^{-1}(\tilde{\tilde{\Phi}}) = \tilde{\tilde{\Phi}}$ and $f^{-1}(\tilde{V}_1,\tilde{V}_2)_{E'} = (\tilde{U}_1,\tilde{U}_2)_E$.*
*(2) If $(F,A)\tilde{\tilde{\subseteq}}(G,B)$, then $f^{-1}(F,A)\tilde{\tilde{\subseteq}}f^{-1}(G,B)$.*
*(3) $f^{-1}((F,A)\tilde{\tilde{\cap}}(G,B)) = f^{-1}(F,A)\tilde{\tilde{\cap}}f^{-1}(G,B)$ and*
*$f^{-1}((F,A)\tilde{\tilde{\cup}}(G,B)) = f^{-1}(F,A)\tilde{\tilde{\cup}}f^{-1}(G,B)$.*
*In general*
*$f^{-1}(\tilde{\tilde{\cap}}_i(F_i,A_i)) = \tilde{\tilde{\cap}}_i f^{-1}(F_i,A_i)$ and $f^{-1}(\tilde{\tilde{\cup}}_i(F_i,A_i)) = \tilde{\tilde{\cup}}_i f^{-1}(F_i,A_i)$.*

*Proof.* (1) This is obvious.
(2) Consider, for $\alpha \in E$, we have

$$\begin{aligned}f^{-1}(F,A)\alpha &= u^{-1}(F(p(\alpha)))\\ &= u^{-1}(F(\beta)), \quad (\textit{ where } \beta = p(\alpha).)\\ &\subseteq u^{-1}(G(\beta)) = u^{-1}(G(p(\alpha)))\\ &= f^{-1}(G,B)\alpha.\end{aligned}$$

This proves (2).
(3) Consider, for $\alpha \in E$

$$\begin{aligned} f^{-1}((F,A)\tilde{\tilde{\cap}}(G,B))\alpha &= f^{-1}(H, A\cap B))\alpha \\ &= u^{-1}(H(p(\alpha))) \qquad (\ p(\alpha)\in A\cap B) \\ &= u^{-1}(F(\beta)\cap G(\beta)) \qquad (\ where\ \beta = p(\alpha)) \\ &= u^{-1}(F(\beta))\cap u^{-1}(G(\beta)) \\ &= u^{-1}(F(p(\alpha)))\cap u^{-1}(G(p(\alpha))) \\ &= (f^{-1}(F,A)\tilde{\tilde{\cap}}f^{-1}(G,B))\alpha. \end{aligned}$$

This proves first part of (3).
Moreover, for $\alpha \in E$.

$$\begin{aligned} & f^{-1}((F,A)\tilde{\tilde{\cup}}(G,B))\alpha = f^{-1}(H, A\tilde{\tilde{\cup}}B)\alpha \\ & = u^{-1}(H(p(\alpha))) \qquad (p(\alpha)\in A\cup B) \\ & = u^{-1}(H(\beta)) \qquad (where\ \beta = p(\alpha)) \\ & = u^{-1}\begin{cases} F(\beta), & if \quad \beta\in A-B\ ; \\ G(\beta), & if \quad \beta\in B-A\ ; \qquad ..(B^*) \\ F(\beta)\cup G(\beta), & if \quad \beta\in A\cap B. \end{cases} \end{aligned}$$

Furthermore, we have

$$\begin{aligned} & (f^{-1}(F,A)\tilde{\tilde{\cup}}f^{-1}(G,B))\alpha = f^{-1}(F,A)\alpha\tilde{\tilde{\cup}}f^{-1}(G,B)\alpha \\ & = u^{-1}(F(p(\alpha)))\cup u^{-1}(G(p(\alpha))) \quad (p(\alpha)\in A\cup B) \\ & = u^{-1}\begin{cases} F(\beta), & if \quad \beta\in A-B \\ G(\beta), & if \quad \beta\in B-A \quad .. \quad (B^{**}) \\ F(\beta)\cup G(\beta), & if \quad \beta\in A\cap B, \end{cases} \end{aligned}$$

where $\beta = p(\alpha)$. From $(B^*)$ and $(B^{**})$, we get the second part of (3).
This completes the proof. □

## 4. An Application in Medical Expert Systems

Keeping in view the medical specialist's opinion, a prime assignment of a medical expert system is patient's problems conversion, symptoms into a set of achievable basis and their respective significance. We may easily encode the case of patient into a binary soft set as:
Consider the patient's description with the medical specialist is as follows:
I have four main problems:
headache, exhaustion, fatigue and dry hair. Because of fatigue, I always feel tired even if I take a long period of rest and I feel unrest in my whole body. I have also some pain in my neck and I suffer from nausea. To a less degree, I also suffer from nose bleeding and eyes pain.

This can be written in the following binary soft set as:

$$(F, A) = \left\{ \begin{array}{c} high\ category = \{(\{headache, exhaustion\}, \{fatigue, dry\ hair\})\}, \\ medium\ category = \{(\{tiredness\}, \{neck\ pain, nausea\})\}; \\ low\ category = \{(\{nose\ bleeding\}, \{eyes\ pain\})\} \end{array} \right\}.$$

The medical knowledge may be encoded in the form of a look-up table. Look-up tables are the computer representation of the notion of mapping in mathematics. Suppose our medical experts have provided us with following knowledge:

$m(headache) = overactivity$,
$m(exhaustion) = lack\ of\ regular\ exercise$,
$m(tiredness) = deficiency\ of\ vitamins$,
$m(nose\ bleeding) = sinus$,

and

$n(fatigue) = stress$,
$n(dry\ hair) = malnutrition$,
$n(neck\ pain) = stifness$,
$n(nausea) = infection$,
$n(eyes\ pain) = bacterial\ Infection$,

also

$p(high\ category) = diabatis\ or\ heart\ disease$,
$p(medium\ category) = Inflammation$,
$p(low\ category) = Inflammation.$

For the case in mathematical manipulation, we denote the symptoms and gradations by symbols as follows:

$h = headache$
$e = exhaustion$
$ns = nose\ bleeding$
$t = tiredness$, $e_1 = high\ category$
$f = fatigue$, $e_2 = medium\ category$
$d = dry\ hair$, $e_3 = low\ category$
$c = neck\ pain$
$a = nausea$
$y = eyes\ pain$

and

$\alpha = overactivity$
$\beta = lack\ of\ regular\ exercise$
$\gamma = deficiency\ of\ vitamins$
$\delta = sinus$, $e'_1 = diabetes\ or\ heartdisease$
$\varepsilon = stress$, $e'_2 = Inflammation$
$\eta = malnutrition$
$\lambda = stiffness$
$\mu = infection$
$\nu = bacterial\ Infection.$

Thus we have two binary soft classes $(U_1, U_2, E)$ *and* $(V_1, V_2, E')$ with $U_1 = \{h, e, t, ns\}$, $U_2 = \{f, d, c, a, y\}, E = \{e_1, e_2, e_3\}$ and $V_1 = \{\alpha, \beta, \gamma, \delta\}, V_2 = \{\varepsilon, \eta, \lambda, \mu, \nu\}$, $E' = \{e'_1, e'_2\}$. $(U_1, U_2, E)$ is the binary soft class of symptoms and their importance for the patient, and $(V_1, V_2, E')$ represents causes and medical preference for treatment. The binary soft set of patient's narration may be given as:

$$(F, A) = \{e_1 = (\{h, e\}, \{f, d\}), e_2 = (\{t\}, \{c, a\}), e_3 = (\{ns\}, \{y\})\}.$$

As a first task of the medical expert system, stored medical knowledge is to be applied on the given case. This knowledge, in the language of computer programming, is given as look-up tables.
Mappings $u : m \times n : P(U_1) \times P(U_2) \to P(V_1) \times P(V_2)\,; \quad m : P(U_1) \to P(V_1)\,,\ n : P(U_2) \to P(V_2)$ and $p : E \to E'$ are defined as:

$$\begin{array}{cccc} m(h) = \alpha, & m(ns) = \delta, & m(t) = \gamma, & m(e) = \beta, \\ n(f) = \varepsilon, & n(a) = \mu, & n(y) = \nu, & n(c) = \lambda, \\ n(d) = \eta, & p(e_1) = e'_1, & p(e_2) = e'_2, & p(e_3) = e'_2, \end{array}$$

for a binary soft set $(F, A)$ in $(U_1, U_2, E)$; $(f(F, A), B)($ where $B = p(A) = \{e'_1, e'_2\})$ is a binary soft set in $(V_1, V_2, E')$ obtained as follows:

$$\begin{aligned} f(F,A)e'_1 &= u(\bigcup F(\{e_1\})) \qquad (since\ p^{-1}(e'_1) \cap A = \{e_1\}) \\ &= u(\{h, e\}, \{f, d\}) \\ &= (\{\alpha, \beta\}, \{\varepsilon, \eta\}). \end{aligned}$$

$$\begin{aligned} f(F,A)e'_2 &= u(\bigcup_{\alpha \in p^{-1}(e'_2) \cap A} F(\alpha)) \qquad (since\ p^{-1}(e'_2) \cap A = \{e_2, e_3\}) \\ &= u(\{F(e_2) \cup F(e_3)\}) \\ &= u((\{t\}, \{c, a\}) \cup (\{n\}, \{y\})) \\ &= u(\{t, n\}, \{c, a, y\}) \\ &= (\{\gamma, \delta\}, \{\lambda, \mu, \nu\}). \end{aligned}$$

Hence

$$\begin{aligned} & f(F, A) \\ &= \{e'_1 = (\{\alpha, \beta\}, \{\varepsilon, \eta\}), e'_2 = (\{\gamma, \delta\}, \{\lambda, \mu, \nu\})\} \\ &= \left\{ \begin{array}{l} diabatis\ or\ heart\ disease \\ = \left( \begin{array}{l} \{overactivity, lack\ of\ \ regular\ exercise\}, \\ \{stress, malnutrition\} \end{array} \right) \\ Inflammation \\ = \left( \begin{array}{l} \{deficiency\ of\ vitamins, sinus\}, \\ \{stiffness, infection, bacterial\ Infection\} \end{array} \right) \end{array} \right\}. \end{aligned}$$

## References


1. B. Ahmad, S. Hussain, *On some structures of soft topology,* Mathematical Sciences **6** (2012), 7 pages.
2. A. Ackgoz, N. Tas, *Binary soft set theory,* European Journal of Pure and Applied Mathematics **9** (2016), 452-463.
3. D. Chen, *The parametrization reduction of soft sets and its applications,* Computers and Mathematics with Applications **49** (2005), 757-763.
4. S. Hussain, *On some structures of binary soft topological spaces,* Hacettepe Journal of Mathematics and Statistics **48** (2019), 644-656.
5. S. Hussain, *A note on soft connectedness,* Journal of Egyptian Mathematical Society **23** (2015), 6-11.
6. S. Hussain, *On some soft functions,* Mathematical Science Letters **4** (2015), 55-61.
7. S. Hussain, *Binary soft connected spaces and an application of binary soft sets in decision making problem,* Fuzzy Information and Engineering (2020), 1-17, DOI: 10.1080/16168658.2020.1773600.
8. S. Hussain, B. Ahmad, *Some properties of soft topological spaces,* Computers and Mathematics with Applications **62** (2011), 4058-4067.
9. S. Hussain, B. Ahmad, *Soft separation axioms in soft topological spaces,* Hacettepe Journal of Mathematics and Statistics **44** (2015), 559-568.
10. A. Kharal, B. Ahmad, *Mappings on soft classes,* New Mathematics and Natural Computation **7** (2011), 471-481.
11. B. Kostek, *Soft set approach to subjective assessment of sound quality,* IEEE Conference **I** (1998), 669-676.
12. Z. Kong, L. Gao, L. Wong, S. Li, *The normal parameter reduction of soft sets and its algorithm,* J. Comp. Appl. Math. **21** (2008), 941-945.
13. P.K. Maji, R. Biswas, R. Roy, *An application of soft sets in a decision making problem,* Computers and Mathematics with Applications **44** (2002), 1077-1083.
14. P.K. Maji, R. Biswas and A.R. Roy, *Soft set theory,* Computers and Mathematics with Applications **45** (2003), 555-562.
15. D. Molodtsov, V.Y. Leonov, D.V. Kovkov, *Soft sets technique and its application,* Nechetkie Sistemy i Myagkie Vychisleniya **1** (2006), 8-39.
16. D. Molodtsov, *Soft set theory-First results,* Computers and Mathematics with Applications **37** (1999), 19-31.
17. M. Mushrif, S. Sengupta, A.K. Ray, *Texture classification using a novel, soft set theory based classification algorithm,* Springer Berlin, Heidelberg, 2006.
18. Z. Pawlak, *Rough sets,* International Journal of Computer Science **1** (1982), 341-356.
19. L.A. Zadeh, *Fuzzy sets,* Information and Control **8** (1965), 338-353.


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