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Original Article

A new methodology for modeling explicit seismic common cause failures for seismic multi-unit probabilistic safety assessment

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ABSTRACT

In a seismic PSA, dependency among seismic failures of components has not been explicitly modeled in the fault tree or event tree. This dependency is separately identified and assigned with numbers that range from zero to unity that reflect the level of the mutual correlation among seismic failures.

Because of complexity and difficulty in calculating combination probabilities of correlated seismic failures in complex seismic event tree and fault tree, there has been a great need of development to explicitly model seismic correlation in terms of seismic common cause failures (CCFs). If seismic correlations are converted into seismic CCFs, it is possible to calculate an accurate value of a top event probability or frequency of a complex seismic fault tree by using the same procedure as for internal, fire, and flooding PSA.

This study first proposes a methodology to explicitly model seismic dependency by converting correlated seismic failures into seismic CCFs. As a result, this methodology will allow systems analysts to quantify seismic risk as what they have done with the CCF method in internal, fire, and flooding PSA.

1. Introduction

1.1. Background

In probabilistic safety assessment (PSA), one of important basic assumptions in the fault tree analysis (FTA) is that all the component failures are independent. The failure probability of a fault tree or minimal cut sets (MCSs) is calculated based on this assumption.

For internal, fire, and flooding PSAs, the dependency among component failures is explicitly modeled by using common cause failures (CCFs). These CCFs explicitly reflect positive dependency among component failures.

In contrast to internal PSA, dependency among seismic failures is not explicitly modeled in a seismic PSA. If two or more components simultaneously fail by a single earthquake ground motion input, failures of these components are considered to be seismically dependent or seismically correlated. Seismic PSA model for core damage frequency (CDF) or conditional core damage probability (CCDP) can be expressed in a general form as shown in Eq. (1), where %S is a seismic event initiator.

$$\begin{array}{c}\text{Fault tree}(\%S,\mathbf{X},\mathbf{Y},\ldots,\mathbf{N},\ \mathbf{R})\\ \mathbf{X}=[X_1,X_2,\ \ldots]\ \text{Correlated seismic failure group 1}\\ \mathbf{Y}=[Y_1,Y_2,\ \ldots]\ \text{Correlated seismic failure group 2}\\ \mathbf{N}=[N_1,N_2,\ \ldots]\text{Non}-\text{correlated seismic failures}\\ \mathbf{R}=[R_1,R_2,\ \ldots]\text{Random failures}\end{array} \tag{1}$$

Complex minimal cut sets (MCSs) are generated by solving this fault tree in Eq. (1). The probabilities of simple AND/OR Boolean combination of correlated seismic failures such as X_{123} and X_{1+2+3} can be calculated by Monte Carlo integrations (see Section 2). If Eq. (1). is factored into $f(\mathbf{X})g(\mathbf{Y})\ldots s(\%\mathbf{S},\mathbf{N},\ \mathbf{R})\ +\ t(\%\mathbf{S},\mathbf{N},\ \mathbf{R})$, the probability of $f(\mathbf{X})$ or $g(\mathbf{Y})$ can be separately calculated by the integration methods in Section 2. However, since $f(\mathbf{X})$ and $g(\mathbf{Y})$ cannot be separated in an actual PSA, probability calculation of arbitrary Boolean combinations among various failures $(\mathbf{X},\mathbf{Y},\ldots,\mathbf{N},\ \mathbf{R})$ in Eq. (1). is impossible. It means the accurate seismic CDF cannot be calculated if a seismic PSA model has complex correlated seismic failures.

If the correlated seismic failures in **X** and **Y** are split into seismic CCFs, the seismic PSA model can be solved in the same manner as

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solved for internal PSA model. It means that an accurate seismic CDF of a complex seismic fault tree can be calculated if the correlated seismic failures can be converted into seismic CCFs.

In a seismic PSA, a failure probability of a single component is expressed in terms of a single ground motion parameter such as peak ground acceleration or spectral acceleration, which is referred to as a seismic fragility (see Section 1.4). Seismic fragility of a component is defined as a conditional probability of failure of the component at a given level of peak ground acceleration [1]. Since seismically induced failures of more than two components in a system are often correlated, there have been attempts to consider such dependency among the failures. There are various methods available to calculate probabilities of the combination failures whose failures are seismically correlated, of which most promising methods would be multivariate normal (MVN) integration method and Reed-McCann integration method [2–4]. The MVN integration was adopted for Seismic Safety Margins Research Program (SSMRP) and is often referred to as SSMRP integration for this reason.

1.2. Objectives

Recently a new method to calculate probabilities of combination failures that are seismically correlated was proposed in 2019 PSA [5]. A basic idea of this new method is converting seismically dependent failures to seismic CCFs. This method was developed based on AND Boolean combination probabilities of correlated seismic failures, which converts correlated seismic failures into seismic CCFs that have approximate seismic CCF probabilities.

This study proposes a new improved method to explicitly model dependency among seismic failures by converting correlated seismic failures into seismic CCFs. This approach is based on OR Boolean combination probabilities of correlated seismic failures, and it generates exact seismic CCF probabilities. A detailed discussion of the proposed methodology and its application are presented in Sections 3 and 4, respectively.

This paper summarizes existing methods to calculate combination probabilities of correlated seismic failures and current practices of seismic PSA in treatment of dependency among seismic failures (see Section 2). Then, this paper proposes a new methodology for explicitly modeling dependency among seismic failures by converting correlated seismic failures into seismic CCFs (see Section 3). Lastly, this paper presents results of an application of this methodology to existing operating nuclear power plants at multi-unit site in South Korea (see Section 4).

1.3. Definitions

Notations and definitions of failures and probabilities used in this study are listed below.

$X_i = \text{Failure of component } i$

$X_{ij} = X_i X_j = \text{Joint failures of components } i \text{ and } j$

$X_{ijk} = X_i X_j X_k = \text{Joint failures of components } i, j, \text{ and } k$

$X_{i+j} = X_i + X_j = \text{OR combination failures of components } i \text{ and } j$

$$X_{i+j+k} = X_i + X_j + X_k = \text{OR combination failures of components } i, j, \text{ and } k$$

$C_i = \text{Common cause failure (CCF) of component } i$

$C_{ij} = \text{CCF of components } i \text{ and } j$

$C_{ijk} = \text{CCF of components } i, j, \text{ and } k$

$X_i = C_i + C_{ij} + C_{ik} + C_{ijk} + \ldots$

$P_i = P(X_i)$

$P_{ij} = P(X_{ij})$

$P_{ijk} = P(X_{ijk})$

$P_{i+j} = P(X_{i+j})$

$P_{i+j+k} = P(X_{i+j+k})$

$Q_i = P(C_i)$

$Q_{ij} = P(C_{ij})$

$Q_{ijk} = P(C_{ijk})$

Equality $P(X_iX_j) = P(X_i)P(X_j|X_i) = P(X_j)P(X_i|X_j)$ is always satisfied for any failures X_i and X_j. Two failures X_i and X_j are independent if and only if $P(X_iX_j) = P(X_i)P(X_j)$. Failures are dependent if they do not satisfy this equality.

A common measure of dependence between two failures X_i and X_j is a correlation $\rho_{ij} = \text{corr}(X_i, X_j) = \text{cov}(X_i, X_j)/\beta_i\beta_j$ where β_i and β_j are standard deviations of X_i and X_j, respectively. The correlation ρ_{ij} is zero if two failures X_i and X_j are independent. The inverse is not true.

If X_i and X_j are positively dependent, the three inequalities $P(X_i|X_j) > P(X_i)$, $P(X_j|X_i) > P(X_j)$, and $P(X_iX_j) > P(X_i)P(X_j)$ hold. It can be said that CCFs reflect positive dependency among failures.

1.4. Seismic fragility model

The seismic fragility of a component is defined as the conditional probability of its failure at a given value of peak ground acceleration [1]. The entire family of fragility curves for a component corresponding to a particular failure mode can be expressed in terms of the best estimate of the median ground acceleration capacity and two random variables. Therefore, the ground acceleration capacity of a component is given by

$$A = A_m \varepsilon_R \varepsilon_U. \quad (2)$$

Here, A_m is a median ground acceleration capacity, ε_R and ε_U are random variables with median values of unity, representing, respectively, randomness (aleatory uncertainty) about the median and the state of knowledge uncertainty (epistemic uncertainty) in the median value. In this model, these two random variables ε_R and ε_U have lognormal distribution with logarithmic standard deviations, β_R and β_U, respectively.

A component failure probability (seismic fragility) that is a conditional failure probability of a component at the ground acceleration *a* can be calculated in two ways according to Eqs. (3) or

(4).

$$P(a) = P(A < a) = P(\ln(A/Am) < \ln(a/Am)) = \Phi\left(\frac{\ln(a/Am) + \beta_U \Phi^{-1}(Q)}{\beta_R}\right) \quad (3)$$

$$P(a) = \Phi\left(\frac{\ln(a/Am)}{\beta}\right) = \int_{-\infty}^{\ln\left(\frac{a}{Am}\right)} \frac{1}{\sqrt{2\pi}\beta} \exp\left(-\frac{1}{2}\frac{x^2}{\beta^2}\right) dx \quad (4)$$

Here, $\Phi()$ is a standard normal cumulative distribution function, and $\Phi^{-1}()$ is a standard normal cumulative distribution inverse function. Q is subjective probability or confidence with a value ranging from 0 to 1, and β is a composite standard deviation defined as $\beta = (\beta_R^2 + \beta_U^2)^{1/2}$.

As mentioned above, there are two ways to calculate component failure probability. First, P(a) in Eq. (3). is calculated many times by randomly selecting Q value from the uniform distribution, and then, their mean value is calculated for $\overline{P}(a)$. Second, P(a) can be calculated one time by Eq. (4). The $\overline{P}(a)$ calculated by Eq. (3). is known to be identical to P(a) in Eq. (4). Since only a point estimate of seismic CCFs is of interest in this study, Eq .(4). is employed for the conversion of correlated seismic failures into seismic CCFs.

2. Dependency in PSA

There are explicit and implicit methods for modeling dependent failures in PSA. Typically, the explicit modeling is employed in PSAs such as internal, fire, and flooding PSAs (see Section 2.1). The dependent failures are explicitly modeled as CCFs in internal, fire, and flooding PSAs. On the other hand, the implicit modeling of dependent failures is employed in seismic PSA (see Section 2.2).

2.1. Explicit dependency modeling in internal, fire, and flooding PSAs

In internal, fire, and flooding PSAs, three dependent failures X_1, X_2, and X_3 are each split into independent CCFs as shown below.

$$\begin{aligned} X_1 &= C_1 + C_{12} + C_{13} + C_{123} \\ X_2 &= C_2 + C_{12} + C_{23} + C_{123} \\ X_3 &= C_3 + C_{13} + C_{23} + C_{123} \end{aligned} \quad (5)$$

Using CCF equations in Eq. (5), MCSs for X_{1+2+3} can be calculated as

$$X_{1+2+3} = C_1 + C_2 + C_3 + C_{12} + C_{13} + C_{23} + C_{123} \quad (6)$$

Here, X_{1+2+3} has seven MCSs and each MCS has one CCF (C_1, C_2, C_3, C_{12}, C_{13}, C_{23}, or C_{123}). In contrast, X_{123} in Eq. (C.3) has eight MCSs and some MCSs have multiple CCFs (C_1C_{23}, C_2C_{13}, C_3C_{12}, $C_{12}C_{13}$, $C_{13}C_{23}$, or $C_{12}C_{23}$). The min cut upper bound (MCUB) probability of Eq. (6). is given in Eq. (7). as shown in Appendix A.

$$P_{1+2+3} = 1 - (1-Q_1)(1-Q_2)(1-Q_3)(1-Q_{12})(1-Q_{13})(1-Q_{23})(1-Q_{123}) \quad (7)$$

As discussed in Appendix A, MCUB probability in Eq. (7). is an exact probability of X_{1+2+3} since there are no duplicated CCFs among seven MCSs in Eq. (6).

2.2. Implicit dependency modeling for seismic PSA

There have been attempts to implicitly calculate AND/OR Boolean combination probabilities of correlated seismic failures by using some integration methods [2–4]. As discussed in NUREG/CR-7237[2] a simple combination probability of correlated seismic failures such as $P_{1+2\ldots+n}(a)$ can be calculated by either MVN or Reed-McCann integration method (see Appendix B). The MVN method is explained in details in the following Sections.

2.2.1. MVN integration

The combination probability of a seismic failure $P_{1+2+\ldots+n}(a) = P(U_{i=1}^{n} A_i < a)$ can be calculated by using Monte-Carlo integration of MVN distribution in Eq (8). Monte Carlo integration is a numerical integration method using random numbers, and it is employed only when the integration has no analytic solution [3,4].

$$P_{1+2\ldots+n}(a) = 1 - \int_{\ln\left(\frac{a}{A_{1m}}\right)}^{\infty} \int_{\ln\left(\frac{a}{A_{2m}}\right)}^{\infty} \cdots \int_{\ln\left(\frac{a}{A_{nm}}\right)}^{\infty} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} \exp\left(-\frac{1}{2}\mathbf{x}^t\Sigma^{-1}\mathbf{x}\right) d\mathbf{x} \quad (8)$$

Here, $\mathbf{x}^t = [x_1x_2\ldots x_n]$. Σ is a symmetric positive definite covariance matrix, $|\Sigma|$is a determinant of Σ, and Σ^{-1} is an inverse matrix of Σ.

$$\Sigma = \begin{bmatrix} \beta_1^2 & \beta_{12}^2 & \ldots & \beta_{1n}^2 \\ \beta_{21}^2 & \beta_2^2 & \ldots & \beta_{2n}^2 \\ \ldots & \ldots & \ldots & \ldots \\ \beta_{n1}^2 & \beta_{n2}^2 & \ldots & \beta_n^2 \end{bmatrix}, \quad \beta_{ij}^2 = \mathrm{cov}(X_i, X_j) \quad (9)$$

Each covariance β_{ij}^2 in Eq. (9) is a composite variability of uncertainty and randomness. The composite standard deviations are calculated as $\beta_i = (\beta_{Ri}^2 + \beta_{Ui}^2)^{1/2}$ and $\beta_{ij} = \beta_{ji} = (\beta_{Rij}^2 + \beta_{Uij}^2)^{1/2}$.

If x_i is replaced with $\beta_i z_i$ as $x_i = \beta_i z_i$, Eq. (8). can be converted into

$$P_{1+2\ldots+n}(a) = 1 - \int_{\frac{\ln(a/A_{1m})}{\beta_1}}^{\infty} \int_{\frac{\ln(a/A_{2m})}{\beta_2}}^{\infty} \cdots \int_{\frac{\ln(a/A_{nm})}{\beta_n}}^{\infty} \frac{1}{\sqrt{|\Sigma_\rho|(2\pi)^n}} \exp\left(-\frac{1}{2}\mathbf{z}^t\Sigma_\rho^{-1}\mathbf{z}\right) d\mathbf{z} \quad (10)$$

Here, $\mathbf{z}^t = [z_1z_2\ldots z_n]$. Σ_ρ is a symmetric positive definite correlation matrix.

$$\Sigma_\rho = \begin{bmatrix} 1 & \rho_{12} & \ldots & \rho_{1n} \\ \rho_{21} & 1 & \ldots & \rho_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \rho_{n1} & \rho_{n2} & \ldots & 1 \end{bmatrix}, \quad \rho_{ij} = \frac{\beta_{ij}^2}{\beta_i\beta_j} \quad (11)$$

2.2.2. Transformation of MVN integration

Reed-McCann integration in Appendix B defines independent variances such as $(\beta_1^-)^2 = \beta_1^2 - (\beta_{12}^2 + \beta_{13}^2)$, and uses multiple integrations over the ${}_nC_2$ integration variables for the covarinces such as β_{12}^2 and β_{13}^2. Although MVN and Reed-McCann integrations are similar methods, MVN and Reed-McCann integrations have different number of integration variables. As shown in Eq. (8) or (10), the MVN integration requires small number of n integration variables. In contrast, Reed-McCann integration in Appendix B requires ${}_nC_2$ integration variables. If n is not small, the number of ${}_nC_2$ is always much bigger than n. It means that the Reed-McCann integration is much more difficult than the MVN integration. So, the MVN integration is recommended in order to drastically reduce the integration difficulty [4].

Difficulty of the MVN integration can be further reduced by integration variable transformations [6]. A sequence of three transformations converts the original integral in Eq. (10) into the integral over a unit hyper-cube. The definite integral in Eq. (10) has lower bounds as

$$\mathbf{b^t} = [b_1,\ b_2,\ \ldots,\ b_n] = \left[\frac{\ln(a/A_{1m})}{\beta_1},\ \ \frac{\ln(a/A_{2m})}{\beta_2},\ \ldots,\ \ \frac{\ln(a/A_{nm})}{\beta_n}\right] \tag{12}$$

Positive definite covariance matrix Σ_ρ can be factored by the Cholesky decomposition as $\Sigma_\rho = CC^t$ and $\Sigma_\rho^{-1} = C^{-t}C^{-1} = (C^{-1})^t C^{-1}$ where C is a lower triangular matrix with real positive diagonal elements. First, this sequence of three transformations begins with transformation of $\mathbf{z} = C\mathbf{y}$ and $\mathbf{z}^t = \mathbf{y}^t C^t$. The integrand and integral variables become

$$\mathbf{z}^t \Sigma_\rho^{-1} \mathbf{z} = \left(\mathbf{y}^t C^t\right)\left(C^{-t}C^{-1}\right)(C\mathbf{y}) = \mathbf{y}^t \mathbf{y} \tag{13}$$

$$d\mathbf{z} = |C| d\mathbf{y} = |\Sigma_\rho|^{\frac{1}{2}}\ d\mathbf{y} \tag{14}$$

First, by using Eqs. (13) and (14), the integral in Eq. (10) is converted into

$$P_{1+2+\ldots+n}(a) = 1 - \frac{1}{\sqrt{(2\pi)^n}} \int_{b_1'}^{\infty} e^{-\frac{y_1^2}{2}} \int_{b_2'}^{\infty} e^{-\frac{y_2^2}{2}} \int_{b_n'}^{\infty} e^{-\frac{y_n^2}{2}}\ d\mathbf{y} \tag{15}$$

$$b_i'(y_1, \ldots, y_{i-1}) = \left(b_i - \sum_{j=1}^{i-1} c_{ij} y_j\right) \Big/ c_{ii} \tag{16}$$

Second, the integration variable y_i is further transformed into t_i using $y_i = \Phi^{-1}(t_i)$, where $\Phi(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{1}{2}t^2} dt$ that is a standard univariate normal distribution function. After this second transformation,

$$P_{1+2+\ldots+n}(a) = 1 - \int_{b_1''}^{1} \int_{b_2''}^{1} \ldots \int_{b_n''}^{1}\ d\mathbf{t} \tag{17}$$

$$b_i''(t_1,\ \ldots,\ t_{i-1}) = \Phi\left(\left(b_i - \sum_{j=1}^{i-1} c_{ij} \Phi^{-1}(t_j)\right) \Big/ c_{ii}\right) \tag{18}$$

Finally, the integration variable t_i in Eq. (17) is transformed into w_i using $t_i = w_i(1 - b_i'') + b_i''$, $w_i = (t_i - b_i'')/(1 - b_i'')$, and $dt_i = (1 - b_i'')dw_i$. The original integral in Eq. (10) is converted into an integral over a unit hyper-cube. Monte Carlo integration is performed over $\mathbf{w}$ in Eq. (19).

$$P_{1+2+\ldots+n}(a) = 1 - \left(1 - b_1''\right) \int_0^1 \left(1 - b_2''\right) \int_0^1 \ldots \left(1 - b_n''\right) \int_0^1\ d\mathbf{w} \tag{19}$$

$$b_i''(w_1, \ldots, w_{i-1}) = \Phi\left(\left(b_i - \sum_{j=1}^{i-1} c_{ij} \Phi^{-1}\left(w_j\left(1 - b_j''\right) + b_j''\right)\right) \Big/ c_{ii}\right) \tag{20}$$

Now, the combination probabilities $P_{1+2+\ldots+n}(a)$ can be calculated by one of integrations in Eqs. (8), (10), (15), (17) and (19).

2.2.3. Discrete correlation assumption in current practices

Dependency among seismic failures in a practical PSA tends to be treated in an approximate manner due to difficulty in calculating combination probabilities of correlated seismic failures, which results from difficulty in determining correlation groups and correlation level. The systems analysts could use a partial correlation level between 0 and 1 among seismic failures that are partially correlated, but practically assume zero or full correlation (0 or 1) depending on the level of correlation. This assumption is for avoiding the difficulty that is explained right after Eq. (1). For example, if identical components are located on the same floor in a building, seismic failure among the components are assumed to be fully correlated [8].

The current practice for treating seismic correlation in a seismic PSA is to screen out seismically rugged components and then develop correlation groups for the screened-in components to represent the simultaneous failure of similar components during earthquake. If components are similar in design, with similar anchorage, and located in the same building and elevation, then their failures are treated as fully correlated failures. Otherwise, it is assumed that there is no correlation among component failures, and failures of similar components in different buildings are not correlated [7].

This practical approach inevitably contributes to the uncertainty in a seismic CDF that could be either overestimated or underestimated depending on AND/OR logic combinations in a seismic fault tree. In order to avoid this uncertainty, there has been a great need to develop a new methodology to explicitly model seismic correlation with seismic CCFs.

3. Explicit dependency modeling for seismic PSA

Combination probabilities of correlated seismic failures cannot be easily calculated by the MVN integration, if correlated seismic failures are combined with (1) other correlated seismic failures (2), non-correlated seismic failures, and (3) random failures in a nested way in a whole seismic fault tree. The methodology developed in this study solves this issue by converting seismic failures into seismic CCFs. A detailed conversion process is presented below.

(Step 1) Collect seismic failures in the same correlation group and make covariance matrix in Eq. (9). Each covariance β_{ij}^2 in Eq. (9) is a composite variability of uncertainty and randomness as $\beta_{ij} = \beta_{ji} = (\beta_{Rij}^2 + \beta_{Uij}^2)^{1/2}$. The covariance matrix in Eq. (9) is converted into correlation matrix in Eq. (11). Then, combination probabilities can be calculated by one of integrations in Eqs. (8), (10), (15), (17) and (19). Seismic PSA model has a general form of a fault tree in Eq. (21) that is a simplified form of Eq. (1). For the clear explanation, it is assumed that there is only one correlation group and this group has three component failures (X_1, X_2, and X_3).

$$\begin{aligned} &\text{Fault tree}(\%S, \mathbf{X}, \mathbf{N}, \mathbf{R}) \\ \mathbf{X} &= [X_1, X_2, \ldots]\text{Correlated seismic failures} \\ \mathbf{N} &= [N_1, N_2, \ldots]\text{Non} - \text{correlated seismic failures} \\ \mathbf{R} &= [R_1, R_2, \ldots]\text{Random failures} \end{aligned} \tag{21}$$

(Step 2) Calculate combination probabilities of correlated seismic failures. By using the MVN integration, 2^3-1 OR combination probabilities ($P_1, P_2, P_3, P_{1+2}, P_{1+3}, P_{2+3}, P_{1+2+3}$) are calculated. As an alternative way, these OR combination probabilities can be calculated from AND combination probabilities ($P_1, P_2, P_3, P_{12}, P_{13}, P_{23}, P_{123}$) by using inclusion-exclusion principle in Appendix A.

(Step 3) Construct 2^3-1 probability equations in Eq. (22). They are nonlinear simultaneous equations based on the MCUB probability equations. These simultaneous equations have 2^3-1 knowns of combination probabilities on the left hand side and 2^3-1 unknowns of seismic CCF probabilities ($Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}$) on the right hand side.

$$\begin{aligned} P_1 &= 1-(1-Q_1)(1-Q_{12})(1-Q_{13})(1-Q_{123}) \\ P_2 &= 1-(1-Q_2)(1-Q_{12})(1-Q_{23})(1-Q_{123}) \\ P_3 &= 1-(1-Q_3)(1-Q_{13})(1-Q_{23})(1-Q_{123}) \\ P_{1+2} &= 1-(1-Q_1)(1-Q_2)(1-Q_{12})(1-Q_{13})(1-Q_{23})(1-Q_{123}) \\ P_{1+3} &= 1-(1-Q_1)(1-Q_3)(1-Q_{12})(1-Q_{13})(1-Q_{23})(1-Q_{123}) \\ P_{2+3} &= 1-(1-Q_2)(1-Q_3)(1-Q_{12})(1-Q_{13})(1-Q_{23})(1-Q_{123}) \\ P_{1+2+3} &= 1-(1-Q_1)(1-Q_2)(1-Q_3)(1-Q_{12})(1-Q_{13})(1-Q_{23})(1-Q_{123}) \end{aligned} \tag{22}$$

Eq. (22) has MCUB probabilities of the Boolean equations in Eq. (23).

$$\begin{aligned} X_1 &= C_1 + C_{12} + C_{13} + C_{123} \\ X_2 &= C_2 + C_{12} + C_{23} + C_{123} \\ X_3 &= C_3 + C_{13} + C_{23} + C_{123} \\ X_{1+2} &= C_1 + C_2 + C_{12} + C_{13} + C_{23} + C_{123} \\ X_{1+3} &= C_1 + C_3 + C_{12} + C_{13} + C_{23} + C_{123} \\ X_{2+3} &= C_2 + C_3 + C_{12} + C_{13} + C_{23} + C_{123} \\ X_{1+2+3} &= C_1 + C_2 + C_3 + C_{12} + C_{13} + C_{23} + C_{123} \end{aligned} \tag{23}$$

(Step 4) Calculate 2^3-1 CCF probabilities ($Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}$) by solving nonlinear simultaneous equations in Eq. (22).

(Step 5) Finally, expand seismic failures (X_1, X_2, X_3) in a seismic fault tree into seismic CCFs ($C_1, C_2, C_3, C_{12}, C_{13}, C_{23}, C_{123}$) as in Eq. (24). If the seismic failures are expanded into seismic CCFs, any complex Boolean equations in the seismic fault tree can be solved in the same manner as solved for internal PSA model.

$$\begin{aligned} &\text{Fault tree}(\%S, \mathbf{C}, \mathbf{N}, \mathbf{R}) \\ \mathbf{C} &= [C_1, C_2, \ldots]\text{Seismic CCFs} \\ \mathbf{N} &= [Y_1, Y_2, \ldots]\text{Non} - \text{correlated seismic failures} \\ \mathbf{R} &= [R_1, R_2, \ldots]\text{Random failures} \\ X_1 &= C_1 + C_{12} + C_{13} + C_{123} \\ X_2 &= C_2 + C_{12} + C_{23} + C_{123} \\ X_3 &= C_3 + C_{13} + C_{23} + C_{123} \end{aligned} \tag{24}$$

4. Applications

The methods in Section 3 are implemented into a new computer software COREX (Correlation Explicit). The method that is developed in this study were applied to small Benchmark problems and a real seismic multi-unit PSA (MUPSA) model. This seismic MUPSA model is for four nuclear power plants. The efficiency and strength of the method developed in this study is validated through these applications. COREX can convert two to twelve correlated seismic failures into seiscmic CCFs. The number of correlated seismic failures that can be converted into seismic CCFs depends on the difficulties (1) to calculate combination failures of correlated seismic failures and (2) to solve simultaneous equations.

A simple OR combination probability of correlated seismic failures such as $P_{1+2+3}(a)$ can be calculated by the MVN integrations. However, it is impossible to calculate the seismic CDF from the complex Boolean equation in Eq. (21) by using MVN integration. After converting Eq. (21) into Eq. (24), that is, converting correlated seismic failures into seismic CCFs by constructing and solving the simultaneous equations in Section 3, it is possible to calculate the seismic CDF by usual methods and tools for the internal PSA.

The number of random samples, running time, and Monte Carlo integration error depend on integrations in Eqs. (8), (10), (15), (17) and (19). The numerical procedure in Ref. [6] with integration in Eq. (19) is recommended for the fast and accurate integration. The running time for each application in Sections 4.1 to 4.4 is less than twenty seconds.

4.1. Non-symmetric correlated seismic failures into seismic CCFs ($n = 2$)

COREX input data and calculation results are summarized in Tables 1 and 2. Combination probabilities (P_1, P_2, P_{1+2}) and (P_1, P_2, P_{12}) of correlated seismic failures are calculated using the MVN integrations in Section 3 and Eq. (C.1), and then, the simultaneous equations in Eqs. (25) and (C.4) are solved to generate seismic CCF probabilities (Q_1, Q_2, Q_{12}). Since the seismic CCF probabilities have no multiple occurrences in any single equation in Eqs. (25) and

Table 1
Non-symmetric correlated seismic failures ($n = 2$).

Ground acceleration	a	1.0
Median capacity	A_{1m}	0.8
	A_{2m}	1.0
Covariance[1/2a]	$\beta_{R1} = \beta_{U1}$	0.4
	$\beta_{R2} = \beta_{U2}$	0.5
	$\beta_{R12} = \beta_{U12}$	0.2

[a] $\beta_i = \sqrt{\beta_{Ri}^2 + \beta_{Ui}^2}$ and. $\beta_{ij} = \beta_{ji} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$

Table 2
Non-symmetric seismic CCFs (n = 2).

AND based method in Eq. (C.4)		OR based method in Eq. (25)	
P_1	0.653381	P_1	0.653381
P_2	0.500000	P_2	0.500000
P_{12}	0.356307	P_{1+2}	0.797074
Q_1	0.594148	Q_1	0.594148
Q_2	0.414555	Q_2	0.414555
Q_{12}	0.145948	Q_{12}	0.145948

Table 3
Symmetric correlated seismic failures (n = 3).

Ground acceleration	a	1.0
Median capacity	A_{1m}	1.0
	A_{2m}	1.0
	A_{3m}	1.0
Covariance[1/2a]	$\beta_{R1} = \beta_{U1}$	0.707107
	$\beta_{R2} = \beta_{U2}$	0.707107
	$\beta_{R3} = \beta_{U3}$	0.707107
	$\beta_{R12} = \beta_{U12}$	0.5
	$\beta_{R13} = \beta_{U13}$	0.5
	$\beta_{R23} = \beta_{U23}$	0.5

[a] $\beta_i = \sqrt{\beta_{Ri}^2 + \beta_{Ui}^2}$ and.$\beta_{ij} = \beta_{ji} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$

Table 4
Symmetric seismic CCFs (n = 3).

AND based method in Eq. (C.2)		OR based method in Eq. (22)	
P_1	0.500000	P_1	0.500000
P_2	0.500000	P_2	0.500000
P_3	0.500000	P_3	0.500000
P_{12}	0.333333	P_{1+2}	0.666667
P_{13}	0.333333	P_{1+3}	0.666667
P_{23}	0.333333	P_{2+3}	0.666667
P_{123}	0.249999	P_{1+2+3}	0.749999
Q_1	0.283142	Q_1	0.249998
Q_2	0.283142	Q_2	0.249998
Q_3	0.283142	Q_3	0.249998
Q_{12}	0.101554	Q_{12}	0.111114
Q_{13}	0.101554	Q_{13}	0.111114
Q_{23}	0.101554	Q_{23}	0.111114
Q_{123}	0.135922	Q_{123}	0.156247

(C.4), the two sets of simultaneous equations in Eqs. (25) and (C.4) generate identical seismic CCF probabilities (Q_1, Q_2, Q_{12}) as shown in Table 2.

$$\begin{aligned} P_1 &= 1-(1-Q_1)(1-Q_{12}) \\ P_2 &= 1-(1-Q_2)(1-Q_{12}) \\ P_{1+2} &= 1-(1-Q_1)(1-Q_2)(1-Q_{12}) \end{aligned} \quad (25)$$

Here, Eq. (25) has MCUB probabilities of the Boolean equations that have seismic CCFs.

$$\begin{aligned} X_1 &= C_1 + C_{12} \\ X_2 &= C_2 + C_{12} \\ X_{1+2} &= C_1 + C_2 + C_{12} \end{aligned} \quad (26)$$

4.2. Symmetric correlated seismic failures into seismic CCFs (n = 3)

COREX input data and results are listed in Tables 3 and 4. From the symmetric seismic variables in Table 3, symmetric combination probabilities (P_1, P_2, P_3, P_{1+2}, P_{1+3}, P_{2+3}, P_{1+2+3}) and (P_1, P_2, P_3, P_{12}, P_{13}, P_{23}, P_{123}) of correlated seismic failures are calculated by the MVN integrations in Section 3 and in Eq. (C.1), and the simultaneous equations in Eqs. (22) and (C.2) are solved to generate seismic CCF probabilities (Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}).

The seismic CCFs have multiple occurrences in a single equation of Eq. (C.3). It means that MCUB probabilities in Eq. (C.2) are approximate probabilities (see Appendix A) and approximate CCF probabilities are generated by solving Eq. (C.2). However, since the seismic CCF probabilities have no multiple occurrences in any single equation of Eq. (23), the exact CCFs probabilities are generated by Eq. (22). That is, it is recommended to use Eq. (22) instead of Eq. (C.2).

Table 5
Non-symmetric correlated seismic failures (n = 3).

Ground acceleration	a	1.0
Median capacity	A_{1m}	0.8
	A_{2m}	1.0
	A_{3m}	1.2
Covariance[1/2a]	$\beta_{R1} = \beta_{U1}$	0.4
	$\beta_{R2} = \beta_{U2}$	0.5
	$\beta_{R3} = \beta_{U3}$	0.6
	$\beta_{R12} = \beta_{U12}$	0.2
	$\beta_{R13} = \beta_{U13}$	0.3
	$\beta_{R23} = \beta_{U23}$	0.4

[a] $\beta_i = \sqrt{\beta_{Ri}^2 + \beta_{Ui}^2}$and.$\beta_{ij} = \beta_{ji} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$

Table 6
Non-symmetric seismic CCFs (n = 3).

AND based method in Eq. (C.2)		OR based method in Eq. (22)	
P_1	0.653381	P_1	0.653381
P_2	0.500000	P_2	0.500000
P_3	0.414935	P_3	0.414935
P_{12}	0.356307	P_{1+2}	0.797074
P_{13}	0.325187	P_{1+3}	0.743130
P_{23}	0.294712	P_{2+3}	0.620224
P_{123}	0.233555	P_{1+2+3}	0.825666
Q_1	0.624438	Q_1	0.540955
Q_2	0.463358	Q_2	0.321314
Q_3	0.089271	Q_3	0.140896
Q_{12}	−0.257155	Q_{12}	0.043566
Q_{13}	0.133201	Q_{13}	0.115877
Q_{23}	0.124950	Q_{23}	0.137386
Q_{123}	0.153039	Q_{123}	0.107046

4.3. Non-symmetric correlated seismic failures into seismic CCFs (n = 3)

The input data and results are summarized in Tables 5 and 6. Similarly to the calculation in Section 4.2, seismic CCF probabilities (Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}) are calculated from the non-symmetric seismic data in Table 5.

With the same reason in Section 4.2, Eq. (C.2) generates approximate seismic CCF probabilities. To make matters worse, one negative CCF probability Q_{12} is calculated from Eq. (C.2) as shown in Table 6. Thus, it is strongly recommended to use Eq. (22) instead of Eq. (C.2) since Eq. (22) generates exact seismic CCF probabilities.

4.4. Correlated seismic failure conversion for seismic MUPSA

Seismic failures of identical components usually have a high level correlation, e.g., 0.7 to 0.9, or full correlation of 1.0. These seismic failures can exist under AND or OR gate in a fault tree. If such a high level correlation is ignored and a full correlation is assumed, the resulting failure probability will increase under AND gate, but it will decrease under OR gate. Therefore, when a high

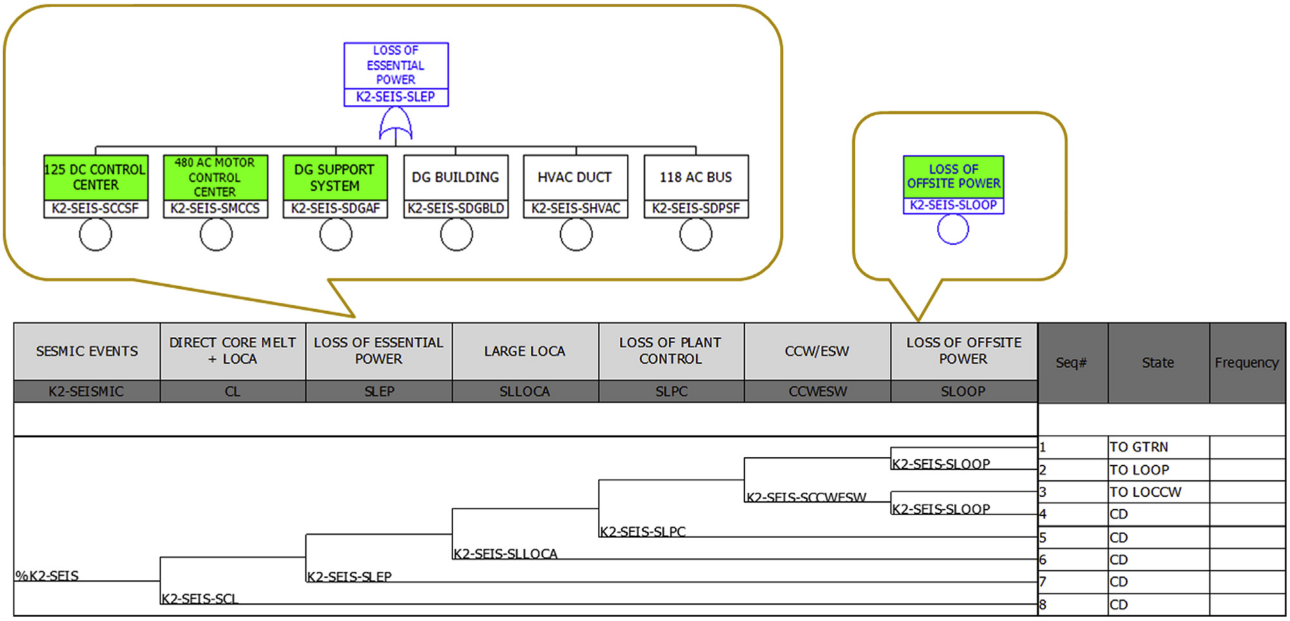

Fig. 1. Event tree for K2 NPP

Table 7
Correlated seismic failures (K2/K3/S1/S2 NPPs).

Description	Seismic events[b]	A_m	β_U				β_R			
125V DC control center (structure failure)	K2-SCCSF	0.85	0.38	(a)	0.05	0.05	0.24	0.22	0.22	0.22
	K3-SIE_SCCSF	1.18	(a)	0.22	0.05	0.05	0.22	0.28	0.22	0.22
	S1-SIE_SDCSF	1.16	0.05	0.05	0.32	0.07	0.22	0.22	0.29	0.22
	S2-SIE_SDCSF	1.16	0.05	0.05	0.07	0.32	0.22	0.22	0.22	0.29
480V AC motor control center (structure failure)	K2-SMCCS	0.85	0.38	(a)	0.05	0.05	0.24	0.22	0.22	0.22
	K3-SIE_SMCCS	1.34	(a)	0.31	0.05	0.05	0.22	0.34	0.22	0.22
	S1-EMBSYMC04-ALL	1.48	0.05	0.05	0.34	0.07	0.22	0.22	0.34	0.22
	S2-EMBSYMC04-ALL	1.48	0.05	0.05	0.07	0.34	0.22	0.22	0.22	0.34
Emergency diesel generator (structure failure)	K2-SDGAF	0.68	0.26	(a)	0.05	0.05	0.24	0.20	0.20	0.20
	K3-EGDGZ-ALL	1.50	(a)	0.17	0.05	0.05	0.20	0.26	0.20	0.20
	S1-EGDGS01-ALL	1.00	0.05	0.05	0.19	0.07	0.20	0.20	0.34	0.20
	S2-EGDGS01-ALL	1.00	0.05	0.05	0.07	0.19	0.20	0.20	0.20	0.34
Offsite power (function failure)[c]	K2-SLOOP	0.30	0.20	0.20	0.20	0.20	0.22	0.22	0.22	0.22
	K3-SIE_SLOOP	0.30	0.20	0.20	0.20	0.20	0.22	0.22	0.22	0.22
	S1-SIE_SLOOP	0.30	0.20	0.20	0.20	0.20	0.22	0.22	0.22	0.22
	S2-SIE_SLOOP	0.30	0.20	0.20	0.20	0.20	0.22	0.22	0.22	0.22

(a) 0.0707107.
[b] Four NPPs (K2/K3/S1/S2) have four groups of correlated seismic failures.
[c] For example, offsite power has $A_m^T = [0.30\ 0.30\ 0.30\ 0.30]$, $\beta_{Uij} = 0.20$, and $\beta_{Rij} = 0.22$.

correlation exists among identical components, the assumption of a full correlation is acceptable only if the correlated failures are under the AND gate.

There are many groups of correlated seismic failures of identical components or functions in a seismic single-unit PSA. The seismic failures in each group are correlated redundant component failures, and they exist under AND gate in a fault tree. In a practical seismic PSA, the high level correlation among the correlated redundant component failures is assumed to be a full correlation, and they are replaced with one component failure in a fault tree. These assumption and replacement guarantee the conservative AND gate failure probability and also conservative CDF.

Nuclear power plants (NPPs) in a single nuclear site can have identical components whose seismic failures have a high correlation. Identical component failures for single-unit PSAs are located under AND gates, and they are logically connected with OR gates at the top level of a seismic MUPSA fault tree when a fault tree solver generates MCSs. It means that the whole identical component failures for a seismic MUPSA are connected in a complex way through the nested AND or OR gates. Due to these OR gates, the assumption of a full correlation for the seismic failures that have a high level correlation does not guarantee the conservative seismic multi-unit CDF (MUCDF). So, the assumption of a full correlation cannot be applied to the seismic MUPSA.

To make matters worse, in the seismic MUPSA, it is impossible to isolate highly correlated seismic failures for the MVN integration since they are combined with various failures through nested AND and OR gates in a seismic MUPSA fault tree, which results in

Table 8
COREX input for EDG (0.5g, K2/K3/S1/S2 NPPs).

NO_COMP	Number of seismic failures n
4	
BU_ACC	Ground acceleration a
0.50 0.50 0.50 0.50	
BU_CAP	Median capacity of components Am
0.85 1.34 1.48 1.48	
BU_COV	β_{Uij} for $\beta_{ij} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$
0.26 (a) 0.05 0.05	
(a) 0.17 0.05 0.05	
0.05 0.05 0.19 0.07	
0.05 0.05 0.07 0.19	
BR_COV	β_{Rij} for $\beta_{ij} = \sqrt{\beta_{Rij}^2 + \beta_{Uij}^2}$
0.24 0.20 0.20 0.20	
0.20 0.26 0.20 0.20	
0.20 0.20 0.34 0.20	
0.20 0.20 0.20 0.34	
EVENTS	Correlated seismic events (X_1, X_2, X_3, X_4)
S05_K2-SDGAF	
S05_K3-SEIS_EGDGR-ALL	
S05_S1-SEIS-EGDGS01-ALL	
S05_S2-SEIS-EGDGS01-ALL	
CCF_PREFIX	Prefix of seismic CCFs ($(Q_i, Q_{ij}, Q_{ijk}, Q_{ijkl})$ (b)
S05_EDG_Q	

(a) 0.0707107.
(b) See Table 9 and Figs. 2 and 3.
(c) Left column has COREX input format, right column has explanations of input.

impossible calculation of the seismic MUCDF. However, if the highly correlated seismic failures are converted into seismic CCFs by COREX, it is possible to calculate the seismic MUCDF by usual methods and tools for the internal PSA. Furthermore, the methods in this study and the COREX code have a great strength that can convert correlated seismic failures regardless of identical or similar components.

The conversion methods from correlated seismic failures into seismic CCFs were applied to the real seismic MUPSA that has four seismic PSAs of Kori-2, Kori-3, Sin-Kori-1, and Shin-Kori-2 (K2, K3, S1, S2) NPPs in one Kori nuclear site. By reviewing individual seismic event trees such as K2 seismic event tree in Fig. 1, four groups of correlated seismic failures in Table 7 were selected. Each group has four seismic failures. Although a single NPP has multiple components in each group (125V DC motor control centers, 480V AC motor control centers, or emergency diesel generators), it is assumed that each NPP has one component in each group in order to demonstrate the strength of the seismic CCF calculation with a simple problem.

(1) Group 1 – Four 125V DC control centers (structure failures) in four NPPs
(2) Group 2 – Four 480V AC motor control centers (structure failures) in four NPPs
(3) Group 3 – Four emergency diesel generators (structure failures) in four NPPs
(4) Group 4 – Four offsite powers (function failures) in four NPPs

Here, each group is separatedly solved by COREX. That is, four seismic failures in each group are converted into $2^4 - 1$ seicmic CCFs by using COREX.

As illustrated in Tables 8 and 9, correlated seismic failures of four emergency diesel generators (EDGs) are converted into $2^4 - 1$ seismic CCFs by COREX. Table 8 has input parameters and Table 9 has calculation results. As listed in Table 8, four NPPs (K2/K3/S1/S2) have four correlated seismic EDG failures (S05_K2-SDGAF, S05_K3-SEIS_EGDGR-ALL, S05_S1-SEIS-EGDGS01-ALL, and S05_S2-SEIS-EGDGS01-ALL). As shown in Table 9, they are converted into $2^4 - 1$ seismic CCFs by the COREX code. The COREX output for S05_K2-SDGAF is depicted in Figs. 2 and 3. As demonstrated in this example, COREX can convert correlated seismic failures into seiscmic CCFs. The number of correlated seismic failures that can be converted into seismic CCFs depends on the difficulties (1) to calculate combination failures of correlated seismic failures in Section 2 and (2) to solve simultaneous equations in Section 3.

Table 9
COREX output for EDG (0.5g, K2/K3/S1/S2 NPPs).

S05_K2-SDGAF +
S05_EDG_Q1
S05_EDG_Q12
S05_EDG_Q13
S05_EDG_Q123
S05_EDG_Q14
S05_EDG_Q124
S05_EDG_Q134
S05_EDG_Q1234
S05_K3-SEIS_EGDGR-ALL +
S05_EDG_Q2
S05_EDG_Q12
S05_EDG_Q23
S05_EDG_Q123
S05_EDG_Q24
S05_EDG_Q124
S05_EDG_Q234
S05_EDG_Q1234
S05_S1-SEIS-EGDGS01-ALL +
S05_EDG_Q3
S05_EDG_Q13
S05_EDG_Q23
S05_EDG_Q123
S05_EDG_Q34
S05_EDG_Q134
S05_EDG_Q234
S05_EDG_Q1234
S05_S2-SEIS-EGDGS01-ALL +
S05_EDG_Q4
S05_EDG_Q14
S05_EDG_Q24
S05_EDG_Q124
S05_EDG_Q34
S05_EDG_Q134
S05_EDG_Q234
S05_EDG_Q1234
S05_EDG_Q1 1.75985e-001
S05_EDG_Q2 3.74139e-005
S05_EDG_Q3 2.54345e-002
S05_EDG_Q4 2.54345e-002
S05_EDG_Q12 7.04162e-005
S05_EDG_Q13 9.14499e-003
S05_EDG_Q23 8.36778e-006
S05_EDG_Q14 9.14499e-003
S05_EDG_Q24 8.36778e-006
S05_EDG_Q34 1.65034e-003
S05_EDG_Q123 2.78606e-005
S05_EDG_Q124 2.78606e-005
S05_EDG_Q134 1.62879e-003
S05_EDG_Q234 3.34388e-006
S05_EDG_Q1234 1.90743e-005

(a) Four correlated seismic failures (S05_K2-SDGAF, S05_K3-SEIS_EGDGR-ALL, S05_S1-SEIS-EGDGS01-ALL, and S05_S2-SEIS-EGDGS01-ALL) are converted into seismic CCFs.

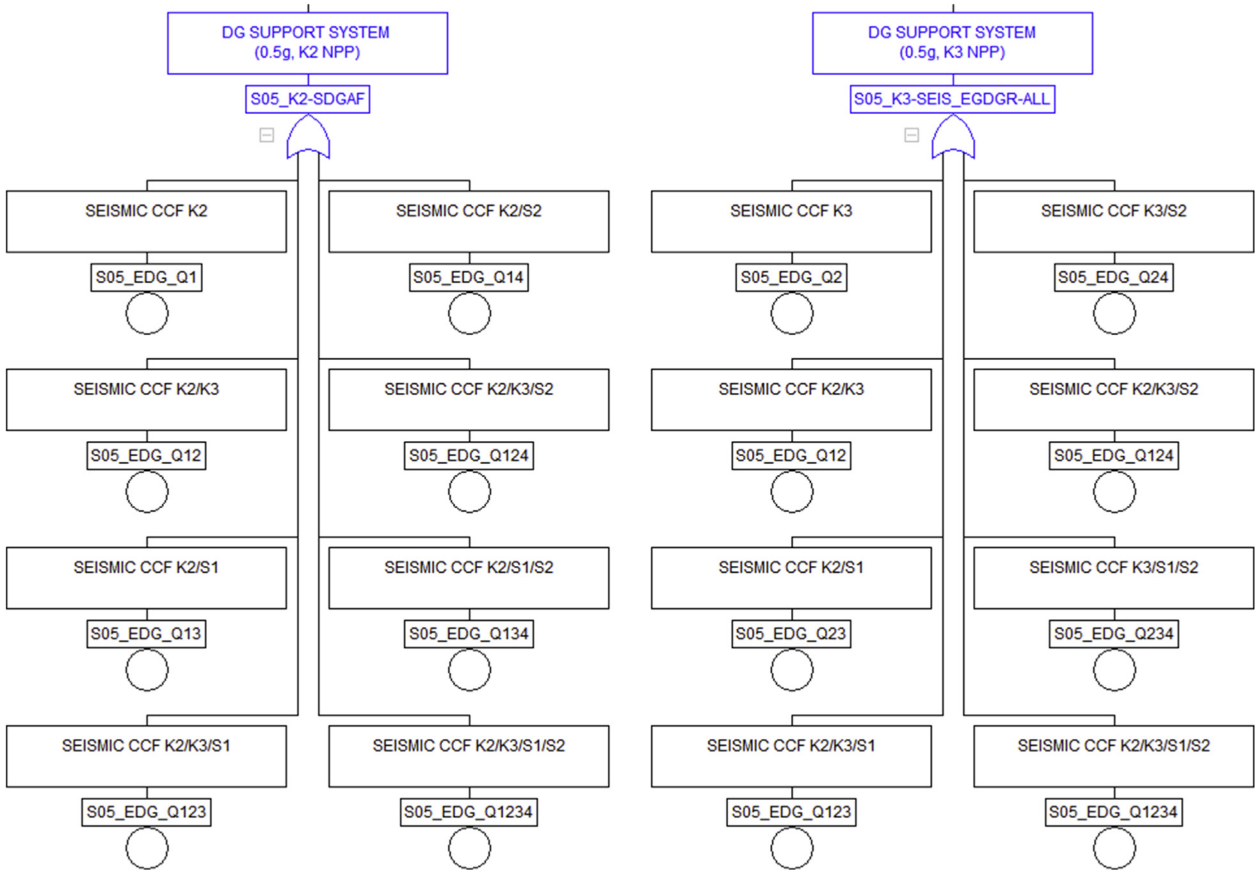

Fig. 2. Seismic CCFs converted from seismic failure of EDGs (0.5g, K2/K3)
(a) S05_K2-SDGAF is converted into seismic CCFs. (b) S05_K3-SEIS_EGDGR-ALL is converted into seismic CCFs.

5. Conclusions

The systems analysts assign an extreme correlation value of 0 and 1 for fully independent and fully dependent failures respectively depending on their judgment on correlation among seismic failures. This practical approach inevitably increases the uncertainty in a seismic CDF. In order to avoid this uncertainty, there has been a great need to develop a new methodology to explicitly model seismic correlation with seismic CCFs.

This paper first proposes a methodology to explicitly model dependency among seismic failures by converting correlated seismic failures into seismic CCFs. In this paper, the approximate and exact conversion methods from correlated seismic failures into seismic CCFs are proposed. The recommended method in this paper is based on OR combination probabilities of correlated seismic failures, which generates exact CCF probabilities. The efficiency and strength of the method developed in this study is validated through small Benchmark problems and a real seismic MUPSA model.

Existing methodologies such as MVN and Reed-McCann methods have some limitation in use for seismic PSA application. If correlated seismic failures are combined with non-correlated seismic failures and/or random failures in a nested way in a whole fault tree, their combination probabilities cannot be calculated by using the existing integration methods. The proposed method in this study removes this difficulty by converting correlated seismic failures into seismic CCFs, and explicitly modeling these CCFs in the seismic fault tree.

If the number of seismically correlated groups is one and correlation exists among a few seismic failures, their combination failure probabilities can be easily calculated and reflected into seismic CDF. However, if the number of seismically correlated groups is greater than one, and the correlated seismic failures spread all over the fault tree or MCSs in a complex way, there is no way to calculate complex combination probabilities among correlated seismic failures. The method in this paper makes it possible to explicitly reflect this complex seismic event correlation into the fault tree or MCSs by converting correlated seismic failures into seismic CCFs.

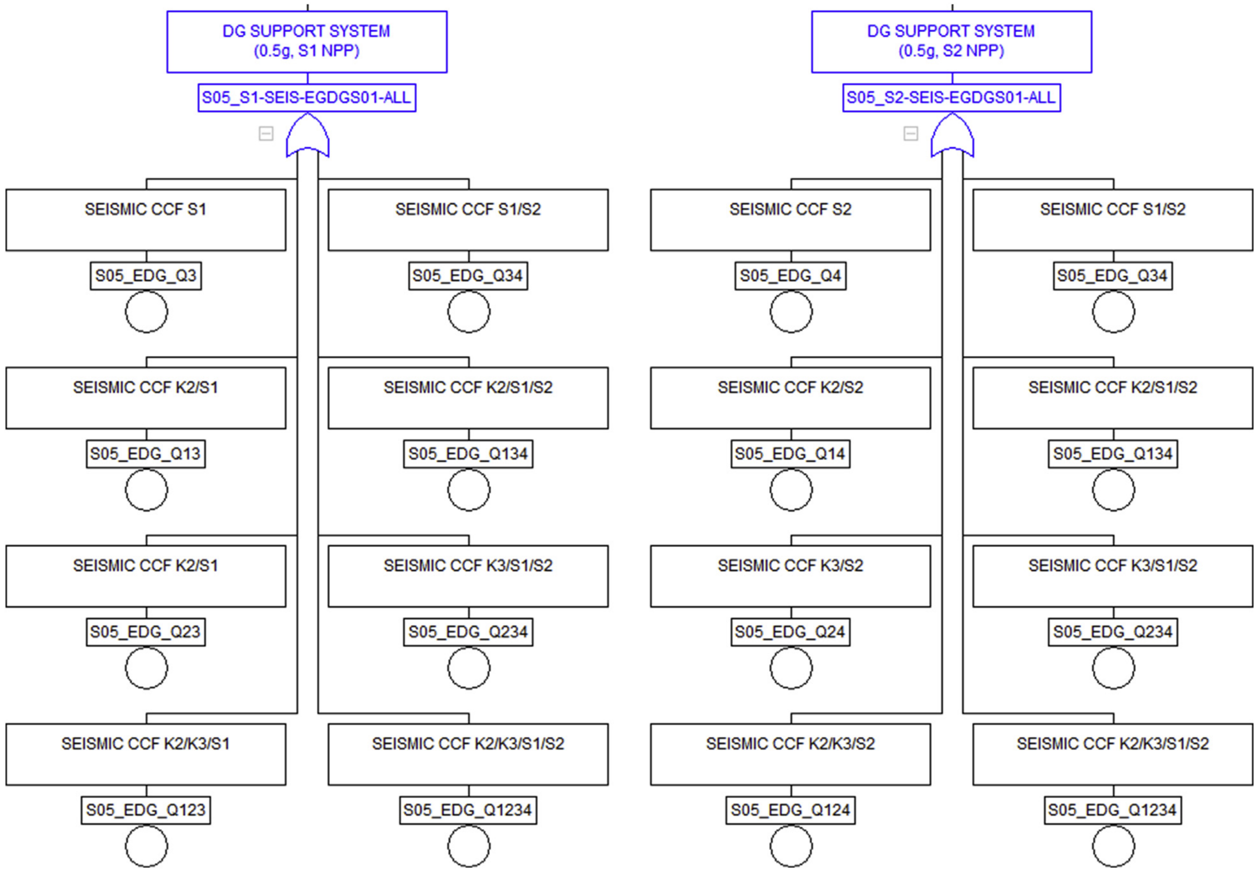

Fig. 3. Seismic CCFs converted from seismic failure of EDGs (0.5g, S1/S2).
(a) S05_S1-SEIS-EGDGS01-ALL is converted into seismic CCFs. (b) S05_S2-SEIS-EGDGS01-ALL is converted into seismic CCFs"

By the application of this methodology, it becomes possible to calculate accurate seismic CDF in a complex seismic PSA model with the same tools and methods that are typically used for internal, fire, and flooding PSAs. As a result, this method will allow systems analysts to quantify seismic risk as what they have done with the CCF method in internal, fire, and flooding PSA.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Probability calculation from MCS

If a fault tree S has MCSs $c_i's$ in Eq. (A.1), the exact inclusion exclusion (IE) probability P_{IE} is calculated as in Eq. (A.2) and an approximate min cut upper bound (MCUB) probability P_{MCUB} is calculated as in Eq. (A.3). If there are duplicated component failures in MCSs $(s_1, s_2, s_3, \ldots)$, P_{MCUB} is an approximate probability. In contrast, if all MCSs do not share any component failures, P_{IE} and P_{MCUB} are identical since $P(s_i s_j)$ is equal to $P(s_i)P(s_j)$. Thus, P_{MCUB} is an exact probability when there are no duplicated component failures in MCSs. It means that MCUB probability can be either an exact or approximate probability depending on MSC structure.

$$S = \bigcup_i s_i = s_1 + s_2 + s_3 \tag{A.1}$$

$$P(S) = P_{IE} = \sum_i P(s_i) - \sum_{1 \le i < j \le n} P(s_i s_j) + \sum_{1 \le i < j < k \le n} P(s_i s_j s_k) \tag{A.2}$$

$$p(S) \approx P_{MCUB} = 1 - \prod_i (1 - P(s_i)) = \sum_i P(s_i) - \sum_{1 \le i < j \le n} P(s_i)P(s_j) + \sum_{1 \le i < j < k \le n} P(s_i)P(s_j)P(s_k) \tag{A.3}$$

Appendix B. Reed-McCann integration

As an alternate method of MVN integration in Section 2, $P_{1+2+\ldots+n}(a) = P(U_{i=1}^{n} A_i < a)$ can be calculated by Reed-McCann integration [4]. Both integrations produce essentially identical results.

$$P_{1+2+\ldots+n}(a) = \int_0^\infty \ldots \int_0^\infty f_{or}(\mathbf{x}) g(\mathbf{x})\, dx_{12} \ldots dx_{(n-1)n} \tag{B.1}$$

$$f_{or}(\mathbf{x}) = 1 - \prod_{i=1}^{n} \left(1 - \Phi\left(\frac{\ln\left(\frac{a}{A_{im} \prod_{j \neq i} x_{ij}} \right)}{\beta_i^-} \right) \right) \tag{B.2}$$

$$g(\mathbf{x}) = \varphi\left(\frac{\ln x_{12}}{\beta_{12}}\right) \frac{1}{\beta_{12} x_{12}} \ldots \varphi\left(\frac{\ln x_{(n-1)n}}{\beta_{(n-1)n}}\right) \frac{1}{\beta_{(n-1)n} x_{(n-1)n}} \tag{B.3}$$

Here $x_{ij} = x_{ji}$, $\beta_i^- = \sqrt{\beta_i^2 - \sum_{j=1,\, j \neq i}^{n} \beta_{ij}^2}$, and $\beta_{ij} = \beta_{ji}$. $\Phi()$ is a standard normal cumulative distribution function, and $\varphi()$ is a standard normal probability density function.

If three failures are correlated, $P_{1+2+3}(a)$ is calculated by using the following equations.

$$P_{1+2+3}(a) = \int_0^\infty \int_0^\infty \int_0^\infty f_{or}(\mathbf{x}) g(\mathbf{x})\, dx_{12} dx_{13} dx_{23} \tag{B.4}$$

$$\begin{aligned} f_{or}(\mathbf{x}) = 1 - &\left(1 - \Phi\left(\frac{\ln\left(\frac{a}{A_{1m} x_{12} x_{13}} \right)}{\beta_1^-} \right) \right) \\ &\left(1 - \Phi\left(\frac{\ln\left(\frac{a}{A_{2m} x_{12} x_{23}} \right)}{\beta_2^-} \right) \right) \\ &\left(1 - \Phi\left(\frac{\ln\left(\frac{a}{A_{3m} x_{13} x_{23}} \right)}{\beta_3^-} \right) \right) \end{aligned} \tag{B.5}$$

$$g(\mathbf{x}) = \varphi\left(\frac{\ln x_{12}}{\beta_{12}}\right) \frac{1}{\beta_{12} x_{12}} \varphi\left(\frac{\ln x_{13}}{\beta_{13}}\right) \frac{1}{\beta_{13} x_{13}} \varphi\left(\frac{\ln x_{23}}{\beta_{23}}\right) \frac{1}{\beta_{23} x_{23}} \tag{B.6}$$

$$\begin{aligned} \beta_1^- &= \sqrt{\beta_1^2 - \left(\beta_{12}^2 + \beta_{13}^2\right)} \\ \beta_2^- &= \sqrt{\beta_2^2 - \left(\beta_{12}^2 + \beta_{23}^2\right)} \\ \beta_3^- &= \sqrt{\beta_3^2 - \left(\beta_{13}^2 + \beta_{23}^2\right)} \end{aligned} \tag{B.7}$$

Appendix C. Simultaneous equations using AND combination probabilities

The combination probability of seismic failures such as $P_{12\ldots n}(a) = P(\cap_{i=1}^{n} A_i < a)$ is calculated by using Monte-Carlo integration of multivariate normal (MVN) distribution [3,4].

$$P_{12\ldots n}(a) = \int_{-\infty}^{\ln\left(\frac{a}{A_{1m}}\right)} \int_{-\infty}^{\ln\left(\frac{a}{A_{2m}}\right)} \ldots \int_{-\infty}^{\ln\left(\frac{a}{A_{nm}}\right)} \frac{1}{\sqrt{|\Sigma|(2\pi)^n}} \exp \times \left(-\frac{1}{2} \mathbf{x}^t \Sigma^{-1} \mathbf{x} \right) d\mathbf{x} \tag{C.1}$$

If three correlated seismic failures are correlated, $2^3 - 1$ probability equations are constructed as in Eq. (C.2). They are nonlinear simultaneous equations based on MCUB probability equations. This system of equations has $2^3 - 1$ knowns of combination probabilities and $2^3 - 1$ unknowns of seismic CCF probabilities (Q_1, Q_2, Q_3, Q_{12}, Q_{13}, Q_{23}, Q_{123}).

$$\begin{aligned} P_1 &= 1 - (1 - Q_1)(1 - Q_{12})(1 - Q_{13})(1 - Q_{123}) \\ P_2 &= 1 - (1 - Q_2)(1 - Q_{12})(1 - Q_{23})(1 - Q_{123}) \\ P_3 &= 1 - (1 - Q_3)(1 - Q_{13})(1 - Q_{23})(1 - Q_{123}) \\ P_{12} &= 1 - (1 - Q_1Q_2)(1 - Q_{12})(1 - Q_{123})(1 - Q_1Q_{23})(1 - Q_2Q_{13})(1 - Q_{13}Q_{23}) \\ P_{13} &= 1 - (1 - Q_1Q_3)(1 - Q_{13})(1 - Q_{123})(1 - Q_1Q_{23})(1 - Q_3Q_{12})(1 - Q_{12}Q_{23}) \\ P_{23} &= 1 - (1 - Q_2Q_3)(1 - Q_{23})(1 - Q_{123})(1 - Q_2Q_{13})(1 - Q_3Q_{12})(1 - Q_{12}Q_{13}) \\ P_{123} &= 1 - (1 - Q_1Q_2Q_3)(1 - Q_{123})(1 - Q_1Q_{23})(1 - Q_2Q_{13})(1 - Q_3Q_{12})(1 - Q_{12}Q_{13}) \\ &\quad (1 - Q_{13}Q_{23})(1 - Q_{12}Q_{23}) \end{aligned} \tag{C.2}$$

Eq. (C.2) has MCUB probabilities of the Boolean equations that have seismic CCFs.

$$\begin{aligned} X_1 &= C_1 + C_{12} + C_{13} + C_{123} \\ X_2 &= C_2 + C_{12} + C_{23} + C_{123} \\ X_3 &= C_3 + C_{13} + C_{23} + C_{123} \\ X_{12} &= C_1C_2 + C_{12} + C_{123} + C_1C_{23} + C_2C_{13} + C_{13}C_{23} \\ X_{13} &= C_1C_3 + C_{13} + C_{123} + C_1C_{23} + C_3C_{12} + C_{12}C_{23} \\ X_{23} &= C_2C_3 + C_{23} + C_{123} + C_2C_{13} + C_3C_{12} + C_{12}C_{13} \\ X_{123} &= C_1C_2C_3 + C_{123} + C_1C_{23} + C_2C_{13} + C_3C_{12} + C_{12}C_{13} \\ &\quad + C_{13}C_{23} + C_{12}C_{23} \end{aligned} \tag{C.3}$$

Similarly, if two correlated seismic failures are correlated, $2^2 - 1$

probability equations are constructed as in Eq. (C.4).

$$
\begin{aligned}
P_1 &= 1-(1-Q_1)(1-Q_{12})\\
P_2 &= 1-(1-Q_2)(1-Q_{12})\\
P_{12} &= 1-(1-Q_1Q_2)(1-Q_{12})
\end{aligned}
\tag{C.4}
$$

Eq. (C.4) has MCUB probabilities of the Boolean equations that have seismic CCFs.

$$
\begin{aligned}
X_1 &= C_1 + C_{12}\\
X_2 &= C_2 + C_{12}\\
X_{12} &= C_1C_2 + C_{12}
\end{aligned}
\tag{C.5}
$$

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