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Estimation of kernel function using the measured apparent earth resistivity

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Abstract

In this paper, we propose a method to derive the kernel function directly from the measured apparent earth resistivity. At this time, the kernel function is obtained through the process of solving a nonlinear system. Nonlinear systems with many variables are difficult to solve. This paper also introduces a method for converting nonlinear derived systems to linear systems. The kernel function is a function of the depth and resistance of the Earth's layer. Being able to derive an accurate kernel function means that we can estimate the earth parameters i.e. layer depth and resistivity. We also use various Earth models as simulation examples to validate the proposed method.

Keywords: *kernel function, apparent earth resistivity, nonlinear system, horizontal r.*

1. Introduction

When installing an underground system, knowledge of the underground structure of the area is absolutely necessary. For underground systems, parameters for stratification are essential for field or circuit level simulation. This is because mis-designed grounding systems cannot guarantee not only high-priced equipment but also safety of life [1].

Apparent earth resistivity is used to estimate earth parameters such as resistivity and thickness of the soil layers. Apparent earth resistivity can be measured and theoretically calculated. The kernel function is used to theoretically calculate apparent earth resistivity and is a function of earth parameters. The most widely used method of measuring apparent earth resistivity is Wenner's four-electrode method. Estimation of the earth parameters is accomplished by repeatedly modifying the earth parameters so that the theoretical calculated apparent resistivity is close to the measured value [2,3]. This results in an optimization problem that minimizes

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the squared value of difference between the calculated value and the measured value as a cost function. Various optimization problem solving methods are used as a way to solve this problem, but the procedure is roughly similar to the following. Initially, the parameters are given a virtual initial value, and the apparent earth resistivity is calculated using these values. The earth structure parameters are repeatedly modified until the calculated apparent earth resistivity becomes similar to the measured apparent earth resistivity [4].

The kernel function is a function of the earth parameters, and it is known that the correct kernel function is not known until the correct earth parameters are determined. In general optimization problem solving methods, a new kernel function is updated every time the parameter values are modified, and the apparent earth resistivity is calculated using the updated kernel function. In other words, the kernel function becomes closer to the correct one as the earth parameters approach to the correct ones. With the conventional method, the valid kernel function is determined gradually as the earth parameters are estimated close to correct value [2,5].

In this paper, we propose a new method to obtain the kernel function directly from the measured apparent resistivity without estimating parameters. It is believed that this obtained kernel function can provide a new method that can estimate the earth parameters faster than other optimization solutions. Wenner electrode configuration was adopted for measuring apparent resistivity. In the simulation, various layers of earth structures were used to evaluate the validity of the proposed method.

2. Apparent Resistivity

Apparent earth resistivity is used to estimate indicators such as the resistance and thickness of the soil layer. Apparent earth resistivity can be measured and theoretically calculated. Three methods are usually used for measuring apparent earth resistivity such as Wenner’s method, Schulumberger’s method and dipole method. Among these methods, Wenner method is most widely used [5].

2.1 Wenner method for measuring apparent resistivity

As mentioned above, the most widely used method for measuring earth resistivity is Wenner's 4-electrode method, published by Frank Wenner in 1915. In this method, to simplify the problem, it is assumed that the ground is composed of N layers horizontally, and each layer has the same resistivity [2,4].

Figure 1 shows the electrode arrangement of Wenner's four-electrode method. In Wenner's method, the four electrodes are placed with the same distance a, and the voltage applied to the two internal electrodes(A, B) is measured when the current is supplied through two external electrodes(C, D). Assuming the depth of last layer is infinite, h_i(i = 1, 2, ..., N-1) and ρ_i (i = 1, 2, ..., N) represent the resistivity and the depth of each soil layer. The data measured with the more wide span a are used for analyzing the more deep earth structure. The measured apparent earth resistivity can be obtained as follows [2,4].

$$\rho_a = 2\pi a \frac{\Delta V}{I} = 2\pi a R \tag{1}$$

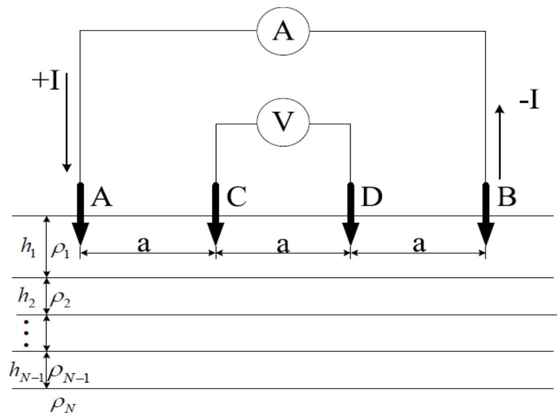

Figure 1. Wenner configuration for measuring apparent soil resistivity of N-layer earth structure

2.2 Kernel function for calculating apparent earth resistivity

The Kernel function is used to calculate apparent earth resistivity theoretically and its equation is [2].

$$\rho_a = \rho_1\left[1 + 2a\int_0^{\infty} f(\lambda)[J_0(\lambda a) - J_0(2\lambda a)]d\lambda\right] \tag{2}$$

where ρ_1 is the soil resistivity of the first layer, a is electrode span, $J_0(\lambda r)$ is the zero order Bessel's function of the first kind and $f(\lambda)$ is the kernel function. In the earth structure composed of N layers, the parameters are the depth of each layer, h_i(i = 1, 2, ..., N-1) and resistivity ρ_i (i = 1, 2, ..., N). The kernel function of earth resistivity is defined as a function of depth and resistivity of earth layer as follows [6,7].

$$f(\lambda) = \alpha_1 - 1 \tag{3}$$

$$\alpha_1 = 1 + \frac{2K_1 e^{-2\lambda h_1}}{1 - K_1 e^{-2\lambda h_1}} \qquad k_1 = \frac{\rho_2\alpha_2 - \rho_1}{\rho_2\alpha_2 + \rho_1}$$

$$\alpha_2 = 1 + \frac{2K_2 e^{-2\lambda h_2}}{1 - K_2 e^{-2\lambda h_2}}, \qquad k_2 = \frac{\rho_3\alpha_3 - \rho_2}{\rho_3\alpha_3 + \rho_2}$$

$$\vdots$$

$$\alpha_{N-1} = 1 + \frac{2K_{N-1} e^{-2\lambda h_{N-1}}}{1 - 2K_{N-1} e^{-2\lambda h_{N-1}}}, \qquad k_{N-1} = \frac{\rho_N + \rho_{N-1}}{\rho_N + \rho_{N-1}}$$

3. Estimation of earth kernel function calculation

The earth kernel function is a function of earth parameters which are used to calculate apparent earth resistivity. If the kernel function can be obtained directly from the measured apparent resistivity, it is believed to use this kernel function to estimate the earth parameters faster than other optimization solutions [8].

3.1 Inducing Kernel function from the measured apparent earth resistivity

It is known that the kernel function $f(\lambda)$ is assumed to be a continuous and smooth function with respect to λ, and the property of its asymptotic function is much similar to the damping exponential function [2]. So the kernel function $f(\lambda)$ can be approximated by a series of exponential function using Prony's method as follows

$$f(\lambda) \simeq \sum_{k=1}^{n} b_k e^{-c_k\lambda} \tag{4}$$

where b_k, c_k are complex numbers, n is sampling number. With the given soil parameters, $f(\lambda)$ is determined using equation (3) and b_k, c_k in (4) are also approximated with the n sampling points using Prony's method. The first step is to make the assumed kernel function $f(\lambda)$ with the updated soil parameters, and the next step is to obtain n sampling points with this $f(\lambda)$. With those sampling points, b_k, c_k in (4) can be approximated using Prony's method. The sampling number n can be set to any enough big number, but according to author's experience, it is suggested to set bigger than the twice of the layer's number (*N*).

Using the following Lipschitz integral [2].

$$\int_0^\infty e^{-\lambda|c|} J_0(\lambda l)\, d\lambda = \frac{1}{\sqrt{c^2 + l^2}} \tag{5}$$

The apparent resistivity ρ_a^c in (2) can be approximated as follows

$$\rho_a^c(a) \simeq \rho_1 \left\{ 1 + 2a \sum_{k=1}^{N} b_k \left[\frac{1}{\sqrt{c_k^2 + a^2}} - \frac{1}{\sqrt{c_k^2 + 4a^2}} \right] \right\} \tag{6}$$

Using the measured apparent earth resistivity, we rearrange Eq. (6) to obtain b_k, replacing ρ_a with the measured apparent resistivity ρ_{ai}, which can be expressed as:

$$\sum_{k=1}^{N} b_k \left[\frac{1}{\sqrt{c_k^2 + {a_i}^2}} - \frac{1}{\sqrt{c_k^2 + 4{a_i}^2}} \right] \cong \frac{1}{2a_i}\left(\frac{\rho_{ai}}{\rho_1} - 1\right), \qquad i = 1, 2, \ldots \ldots m \tag{7}$$

If the surface layer resistivity(ρ_1) is known, the right side of Eq. (7) is determined, so that it becomes a nonlinear system for two variables b_k , c_k. As can be seen from Eq. (2), the surface layer resistivity(ρ_1) can be replaced with the measured apparent resistivity at very close electrode distance. The author 's experience showed that the apparent resistivity measured at about 0.1 [m] was very close to the surface layer resistivity(ρ_1). Thus, Eq. (7) can be expressed as a nonlinear system and the solution of a nonlinear system is obtained using various iterative methods such as the Newton-Raphson method [9].

3.2 Linearized system for determining Kernel function

However, equation (7) is a nonlinear system, and it is not easy to obtain by a general iterative method because it requires a large number of b_k, c_k to obtain a kernel function. Assuming that $f(\lambda)$ in Eq. (4)

consists of a number of exponential functions whose exponential exponents are increased at regular intervals as follows.

$$f(\lambda) \cong \sum_{k=1}^{n} b_k e^{-dk\lambda} \qquad (8)$$

Since the value in parentheses on the left side of equation (7) is determined, it becomes a linear system. Here, d is a very small constant value, and according to the author's experience, it is enough to set it to about 0.1. Then it can be expressed as a linear system as follows.

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & & A_{2N} \\ \vdots & & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mN} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} = \begin{bmatrix} \frac{1}{2a_1}\left(\frac{\rho_1}{\rho_1} - 1\right) \\ \frac{1}{2a_2}\left(\frac{\rho_2}{\rho_1} - 1\right) \\ \vdots \\ \frac{1}{2a_m}\left(\frac{\rho_m}{\rho_1} - 1\right) \end{bmatrix} \qquad (9)$$

Where A_{ik} is

$$A_{ik} = \frac{1}{\sqrt{d^2k^2 + a_i{}^2}} - \frac{1}{\sqrt{d^2k^2 + 4a_i{}^2}} \qquad (10)$$

However, since equation (9) does not match the number of variables (N) and the number of equations (M), that is, M ≠ N, the only solution is not found. If the number of equations is greater than the number of variables, that is, M> N, then an overdetermined system cannot find a solution satisfying all of the equations and find an approximate value that minimizes the error. And in the opposite case, there are many kinds of solutions in the underdetermined system. Here, we have found through trial and error that the kernel function is obtained correctly only when N >> M. The authors' experience shows that the value of dn (the last value of dk) should be 4 to 5 times the value of a_m (the longest interelectrode span). Since the apparent surface resistivity measure number (M) is usually 10 to 20, Eq. (9) is generally an underdetermined linear system. There are various methods for solving the underdetermined linear system, but QR decomposition is widely used. In this paper, QR decomposition method is used [9].

4. Simulation and Results

In order to display the robustness of the proposed method, two typical models with some ideal resistivity profile of geological relevance are chosen. The first model consists of two layers with three earth parameters as shown in Table 1-1. The measured apparent resistivity consisting of 9 data points are used to calculate the kernel function in Table 1-2. The results from the proposed method are illustrated by the dotted line in Figure 2.a, showing best match between the exact kernel function and the estimated one. The second model has been chosen for more complicated example consisting of four layers with seven earth parameters in Table 2-1. The measured apparent resistivity in Table 2-2 are used to calculate the kernel function. The solid line in Figure 2.b is the exact kernel functions of the earth structure shown in Table 2-1. The dotted line is estimated using the proposed method, and showing it agrees with exact kernel function.

Table 1-1. Parameters of a two-layer earth structure

Layer No.	Resistivity ρ_i $(\Omega \cdot m)$	Depth h(m)
1	352	14
2	1600	∞

Table 1-2. The measured apparent resistivity of a two-layer earth structure

No	1	2	3	4	5	6	7	8	9
Electrode Span a(m)	0.1	1	2	3	5	7	10	15	20
Apparent resistivity ρ_a $(\Omega \cdot m)$	250.0	250.3	252.7	258.3	280.9	314.9	374.5	469.6	547.9

Table 2-1. Parameters of a four-layer earth structure

Layer No.	Resistivity ρ_i $(\Omega \cdot m)$	Depth h(m)
1	68	1.08
2	627.9	5.64
3	7.3	3.98
4	125.4	∞

Table 2-2. The measured apparent resistivity of a four-layer earth structure

No	1	2	3	4	5	6	7	8	9
Electrode Span a(m)	0.1	1	2	3	5	7	10	12	14
Apparent resistivity ρ_a $(\Omega \cdot m)$	68.0	89.2	140.2	185.3	243.1	266.1	259.8	241.4	218.5

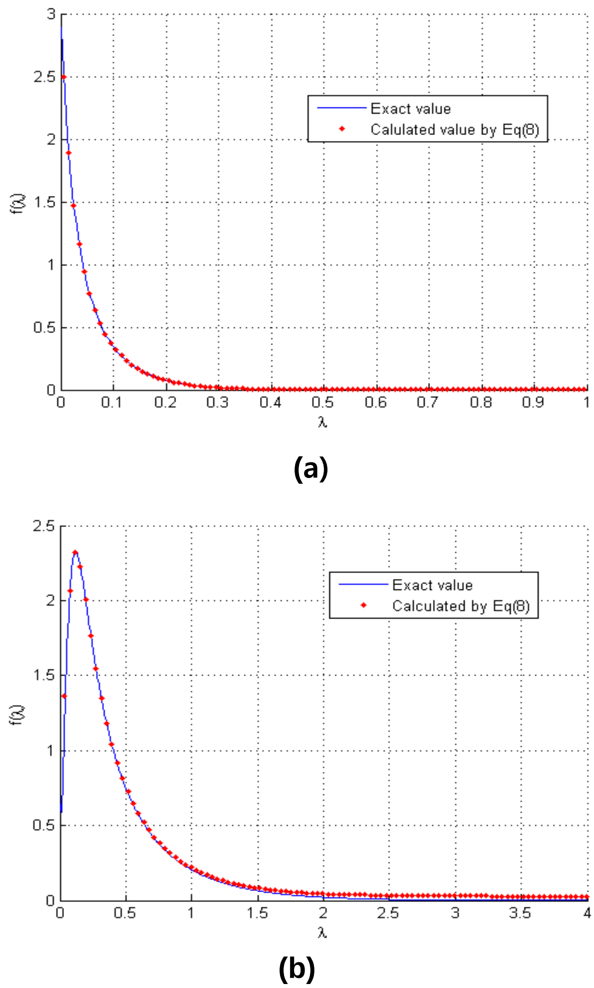

Figure 2. The kernel functions of earth structures (a) 2-layer and (b) 4-layer

5. Conclusion

A method has been proposed to derive a resistivity kernel function from the measured apparent earth resistivity for an arbitrary number of layers. The resistivity kernel function is a function of the earth layer's depth and resistivity. Being able to derive an accurate kernel function means that you can estimate the earth parameters, i.e. the layer depth and resistivity. In this paper, a method for deriving the kernel function is presented, and the next study intends to present a new method for estimating parameters using this kernel function. The numerical results show the validity of the proposed approach compared to the results obtained from the existing method.

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References

[1] A. I. Aderibigbe, Isaac Samuel, B. Adetokun and S. Tobi, “Monte Carlo Simulation Approach to Soil Layer Resistivity Modelling for Grounding System,” International Journal of Applied Engineering Research, vol. 12, pp. 13759-13766, 2017. Orcid: 0000-0002-9998-549X

[2] B. Zhang, X. Cui, L. Li, and J. L. He, “Parameter estimation of horizontal multilayer earth by complex image method,” IEEE Trans. on Power Delivery, vol. 20, pp. 1394—1401, 2005. DOI: 10.1109/TPWRD.2004.834673

[3] U.K. Singh, R.K. Tiwari and S. B. Singh, “One-dimensional of geo-electrical resisitivity sounding data using artificial neural networks- a case study,” Computer & Geoscience, vol. 31, pp. 99—108, 2005. https://doi.org/10.1016/j.cageo.2004.09.014

[4] K. H. Jin, M. T. Mccann, E. Froustey, and M. Unser, “Deep convolutional neural network for inverse problems in imaging,” IEEE Trans. Image Process, vol. 26, no. 9, pp. 4509–4522, Sep. 2017. DOI: 10.1109/TIP.2017.2713099

[5] G. Cheng, C. Yang, X. Yao, L. Guo, and J. Han, “When deep learning meets metric learning: Remote sensing image scene classification via learning discriminative CNNs,” IEEE Trans. Geosci. Remote Sens., vol. 56, no. 5, pp. 2811–2821, 2018. DOI: 10.1109/TGRS.2017.2783902

[6] T. Takahashi and T. Kawase, “Analysis of apparent resistivity in a multi-layer earth structure,” IEEE Trans. on Power Delivery, vol. 5, pp. 604—612, 1990. DOI: 10.1109/61.53062

[7] F. Dawalibi, “Earth resistivity measurement interpretation techniques,” IEEE Trans. on Power Apparatus Systems, vol. 103, pp. 374—382, 1984. DOI: 10.1109/TPAS.1984.318254

[8] J. C. Egbai, “Analysis of kernel function by the transformation of field data”, Advanced in Applied Science Research, vol. 3, pp. 508—519, 2012.

[9] S. C. Chapra, Applied Numerical Methods with MATLAB for Engineers and Scientists, 2nd ed. New York: McGrow-Hill, pp. 270-276, 2008.