

## A New Hybrid Algorithm for Invariance and Improved Classification Performance in Image Recognition

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### Abstract

*It is important to extract salient object image and to solve the invariance problem for image recognition. In this paper we propose a new hybrid algorithm for invariance and improved classification performance in image recognition, whose algorithm is combined by FT(Frequency-tuned Salient Region Detection) algorithm, Guided filter, Zernike moments, and a simple artificial neural network (Multi-layer Perceptron). The conventional FT algorithm is used to extract initial salient object image, the guided filtering to preserve edge details, Zernike moments to solve invariance problem, and a classification to recognize the extracted image. For guided filtering, guided filter is used, and Multi-layer Perceptron which is a simple artificial neural networks is introduced for classification. Experimental results show that this algorithm can achieve a superior performance in the process of extracting salient object image and invariant moment feature. And the results show that the algorithm can also classifies the extracted object image with improved recognition rate.*

**Keywords:** Salient object extraction, Invariance, Guided filtering, Zernike moments, Multi-layer Perceptron

### 1. Introduction

With the development of technologies and electronic equipment, a large amount of image information to be processed is given to deal with. The technologies and equipment makes the information processing speed slow and inefficient. Therefore, how to extract quickly the interest objects from massive image information is one of key technical issues in the field of image recognition. The salience of an item is the state or quality by which it stands out from its neighbors. Thus it is important to take culling redundant information in the original image and to make extracting areas that attract human attention.

In recent years, many researchers have concerned in salience image detection. In 2009, Achanta et al proposed FT model, which is a salient image detection model based on the center-periphery contrast [1]. The

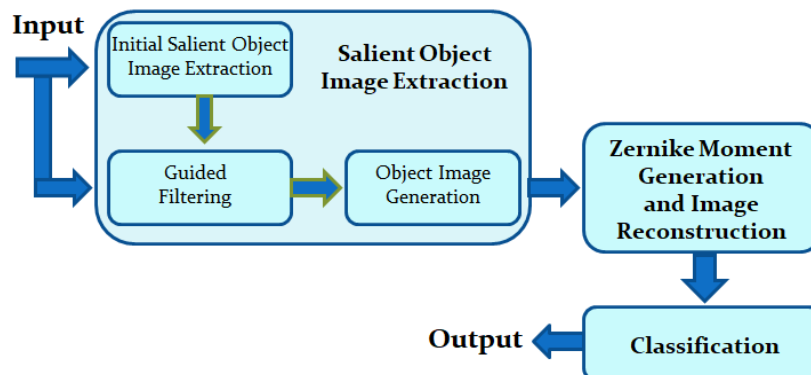
FT model performs local comparison at multiple scales, and measures the saliency of the image by calculating the distance between the pixel and the average feature vector of the close area. In 2011, Cheng et al proposed a method based on histogram contrast to detect the saliency of an image [2]. The algorithm determines the saliency value of the pixel by solving the color difference with other pixels, and finally generates a saliency map. In 2013, Yang proposed a bottom-up saliency detection model using contrast, center and smoothness prior [3]. This model can effectively enhance the target area while suppressing the background area. In 2018, Liu proposed a recursive convolutional neural network with a deep spatial context for saliency detection [4,5]. However, the salient objects obtained by the above methods are not complete, and introduce some disturbing by background areas. As another method, the use of deep learning neural networks to obtain salient object images requires the large amount of workload and the electronic equipment with high processing speed.

In this paper, we propose a new hybrid algorithm for extracting salient object image, shift/rotation invariance problem, and improved performance for image recognition. The proposed algorithm is the one to combine the conventional FT algorithm, guided filtering, Zernike moments, and a simple artificial neural network for classification. The proposed algorithm has the two advantages compared with conventional algorithms. First, this algorithm with guided filtering is performed to maintain the integrity and edge details of object image but also to have robustness for interference area. Second, through cropping the object image and using Zernike moments, the features of the object image has invariance characteristic. This proposed algorithm not only reduces the processing time of features extraction, but also makes the extracted feature more accurately express the object image.

## 2. Proposed Algorithm

The figure 1 shows the structure of our proposed algorithm, which consists of its key function modules: Salient object image extraction, Zernike moment generation and image reconstruction, and Classification.

First module of the proposed algorithm functions is the part for the extraction of initial salient object image, guided filtering, and object image generation. The final output of this module is the object image to be concerned. This module is divided into three subparts again: initial salient object image extraction, guided filtering, and object image generation. Here the initial salient object image is used as the filtering input of guided filter, whose filter is utilized to make salient image map. The guided filter can performs the function of the edge-preserving smoothing for salient image.



**Figure 1. The structure of proposed algorithm**

The second module performs the generation of Zernike moments and the image reconstruction from Zernike

moments. The Zernike moments have good rotation invariability and they can construct arbitrary high-order moments, so their application to the recognition of rotated object image shows good effects. The last module performs the function of classifying the salient object image. In this module, multi-layer Perceptron, which belongs to artificial neural network models, is used.

## 2.1 Salient Object Image Extraction

Generally, the human brain and visual system pay more attention to certain parts of the image. Visual attention mechanisms have been extensively studied in the fields of physiology, psychology nervous system, and computer vision [6, 7]. Recent research shows that the visual attention model can help us to detect, track [8], and recognize objects. Saliency is the perception of pixels in an object, person, or image that stand out relative to the field, thus drawing our visual attention [9]. In general, humans are attracted to object regions, not individual pixels. Therefore, it is of great significance to detect the salient region of the image. In this paper, FT algorithm is used to obtain the initial salient object image.

### 1) Initial Salient Image Extraction

#### Frequency-Tuned Salient Region Detection

The FT algorithm is introduced for salient region detection. The algorithm is very simple one in saliency calculation and it has a high reference rate, because it elevates the image saliency to the application level. People pay more attention to the salient region in the whole image rather than just some salient points. The algorithm first performs Gaussian blur (or Gaussian smoothing) on the image by software. In image processing, a Gaussian blur, also known as Gaussian smoothing, gives us the result of blurring an image by Gaussian function. It is a widely used in graphics software, typically to reduce image. The visual effect of this blurring is a smooth blur resembling that of viewing the image through a translucent screen. Mathematically, applying a Gaussian blur to an image is the same as convolving the image with a Gaussian function. It can also be used as a pre-processing stage in computer vision algorithms to enhance image structures at different scales.

The method of finding the saliency  $S(x,y)$  from an original source image  $I$  can be performed as

$$S(x,y) = |I_g(x,y) - I_u| \quad (1)$$

Where the  $I_g(x,y)$  is the Gaussian blurred version of original source image  $I$  which includes texture details as well as noise and coding artifacts, and the  $I_u$  is the arithmetic average pixel value of the input image  $I$ .

#### Initial Salient Image Extraction

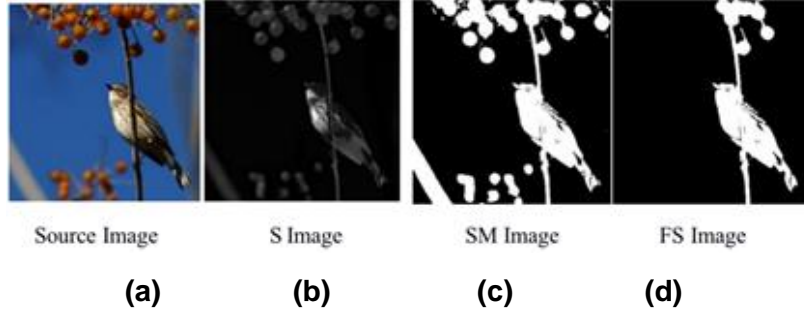
The figure 2 (a) and (b) show the images of initial source image and  $S$  image of Eq. (1) respectively. To extract initial salient image, we first need to take binarization of saliency image  $S$  by using a threshold value  $T$ . The binarization is performed to obtain  $SM(x,y)$  image as following:

$$SM(x,y) = \begin{cases} 1 & S(x,y) > T \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

We usually think of the salient region  $S$  as a complete connected region, so we need to eliminate some interference areas. Thus the second step for extracting initial salient image is to get the number of salient regions and to calculate the area of each region in the  $SM$ . And then we have to choose one of the regions. Generally the largest area in  $SM$  is chosen as the one we are most interested in, while other areas are considered as interference ones. Therefore, the area with the largest one in  $SM$  is taken as  $FS$ :

$$FS = \max(SMA_i) \quad i = 1,2,3 \dots N \quad (3)$$

where the  $N$  is the number of regions and the  $SMA_i$  is the area of the  $i$ th region. figure 2 (c) and (d) show the SM image and the FS image for the initial source image of figure 2 (a), respectively.



**Figure 2. An example of extracting initial salient image**

In the figure 2, we know that the salient FS image to be obtained can roughly locate the salient object image  $S$  but the information of the salient object image is not fully expressed such as the edge detail information. The FS image is the initial salient image, which is fed to the filtering input of the Guided filter to be followed in next

## 2) Guided Filtering and Salient Image Map Generation

In order to obtain a refined salient image map with more detailed information, the guided filter which is inspired by Rolling Guidance [10] is adopted in this work.

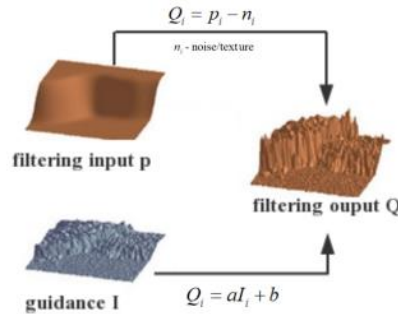
### Guided Filtering

The guided filter is an edge-preserving smoothing operator such as the popular bilateral filter [11,12], but it has better behavior near the edges. Thus the Guided filtering is an image filtering technique that uses a guidance image  $I_i$  to filter the input image  $p_i$ . The filter produces the output image  $Q_i$  of the guided filter, which is similar to the input image  $p_i$ , but it contains more detail information of guidance image. The relationship between input image  $p_i$  and output image  $Q_i$  with the guidance image  $I_i$  has the following:

$$Q_i = a_k I_i + b_k, \forall_i \in \omega_k, \quad a_k = \frac{1}{|\omega|} \frac{\sum_{i \in \omega_k} I_i p_i - u_k \bar{p}_k}{\delta_k^2 + \varepsilon}, \quad b_k = \bar{p}_k - a_k u_k \quad (4)$$

where the  $(a_k, b_k)$  are constant coefficients in  $\omega_k$ , and the  $\omega_k$  is the size of local window centered at pixel  $k$ , the  $|\omega|$  is the number of pixels in  $\omega_k$ , the  $\bar{p}_k$  is the mean of  $P$  in  $\omega_k$ , the  $u_k$  and  $\delta_k^2$  are the mean and variance of  $I_i$  in  $\omega_k$  respectively. The  $Q_i$  is a linear transform of  $I_i$  in a window  $\omega_k$  centered at pixel  $k$ . In this work, the initial input of the guided filter utilizes the FS image of the initial salient image. Figure 3 shows the signal processing steps of the guided filtering.

The guided filter has also a more generic concept beyond smoothing. It can transfer the structures of the guidance image to the filtering output, which enables new filtering applications like dehazing and feathering.



**Figure 3. Signal processing steps of guided filter (referred from Ref. [12])**

### Salient Image Map Generation

The gray level initial source image  $I$  is used as the guidance image and the salient image map to be binarized from the filtered output. Iterative filtering is done to obtain final salient image map, and the detailed steps are shown in table 1.

**Table 1. Steps for salient image map generation**

Step No.	Contents
<b>Step 1:</b>	Obtain the gray level image $I$ from the original source image and extract the initial salient image FS.
<b>Step 2:</b>	Use gray level image $I$ as the guidance image, and the FS of the initial salient image as the filtering input $p$ of the guided filter.
<b>Step 3:</b>	Obtain the filtering output $Q$ of guided filter by guidance image $I$ and filtering input $p$ .
<b>Step 4:</b>	Iterate the step 3 to get final salient image map until the required iteration is satisfied. In this step, the filtering output $Q$ is used as the next filtering input $p$ of the guided filter.

### Evaluation of Salient Object Image

In this algorithm, the different types of images were selected from the Corel-1000 data set, whose images include flowers, birds and so on. The number of iteration of guided filter was set to 10. Then we compared with two algorithms, which include FT model and GR model to get salient object [3]. And we also adopt objective evaluation index which includes the rate of precision, the rate of recall, and F-measure [10, 13]. They are given as follows:

$$precision = \frac{TP}{TP + FP} \quad (5)$$

$$recall = \frac{TP}{TP + FN} \quad (6)$$

$$F - measure = \frac{(1 + \beta) * precision * recall}{\beta * precision + recall} \quad (7)$$

where the TP is positive, the FP is false positive, the FN is false negative, and the  $\beta$  is a constant value.

### 3) Object Image Generation

Tailoring in image processing means creating communications, in which information about a given individual image is used to determine what the image will express. The claims about how to enhance the communication by tailoring fall into two types of classes mechanisms: (i) the tailoring enhances cognitive preconditions for image (or message) processing, and (ii) it tailors message impacts by selectively modifying initial behavioral determinants of desired outcomes.

Our proposed algorithm detects the salient object/region in an image and then tailors the salient object region to obtain the object image. Here the information expressed by the salient object is what the whole image wants to express. At the same time, some information that we do not need to pay attention to is discarded. This process not only reduces our field of vision that we need to pay attention, but also facilitates subsequent accurate identification. The main concern is to calculate the peripheral circle centered on the centroid of the salient object and to carry out adaptive clipping.

## 2.2 Zernike Moments Generation and Image Reconstruction

### Zernike Moments Generation

The proposed algorithm uses Zernike moments for the extraction of the invariant moment features of the object image after clipping. This process not only reduces the amount of computation time, but also reduces background noise during feature extraction.

Zernike moments is a relatively mature orthogonal invariant moments, which was introduced by Teague at 1980 [14]. It is based on orthogonal radial polynomial. Therefore, the extracted features have low correlation, less redundancy and strong anti-noise ability. Zernike moments have good rotation invariability, and can construct arbitrary high-order moments.

Zernike moments functions the projections of image functions  $f(x,y)$  onto orthogonal polynomials  $\{V_{nm}(x,y)\}$ . Where the  $\{V_{nm}(x,y)\}$  are orthogonal in the unit circle, they are calculated as  $V_{nm}(x,y) = V_{nm}(\rho, \theta) = R_{nm} \exp(jm\theta)$ , where the  $n$  is a positive integer or zero, the  $m$  is a positive integer or negative integer,  $n - |m| = \text{even number}$ , and  $|m| < n$ , the  $\rho$  is the length between the center point and  $(x, y)$ , the  $\theta$  is the angle between  $\rho$  vector and  $x$ , and the  $R_{nm}(\rho)$  is the radial polynomial. The  $N$  order Zernike moments are defined as follows:

$$A_{nm} = \frac{n+1}{\pi} \iint f(x,y) V_{nm}^*(\rho, \theta) \quad x^2 + y^2 \leq 1 \quad (8)$$

For two-dimensional images, Zernike moments  $A_{nm}$  are complex numbers. For digital images, integration is replaced by summation, which can be expressed as follows:

$$A_{nm} = \frac{n+1}{\pi} \sum_x \sum_y f(x, y) V_{nm}^*(\rho, \theta) \quad x^2 + y^2 \leq 1 \quad (9)$$

Usually, we use  $|A_{nm}|$  as a moment-invariant feature. It has the characteristic of rotation invariance. In this work, we adopt the scale normalization and use  $Z_{nm}$  as a feature [15]

$$|A_{nm}|' = \frac{|A_{nm}|}{m_{00}} \quad (10)$$

$$m_{00} = \iint f(x, y) dx dy \quad (11)$$

$$Z_{nm} = \log(|A_{nm}|') \quad (12)$$

Above process not only solves the translation invariance of the object, but also effectively avoids the background interference in the process of feature extraction, and reduces the feature extraction time.

## Image Reconstruction

One of the most important things for image recognition is the appropriate choice of numbers to represent an image. A major goal is to establish a system to be able to distinguish the localization in many parts of images, not just all image patterns.

Since the Zernike polynomials are an orthogonal basis set, it is possible to use the Zernike moments, which are calculated for a particular salient image, to reconstruct that image. In theory, the reconstruction of a continuous image without error requires an infinite number of Zernike moments. Since only limited moments is used to describe the images, it is of interest to examine representative images to be reconstructed from those moments. The reconstructions provide some insight into the amount of information that is included in the limited Zernike moments for classification. It is clear that much of the detailed information in each image is not preserved in the low degree moments.

### 2.3 Classification for Image Recognition

For the classification, we used multi-layer Perceptron which is one of artificial neural networks [16]. The multi-layer Perceptron is trained by BP(Back Propagation) algorithm, and it classifies any input image for recognition.

Through training process, the multi-layer Perceptron modifies its connection weights and threshold by the method of reducing the error between the output value and expected value. The training for multi-layer Perceptron belongs to supervised learning. This algorithm is based on the least squares method. For the training, we first prepare the training images  $y_i$  and the corresponding desired output  $d_i$ . for image pattern  $p$ . Here the error function E is defined as

$$E = \sum_{p=1}^M \sum_{i=1}^N (d_i^p - y_i^p)^2 \quad (13)$$

where the M is the number of input image sets, the N is the number of output neurons corresponding to the

number of training image patterns, the  $i$  is the index for the  $i$ th output neuron.

Training (or learning) is the process to adjust connection weights of the artificial neural network. The weight adjustment has been processed to minimize the error  $E$ , and accomplished by updating the connection weights as following.

$$\omega_{i,j}(t+1) = \omega_{i,j}(t) + \Delta\omega_{i,j}(t), \quad \Delta\omega_{i,j}(t) = -\eta\Delta\frac{\partial E}{\partial t} \quad (14)$$

where the  $\eta$  is learning rate. Repeat the above steps until the  $E$  reaches to sufficiently small value, whose value is predetermined by users. After finishing the training, we fix all the connection weights.

After training, the recall process has been done to classify the input images for recognition.

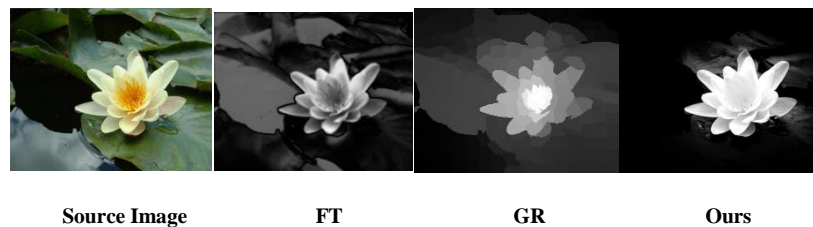
### 3. Simulation Results

The purpose of the computer simulation is to show how well the proposed algorithm works and how well it performs relative to conventional algorithms. The simulation is conducted on three parts: first, whether the salient image is extracted normally, second, whether the Zernike image is extracted properly through the Zernike moments, and finally, what the recognition rate of the source image through the proposed algorithm is and what the recognition rate through other algorithms are.

In our simulation, we used MATLAB R2018b software

#### 3.1 Salient Image Extraction

The experimental simulation results by Eq. (6) and Eq. (7) are shown in figure 4 and figure 5.



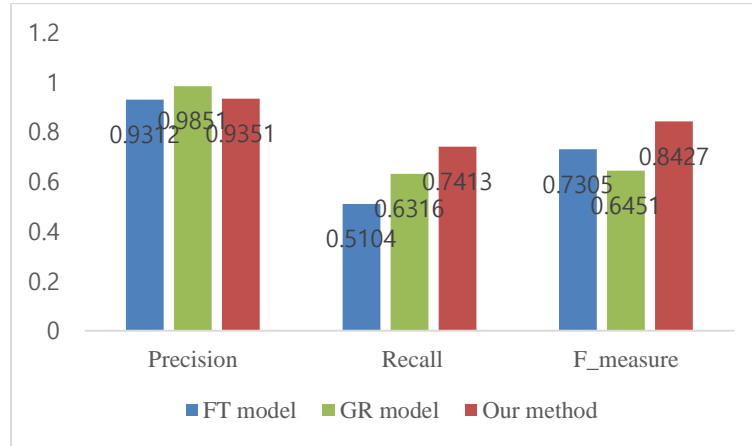
**Figure 4. An example for the extracted salient images according to different types of algorithm**

It can be seen from figure 4 and figure 5 that the FT model can get salient object with rich detail information, which more in line with human vision. But it introduce more background information.

The GR model get salient object with less detail information, and it misinterpret part of the background as a salient area. The result of salient object not only maintain the integrity and edges detail of object but also avoids the induction of the Interference area. Because of the FT model and GR model introduced more background information, so their recall rate is lower comprehensive and objective evaluation. But our



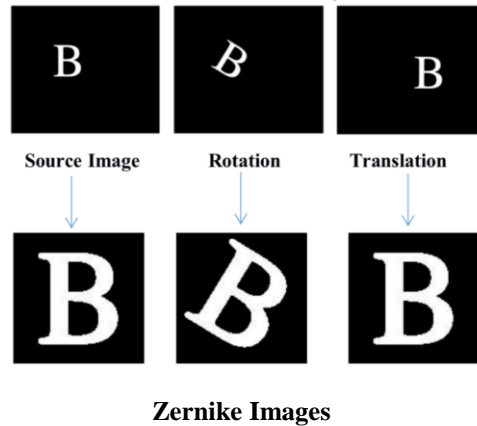
algorithm is superior to other algorithms in the view points of integrity and edges detail.



**Figure 5. Precision, Recall, and F measure for the extracted salient object image according to different types of models**

### 3.2 Zernike Images

In this simulation, firstly the geometric transformation (rotation, translation) of B-letter images was made. Secondly Zernike images were obtained. Finally the invariant characteristics of Zernike images were extracted. The simulation results were shown in figure 6.



**Figure 6. The Zernike images for different types of image ‘B’**

From figure 6, the features of Zernike moments remained unchanged after geometric transformation

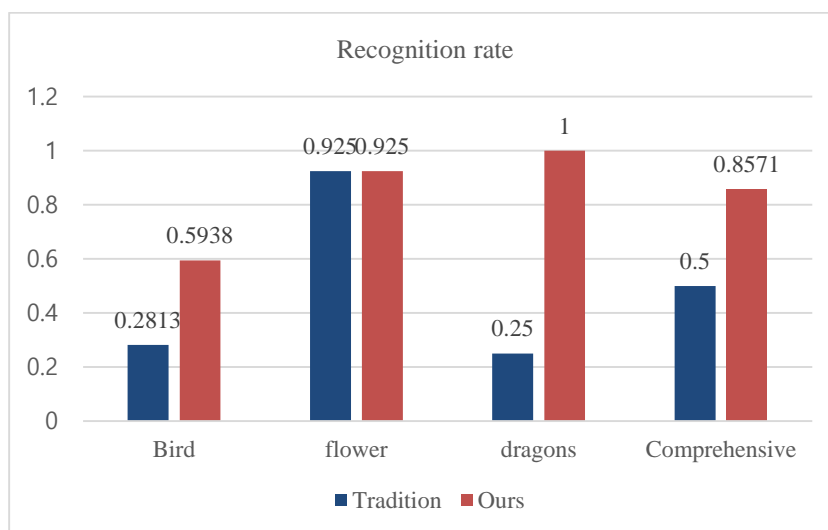
### 3.3 Classification Performance

In this simulation, the recognition rate of the proposed algorithm is compared with the rates of the conventional algorithm. Here Zernike moment features of the original image directly extracted directly and then they are used to train.

For the simulation, 3 different types of images were selected from the Corel-1000 data set, which include flowers, birds, and dinosaurs. There were 40 images for each type. Among the 40 images, the 30 images were selected for the training artificial neural network (Multi-layer Perceptron). And the total of 94 images were

obtained by geometric transformation (rotation, translation) of remaining 10 images at each type as the test set. And the neural network consists of 72 neurons at input, 35 neurons at the one hidden layer, 3 neurons at output layer, and the error E was set to 0.001.

Figure 7 shows the recognition rates (classification performances) for the proposed algorithm and conventional algorithm.



**Figure 7. Classification performance for the proposed and traditional algorithm: “Comprehensive” refers to the average of birds, flowers, and dragons**

From the figure 7, we can see that the recognition rate of bird is low. The reason why it is low is in the size of the bird in the test set images and posture of birds with inconsistency. The conventional algorithm is interfered by the background so that the recognition rate is much lower than that of ours. The recognition rate of flowers is similar each other, because the background of flowers is relatively single and they does not have the background noise, so the recognition rates are higher. The recognition rates of the dragons by ours is high, because the size and shape of the dragon are basically the same, and the algorithm will kick out the background noise, so the rate reaches 100%. However, the rate of conventional algorithm for dragons is very lower than that of ours because it is susceptible to background noise and/or interference. Thus some noise will be generated during rotation and translation, so the rate of conventional algorithm is lower than that of ours.

Because of the proposed hybrid characteristics, the classification performance of our algorithm is higher than that of the conventional algorithm, that is, the recognition rate of the proposed algorithm is improved effectively.

#### 4. Conclusion

In this paper we proposed a new hybrid algorithm for invariance and improved classification performance in image recognition. The proposed algorithm is the one to combine the FT algorithm, guided filtering, Zernike moments, and a simple artificial neural network. The Multi-layer Perceptron as a simple artificial neural network is adopted for image recognition. The algorithm has the two advantages compared with conventional algorithms. First, the algorithm with guided filter performs to maintain the integrity and edges detail of object image but also to have robustness for interference. Second, through cropping the object image and using Zernike moments, the features of the object image has invariance characteristic. Thus the algorithm reduces background noise, so does to improve the classification performance in the entire recognition process.

To certify the performance of our proposed algorithm, we performed computer simulations on the recognition rate for various image patterns. As test patterns, the total of 94 images were obtained by geometric transformation (rotation, translation). The test patterns are about birds, flowers, and dragons. For the comparison, the performance of the proposed algorithm is compared with the one of conventional algorithm. Experimental results showed that the performance of the proposed algorithm with about 86% was superior to that of the conventional algorithm with about 50%. The proposed algorithm could be usefully applied to the image recognition with arbitrary size and orientation.

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