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On Power Calculation for First and Second Strong Channel Users in *M*-user NOMA System

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Abstract

Non-orthogonal multiple access (NOMA) has been recognized as a significant technology in the fifth generation (5G) and beyond mobile communication, which encompasses the advanced smart convergence of the artificial intelligence (AI) and the internet of things (IoT). In NOMA, since the channel resources are shared by many users, it is essential to establish the user fairness. Such fairness is achieved by the power allocation among the users, and in turn, the less power is allocated to the stronger channel users. Especially, the first and second strong channel users have to share the extremely small amount of power. In this paper, we consider the power optimization for the two users with the small power. First, the closed-form expression for the power allocation is derived and then the results are validated by the numerical results. Furthermore, with the derived analytical expression, for the various channel environments, the optimal power allocation is investigated and the impact of the channel gain difference on the power allocation is analyzed.

Keywords: *NOMA, Superposition coding, User-fairness, Successive interference cancellation, Power allocation.*

1. Introduction

The advanced smart convergence of the state-of-the-art technologies has been enabled with the internet of things (IoT) and the fifth generation (5G) and beyond mobile communications [1]. One of the promising multiple access (MA) in 5G mobile networks is non-orthogonal multiple access (NOMA) [2-4]. NOMA will be able to implement 1000 faster networks than orthogonal multiple access (OMA) in the fourth generation (4G) mobile communications, such as long term evolution advanced (LTE-A) [5-7]. Such improvement is possible owing to sharing the channel resources, such as frequency and time [8]. By sharing the channel resources, the user-fairness should be established between the users [9]. Also, NOMA-assisted underwater visible light communication was studied in [10]. A modified NOMA scheme for a multi-antenna base station was investigated, based on the absolute correlation coefficient between the channels [11]. The bit-error rate (BER) is the practical performance measure for NOMA [12], while the achievable data rate is provided as the ultimate bound. The impact of local oscillator imperfection for NOMA was considered in [13]. In this paper, we consider the power optimization for the first and second strong channel users, who have to share the extremely small amount of power. In this case, the power optimization of the two users of the stronger channel

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conditions is more significant than that of the other users with the more power. The remainder of this paper is organized as follows. In Section 2, the system and channel model are described. The closed-form expression for the power optimization is derived in Section 3. The results are presented and discussed in Section 4. Finally, the conclusions are presented in Section 5.

1.1 Our Contribution Summary

The main contributions of this paper is summarized as follows: 1. For *M*-user NOMA, we derive an analytical expression of the power allocation for the first and second strong channel users, which achieves the equal BERs, to ensure the user fairness. 2. We validate the result of the closed-form expression by simulations, and it is shown that they agree exactly. 3. Furthermore, with application to various channel environments, we also calculate the power allocation of the equal BERs, and we show that the results of the power allocation follow the main principle of NOMA, the more power is allocated to the weaker channel user.

2. System and Channel Model

We consider a cellular downlink NOMA transmission system, in which a base station and *M* users within the cell. The Rayleigh fading channel between the *m*th user and the base station is denoted by $h_m \sim \mathcal{CN}(0,\Sigma_m)$, $1 \le m \le M$, where $\mathcal{CN}(\mu,\sigma^2)$ is represents the distribution of circularly-symmetric complex Gaussian (CSCG) random variable (RV) with mean μ and variance σ^2. The base station transmits the superimposed signal $x = \sum_{m=1}^{M} \sqrt{\alpha_m P} s_m$, where s_m is the message for the *m*th user with unit power, α_m is the power allocation coefficient, with $\sum_{m=1}^{M} \alpha_m = 1$, $\alpha_1 \le \cdots \le \alpha_M$, and P is the constant total transmitted power at the base station. The power of s_m is normalized as unit power, $\mathbb{E}[s_m s_m^*] = \mathbb{E}[|s_m|^2] = 1$. The observation at the *m*th user is given by

$$r_m = |h_m| x + n_m, \tag{1}$$

where $n_m \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian noise (AWGN). We consider the binary phase shift keying (BPSK) modulation, with $s_m \in \{+1,-1\}$. Let the information input bit for the *m*th user be $b_m \in \{0,1\}$. Then, the bit-to-symbol mapping is given as

$$\begin{cases} s_m(b_m = 0) = +1 \\ s_m(b_m = 1) = -1 \end{cases}, \quad \text{or} \quad s_m(b_m) = (-1)^{b_m}. \tag{2}$$

Additionally, we use the binary representation notation for the information index i, $0 \le i \le 2^{m-1} - 1$,

$$(i)_2 = b_{m-1} b_{m-2} \cdots b_2 b_1 \tag{3}$$

where each bit b_j, $1 \le j \le m-1$, corresponds to the *j*th user. In this paper, under the sort by the channel gain variances, i.e., $\Sigma_1 \geq \cdots \geq \Sigma_M$, the BER of the mth user can be expressed by [12]

$$P_e^{(m;\,\text{SIC NOMA})} = \sum_{i=0}^{2^{m-1}-1} \frac{1}{2^{m-1}} F\left(\frac{\Sigma_m P \alpha_{m,i}(i)_2}{N_0}\right) \tag{4}$$

where $\alpha_{m,i}(i)_2$ is given by

$$\alpha_{m,i}(i)_2 = \left(\sqrt{\alpha_m} + (-1)^{b_{m-1}} \sqrt{\alpha_{m-1}} + \ldots + (-1)^{b_1} \sqrt{\alpha_1}\right)^2 \tag{5}$$

and for Rayleigh fading BER performance, we use the following notation:

$$F(\gamma_b) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \tag{6}$$

Then, the BER performance of the first user is given by

$$P_e^{(1;\,\text{SIC NOMA})} = F\left(\frac{\Sigma_1 P \alpha_1}{N_0}\right), \tag{7}$$

and, the BER performance of the second user is given by

$$P_e^{(2;\,\text{SIC NOMA})} = \frac{1}{2} F\left(\frac{\Sigma_2 P\left(\sqrt{\alpha_2} - \sqrt{\alpha_1}\right)^2}{N_0}\right) + \frac{1}{2} F\left(\frac{\Sigma_2 P\left(\sqrt{\alpha_2} + \sqrt{\alpha_1}\right)^2}{N_0}\right). \tag{8}$$

3. Power Calculation for First and Second Strong Channel Users in *M*-user NOMA

The power calculation problem can be formulated as

$$\text{Find } \alpha_1,$$

$$P_e^{(1;\,\text{SIC NOMA})} = P_e^{(2;\,\text{SIC NOMA})} \tag{9}$$

$$\text{subject to} \quad \alpha_1 < \frac{1 - \sum_{m=3}^{M} \alpha_m}{2}, \quad \text{for user fairness.}$$

When we solve the above-mentioned equation, the following condition is used

$$\sum_{m=1}^{M} \alpha_m = 1. \tag{10}$$

Then, by using the above-mentioned equation, α_2 is canceled as

$$\alpha_2 = \underbrace{1 - \sum_{m=3}^{M} \alpha_m}_{=\Omega} - \alpha_1, \tag{11}$$

where $\Omega = 1 - \sum_{m=3}^{M} \alpha_m$. Finally, by algebraic manipulation, we have the following quartic function:

$$f\left(\alpha_1\right) = \alpha_1^4 + p\alpha_1^3 + q\alpha_1^2 + r\alpha_1 + s = 0, \tag{12}$$

where

$$p = \frac{\left(-3\Sigma_1 P N_0 - 4\Sigma_1 P \Sigma_2 P\left(1-\alpha_3\right) - 32\left(\Sigma_2 P\right)^2\left(1-\alpha_3\right)\right)}{16\left(\Sigma_2 P\right)^2}, \tag{13}$$

$$q = \frac{\left(+6N_0^2 + \Sigma_1 P\left(3N_0 + 4\Sigma_2 P\left(1-\alpha_3\right)\right)\left(1-\alpha_3\right) + 12\Sigma_2 P N_0\left(1-\alpha_3\right) + 24\left(\Sigma_2 P\right)^2\left(1-\alpha_3\right)^2\right)}{16\left(\Sigma_2 P\right)^2}, \tag{14}$$

$$r = \frac{\Sigma_1 P\left(N_0 + \Sigma_2 P\left(1-\alpha_3\right)\right)^3\left(-1\right)}{16\left(\Sigma_2 P\right)^4} + \frac{\left(\begin{array}{l} -12\Sigma_1 P\Sigma_2 P N_0\left(1-\alpha_3\right)\left(1-\alpha_3\right) - 8\Sigma_1 P\left(\Sigma_2 P\right)^2\left(1-\alpha_3\right)^2\left(1-\alpha_3\right) \\ -6\Sigma_1 P N_0^2\left(1-\alpha_3\right) \\ +N_0^3 \end{array}\right)}{16\Sigma_1 P\left(\Sigma_2 P\right)^2}, \tag{15}$$

and

$$s = \frac{+\Sigma_1\left(1-\alpha_3\right)\left(N_0 + \Sigma_2 P\left(1-\alpha_3\right)^1\right)^3 - \Sigma_2 N_0^3\left(1-\alpha_3\right)}{16\Sigma_1\left(\Sigma_2 P\right)^3}. \tag{16}$$

Then, for $0 \leq \alpha_1 \leq \dfrac{1-\sum_{m=3}^{M}\alpha_m}{2}$, we have one real root only,

$$\alpha_{\text{opt1}} = -\frac{p}{4} + \frac{1}{2}\sqrt{\frac{1}{4}p^2 - q + z_1} - \frac{1}{2}\sqrt{\frac{3}{4}p^2 - \left(\frac{1}{4}p^2 - q + z_1\right) - 2q + \frac{1}{4}\left(4pq - 8r - p^3\right)\frac{1}{\sqrt{\frac{1}{4}p^2 - q + z_1}}}, \tag{17}$$

where

$$z_1 = -\frac{\left(-q\right)}{3} + \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} + \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}, \tag{18}$$

with

$$a = \frac{1}{3}\left(3\left(pr - 4s\right) - \left(-q\right)^2\right),$$

$$b = \frac{1}{27}\left(2\left(-q\right)^3 - 9\left(-q\right)\left(pr - 4s\right) + 27\left(4qs - r^2 - p^2 s\right)\right). \tag{19}$$

4. Numerical Results and Discussions

We consider the 4-user NOMA system, with $\Sigma_1 + \Sigma_2 = 3$ and $\Sigma_3 + \Sigma_4 = 1$, which guarantees the unit average channel gain of the energy conservation of the given system, i.e., $\frac{\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4}{4} = 1$. Assume the constant total transmitted signal power to noise power ratio $P / N_0 = 40 \text{ dB}$, and $\sum_{m=3}^{M} \alpha_m = 0.7$.

First, in Fig. 1, we depict the quartic function $f(\alpha)$, and verify only one real root α_{opt1}, i.e., the exact analytical optimal power allocation, in $0 \le \alpha_1 < \left(1 - \sum_{m=3}^{M} \alpha_m\right) / 2 = 0.3$,

$$\alpha_{\text{opt1}} = 0.0664737282166938. \tag{20}$$

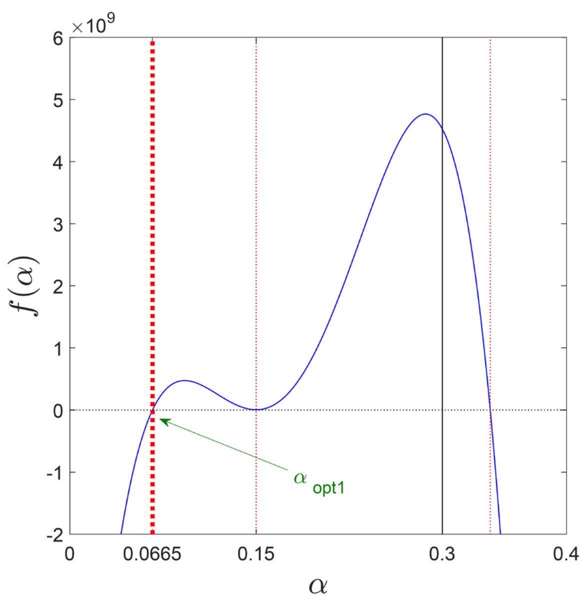

Figure 1. Quartic function $f(\alpha)$ and real root α_{opt1}.

As shown in Fig. 1, only one real root α_{opt} is in $0 \le \alpha_1 < \left(1 - \sum_{m=3}^{M} \alpha_m\right) / 2 = 0.3$, the other real root is not in $0 \le \alpha_1 < 0.3$, which does not satisfy the assumption of the optimization problem statement. In addition, for all channel gains (Σ_1, Σ_2), only one real root is in the power allocation range, because the following condition is satisfied,

$$\begin{aligned} &256s^3 - 192prs^2 - 128qs^2 + 144qr^2s - 27r^4 \\ &+144p^2qs^2 - 6p^2r^2s - 80pq^2rs + 18pqr^3 + 16q^4s \\ &-4q^3r^2 - 27p^4s^2 + 18p^3qrs - 4p^3r^3 - 4p^2q^3s + p^2q^2r^2 < 0 \end{aligned} \tag{21}$$

Then, in Fig. 2, we validate α_{opt1} with the value $\alpha_{\text{opt1}}^{(simulation)} = 0.0665$ of the optimal power allocation, which is obtained by the simulations.

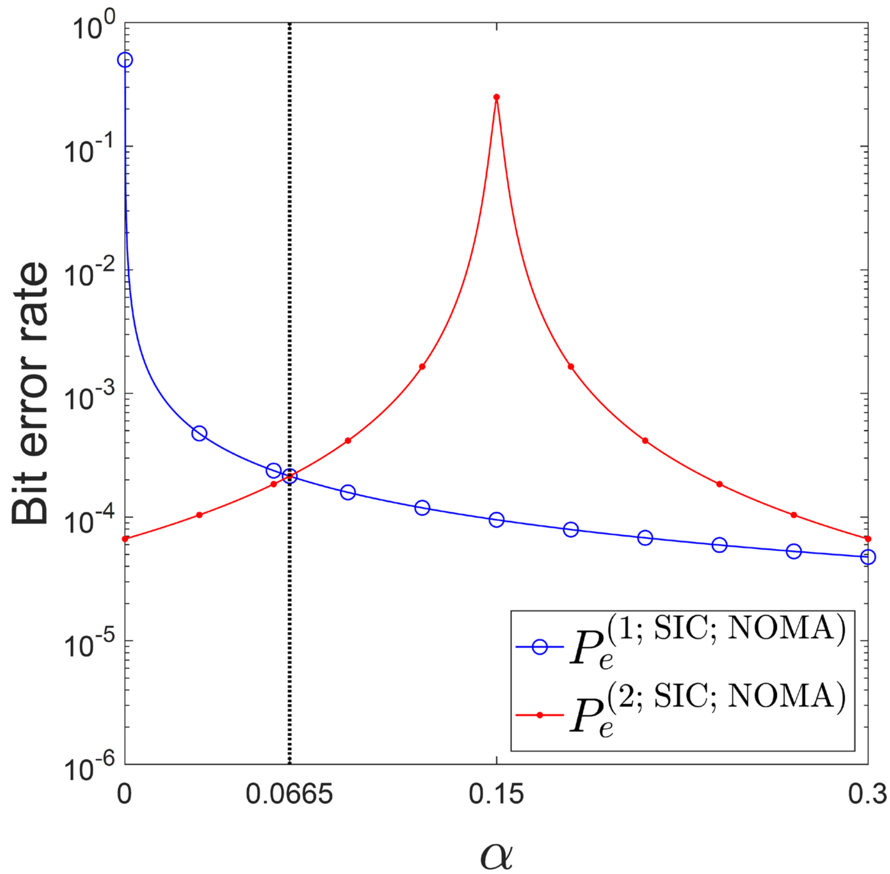

Figure 2. Optimal power allocation $\alpha_{\text{opt1}}^{(simulation)}$ by simulation.

As shown in Fig. 2, $\alpha_{\text{opt1}}^{(simulation)}$ is well consistent with α_{opt1}, which is obtained analytically.

Also, in Fig. 3, for the various channel gains with the energy conservation of the unit average variance, we depict α_{opt1}, versus Σ_1, from which the channel gain variance of the second user is obtained by $\Sigma_2 = 3 - \Sigma_1$.

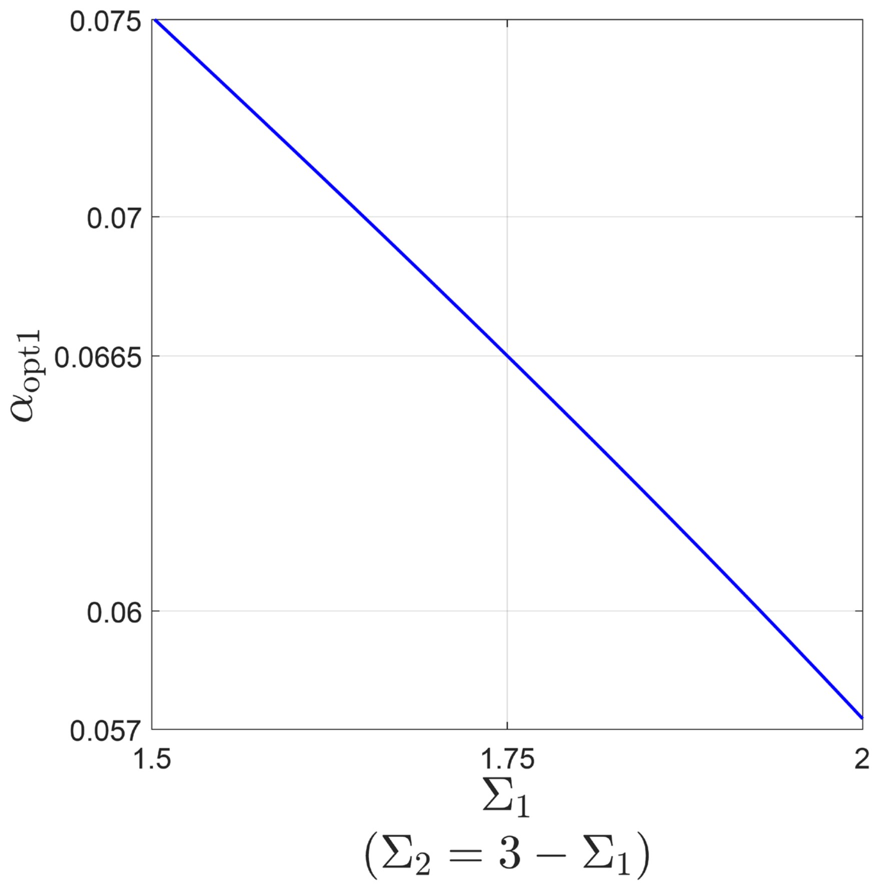

Figure 3. Optimal power allocation α_{opt1}, for various channels.

As shown in Fig. 3, for the equal channel gains, i.e., $\Sigma_1 = \Sigma_2 = 1.5$, we obtain $\alpha_{\mathrm{opt1}} = 0.075 = \frac{1}{2} \times \frac{1}{2} \times 0.3$, and as the difference of the channel gains increases, α_{opt} decreases, i.e., the more power is allocated to the weaker channel user, in order to establish the user fairness between the users.

In addition, for $\sum_{m=3}^{M} \alpha_m = 0.6$, we present the above-mentioned results, as follows: only one real root α_{opt1}, is calculated as

$$\alpha_{\mathrm{opt1}} = 0.0886278360618686. \tag{22}$$

Then, in Fig. 4, we validate α_{opt1} with the value $\alpha_{\mathrm{opt1}}^{(simulation)} = 0.0886$ of the optimal power allocation, which is obtained by the simulations.

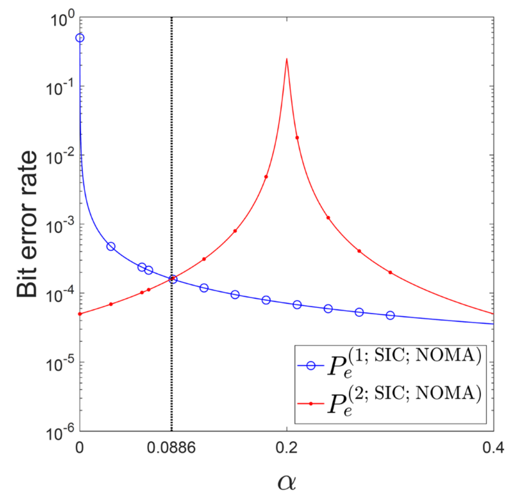

Figure 4. Optimal power allocation $\alpha_{\text{opt1}}^{(simulation)}$ by simulation.

As shown in Fig. 4, $\alpha_{\text{opt1}}^{(simulation)}$ is well consistent with α_{opt1}, which is obtained analytically.

Also, in Fig. 5, for the various channel gains, we depict α_{opt1}, versus Σ_1.

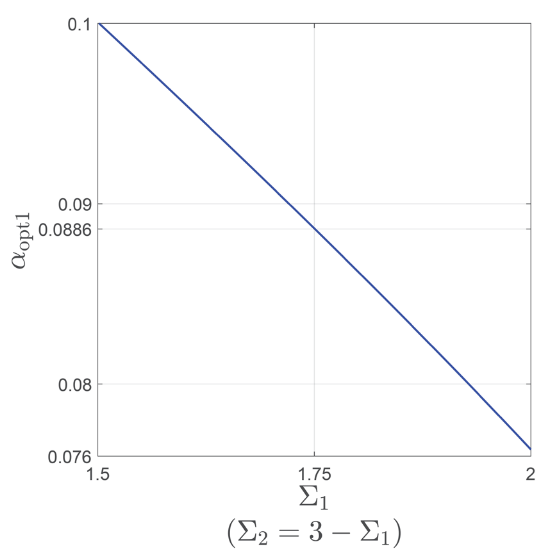

Figure 5. Optimal power allocation α_{opt1}, for various channels.

As shown in Fig. 5, for the equal channel gains, i.e., $\Sigma_1 = \Sigma_2 = 1.5$, we obtain $\alpha_{\text{opt1}} = 0.1 = \frac{1}{2} \times \frac{1}{2} \times 0.4$, and as the difference of the channel gains increases, α_{opt1} decreases.

5. Conclusion

In this paper, we derived the analytical expression of the optimal power allocation for the first and second strong channel users in M-user NOMA. Then, we validated the derived results by simulations and it was shown that the analytical results are consistent with the simulation results. Additionally, for various channel environments, we applied the analytical results, the power allocation of the equal BERs were calculated, and then, we showed that the results of the power allocation follow the main principle of NOMA, the more power is allocated to the weaker channel user, whereas the less power is allocated to the stronger channel user. As a consequence, the user fairness of the equal BERs could be established analytically, under the various channel surroundings.

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