

COMMUTATORS AND ANTI-COMMUTATORS HAVING AUTOMORPHISMS ON LIE IDEALS IN PRIME RINGS

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ABSTRACT. In this manuscript, we discuss the relationship between prime rings and automorphisms satisfying differential identities involving commutators and anti-commutators on Lie ideals. In addition, we provide an example which shows that we cannot expect the same conclusion in case of semiprime rings.

1. Motivation

This work is inspired by the work of several algebraist in which they have evaluated certain identities having commutators and anti-commutators with derivations or automorphisms. In the last few decades, there has been a continuing interest pertaining to the relationship between the commutativity of a ring and the existence of certain specific types of mappings viz derivations, automorphisms etc. Herstein [13] has proven that if \mathcal{R} is a prime ring with characteristic different from 2 and \mathcal{R} admits a nonzero derivation d such that $[x^d, y^d] = 0$ for all $x, y \in \mathcal{R}$, then \mathcal{R} is commutative. While in the year 1992, Daif and Bell [9] proved that if \mathcal{R} is a semiprime ring and d is a nonzero derivation of \mathcal{R} such that $[x, y]^d = [x, y]$ for all $x, y \in \mathcal{R}$, then \mathcal{R} is commutative. In another study Ashraf and Rehman [3] proved that if \mathcal{R} is a prime ring, \mathcal{I} is a nonzero ideal of \mathcal{R} and d is a nonzero derivation of \mathcal{R} such that $(x \circ y)^d = x \circ y$

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for all $x, y \in \mathcal{I}$, then \mathcal{R} is commutative. Again in the year 1994, Bell and Daif [6] proved that if \mathcal{R} is a semiprime ring, \mathcal{I} is a nonzero ideal of \mathcal{R} and \mathcal{R} admits a derivation d such that $[x^d, y^d] = [x, y]$ for all $x, y \in \mathcal{I}$, then $\mathcal{I} \subseteq \mathcal{Z}(\mathcal{R})$. Furthermore, when \mathcal{R} is prime, it is considered \mathcal{R} to be commutative. Later on, in an attempt to generalize the theorem proved by Bell and Daif [6], Deng and Ashraf [11] proved that if \mathcal{R} is a semiprime ring and \mathcal{I} a nonzero ideal of \mathcal{R} and \mathcal{R} admits a mapping f and a derivation d such that $[x^f, y^d] = [x, y]$ for all $x, y \in \mathcal{I}$, then \mathcal{R} contains a nonzero central ideal of \mathcal{R} . Henceforth in 2002, Ashraf and Rehman [3] replaced commutator by anti-commutator and proved that if \mathcal{R} is a 2-torsion free prime ring, \mathcal{I} a nonzero ideal of \mathcal{R} and d a nonzero derivation of \mathcal{R} such that $x^d \circ y^d = x \circ y$ for all $x, y \in \mathcal{I}$, then \mathcal{R} is commutative.

On other hand, many researchers have studied and made an effort to generalize the results obtained on derivations to automorphisms. In [20], Mayne studied Posner's second theorem on derivations [21] for automorphisms of prime rings. Precisely, he proved that let \mathcal{R} be a prime ring with center $Z(\mathcal{R})$ and ξ be a nontrivial automorphism of \mathcal{R} . If $[x^\xi, x] \in Z(\mathcal{R})$ for every $x \in \mathcal{R}$, then \mathcal{R} is a commutative integral domain. In [16], Lee and Lee established that if $\text{char}(\mathcal{R}) \neq 2$ and $[x^d, x] \in Z$ for all x in a non-central Lie ideal \mathcal{L} of \mathcal{R} , then \mathcal{R} is commutative. An analogous extension for Lie ideals in the automorphism case was obtained by Mayne [18]. He was able to accurately draw a conclusion that let \mathcal{R} be a prime ring of characteristic not equal to 2 and ξ be an automorphism of \mathcal{R} . If \mathcal{L} is a Lie ideal of \mathcal{R} such that ξ is nontrivial on \mathcal{L} and $[x^\xi, x]$ is in the center of \mathcal{R} for every x in \mathcal{L} , then \mathcal{L} is contained in the center of \mathcal{R} . Since then a lot of work has been done in this direction on prime and semiprime rings involving automorphisms ([1, 2, 10, 22–25] and references therein).

Persuaded by the above mentioned works, our aim is to discuss the relationship between prime rings and automorphisms satisfying differential identities having commutators and anti-commutators on Lie ideals.

2. Preliminaries

For a given $x, y \in \mathcal{R}$, the commutator (anti-commutator) of x, y is denoted by $[x, y]$ ($x \circ y$) and defined by $[x, y] = xy - yx$ ($x \circ y = xy + yx$)

respectively. Recall that a ring \mathcal{R} is prime, if for any $a, b \in \mathcal{R}$, $a\mathcal{R}b = (0)$ implies either $a = 0$ or $b = 0$. Throughout, \mathcal{R} is a prime ring with center Z and $Q = Q_{mr}(\mathcal{R})$ is the maximal right ring of quotient of \mathcal{R} . To be noted that Q is also a prime ring and the center C of Q , which is called the extended centroid of \mathcal{R} , is a field. Moreover, $Z \subseteq C$ (further explanation refer to [5]). It is well known that any automorphism of \mathcal{R} can be uniquely extended to an automorphism of Q . An automorphism ξ of \mathcal{R} is called Q -inner if there exists an invertible element $g \in Q$ such that $x^\xi = gxg^{-1}$ for all $x \in \mathcal{R}$. Otherwise, ξ is called Q -outer. We symbolize by G the group of all automorphisms of \mathcal{R} and by A_i the group consisting of all Q -inner automorphisms of \mathcal{R} . Recollect that a subset \mathfrak{A} of G is considered independent (modulo A_i) if for any $a_1, a_2 \in \mathfrak{A}$, $a_1 a_2^{-1} \in A_i$ implies $a_1 = a_2$. In the same manner, if a is an outer automorphism of \mathcal{R} , then 1 and a are independent (modulo A_i). Herein, this work we present some well-known facts that will be used in the follow-up.

FACT 2.1. ([8, Theorem 3]) Suppose that \mathcal{R} is a prime ring and \mathfrak{A} an independent subset of G modulo A_i . Let $\phi = \chi(x_i^{a_j}) = 0$ be a generalized identity with automorphisms of \mathcal{R} reduced with respect to \mathfrak{A} . If for all $x_i \in X$, $a_j \in \mathfrak{A}$, the $x_i^{a_j}$ -degree of $\phi = \chi(x_i^{a_j})$ is strictly less than $char(\mathcal{R})$ when $char(\mathcal{R}) \neq 0$, then $\chi(z_{ij}) = 0$ is also a generalized polynomial identity of \mathcal{R} .

FACT 2.2. Let \mathcal{R} be a prime ring and \mathcal{L} a non-central Lie ideal of \mathcal{R} . If $char(\mathcal{R}) \neq 2$, then there exists a nonzero ideal I of \mathcal{R} such that $0 \neq [I, \mathcal{R}] \subseteq \mathcal{L}$. If $char(\mathcal{R}) = 2$ and $dim_C \mathcal{R}C > 4$, then there exists a nonzero ideal I of \mathcal{R} such that $0 \neq [I, \mathcal{R}] \subseteq \mathcal{L}$. Thus if either $char(\mathcal{R}) \neq 2$ or $dim_C \mathcal{R}C > 4$, then we may conclude that there exists a nonzero ideal I of \mathcal{R} such that $[I, I] \subseteq \mathcal{L}$.

FACT 2.3 ([4, Lemma 7.1]). Let $\mathcal{V}_{\mathcal{D}}$ be a vector space over a division ring \mathcal{D} with $dim \mathcal{V}_{\mathcal{D}} \geq 2$ and $\mathcal{S} \in End(\mathcal{V})$. If s and $\mathcal{S}s$ are \mathcal{D} -dependent for every $s \in \mathcal{V}$, then there exists $\chi \in \mathcal{D}$ such that $\mathcal{S}s = \chi s$ for every $s \in \mathcal{V}$.

3. The results in Prime Rings

We begin with the following results which are indispensable to establish our principle theorem.

PROPOSITION 3.1. Let ξ be an automorphism of $End(\mathcal{V}_{\mathcal{D}})$ such that for every $x_1, y_1, x_2, y_2 \in End(\mathcal{V}_{\mathcal{D}})$, $[x_1, x_2]^{\xi} \circ [y_1, y_2]^{\xi} = [[x_1, x_2]^{\xi}, [y_1, y_2]^{\xi}]$. If $dim(\mathcal{V}_{\mathcal{D}}) \geq 2$, then ξ is identity map of $End(\mathcal{V}_{\mathcal{D}})$.

Proof. By a theorem of Jacobson [14, Isomorphism Theorem, p.79], there exists an invertible semilinear transformation $T : \mathcal{V} \rightarrow \mathcal{V}$ such that $x^{\xi} = PxP^{-1}$ for all $x \in End(\mathcal{V}_{\mathcal{D}})$. In particular, there exists an automorphism ζ of \mathcal{D} such that $P(v\gamma) = (Pv)\zeta(\gamma)$ for all $v \in \mathcal{V}$ and $\gamma \in \mathcal{D}$. Using our hypothesis $[x_1, x_2]^{\xi} \circ [y_1, y_2]^{\xi} = [[x_1, x_2]^{\xi}, [y_1, y_2]^{\xi}]$, we find that

$$P[x_1, x_2]P^{-1} \circ P[y_1, y_2]P^{-1} = [P[x_1, x_2]P^{-1}, P[y_1, y_2]P^{-1}]$$

for all $x, y, z \in End(\mathcal{V}_{\mathcal{D}})$. We could divide our proof into the following cases:

There exists $v \in \mathcal{V}$ such that v and $P^{-1}v$ are \mathcal{D} -independent. Let's first, suppose that $\{v, Pv, P^{-1}v\}$ is \mathcal{D} -independent. Let $x, y, z \in End(\mathcal{V}_{\mathcal{D}})$ such that

$$\begin{aligned} x_1v &= v, & x_1P^{-1}v &= 0, & y_1Pv &= P^{-1}v \\ y_1v &= 0, & y_1P^{-1}v &= P^{-1}v \\ y_2v &= Pv, & x_2P^{-1}v &= v \end{aligned}$$

Then $[y_1, y_2]v = P^{-1}v$, $[x_1, x_2]P^{-1}v = v$, and hence

$$\begin{aligned} 0 &= (P[x_1, x_2]P^{-1} \circ P[y_1, y_2]P^{-1} - [P[x_1, x_2]P^{-1}, P[y_1, y_2]P^{-1}])v \\ &= 2v, \text{ a contradiction} \end{aligned}$$

Suppose next that $\{v, Pv, P^{-1}v\}$ is \mathcal{D} -dependent. Then there exist $\mu, \chi \in \mathcal{D}$ such that $Pv = v\mu + P^{-1}v\chi$. Moreover, we claim that $\chi \neq 0$. Indeed, if $\chi = 0$, then $Tv = v\mu$ and $v = P^{-1}v\mu$, a contradiction. Let $x, y, z \in End(\mathcal{V}_{\mathcal{D}})$ such that

$$\begin{aligned} x_1v &= v, & x_1P^{-1}v &= 0 \\ y_1v &= 0, & y_1P^{-1}v &= P^{-1}v \\ y_2v &= v\mu + P^{-1}v\chi, & x_2P^{-1}v &= v \end{aligned}$$

We can easily see that

$$\begin{aligned} 0 &= (P[x_1, x_2]P^{-1} \circ P[y_1, y_2]P^{-1} - [P[x_1, x_2]P^{-1}, P[y_1, y_2]P^{-1}])v \\ &= 2v\chi, \text{ a contradiction} \end{aligned}$$

We have that v and $P^{-1}v$ are \mathcal{D} -dependent for every $v \in \mathcal{V}$. By Fact 2.3 $P^{-1}v = v\alpha$ for all $v \in \mathcal{V}$, where $\alpha \in \mathcal{D}$. Therefore $P^{-1}(xv) = xv\alpha$ for

all $x \in \text{End}(\mathcal{V}_{\mathcal{D}})$, the same for $xv = P(xv\alpha) = P(x(v\alpha)) = PxP^{-1}(v) = x^{\xi}v$ for all $x \in \text{End}(\mathcal{V}_{\mathcal{D}})$ and $v \in \mathcal{V}$. In particular, $(x^{\xi} - x)V = 0$ for all $x \in \text{End}(\mathcal{V}_{\mathcal{D}})$. Thus $x^{\xi} = x$ for all $x \in \text{End}(\mathcal{V}_{\mathcal{D}})$. This implies ξ is the identity map of $\text{End}(\mathcal{V}_{\mathcal{D}})$, proving the proposition. \square

THEOREM 3.1. *Let \mathcal{R} be a prime ring of characteristic different from 2 and ξ be an automorphism of \mathcal{R} such that $x^{\xi} \circ y^{\xi} = [x^{\xi}, y^{\xi}]$ for all $x, y \in \mathcal{L}$, a nonzero Lie ideal of \mathcal{R} . Then \mathcal{L} contained in the center of \mathcal{R} .*

Proof. On the contrary suppose that \mathcal{L} is non-central. By given hypothesis and Fact 2.2, there exists a nonzero ideal I of \mathcal{R} such that $0 \neq [I, I] \subseteq \mathcal{L}$. Also, \mathcal{R} is non-commutative as \mathcal{L} is non-central Lie ideal of \mathcal{R} . Accordingly, we have

$$(3.1) \quad [x_1, x_2]^{\xi} \circ [y_1, y_2]^{\xi} = [[x_1, x_2]^{\xi}, [y_1, y_2]^{\xi}]$$

for all $x_1, y_1, x_2, y_2 \in I$. Firstly, we suppose that ξ is an identity automorphism and hence we can easily observe that $2[y_1, y_2][x_1, x_2] = 0$ for all $x_1, y_1, x_2, y_2 \in I$. This gives,

$$[y_1, y_2][x_1, x_2] = 0$$

for all $x_1, y_1, x_2, y_2 \in I$. Replace x_1 by x_1r , where $x_1 \in I$ and $r \in \mathcal{R}$, we have

$$[y_1, y_2]x_1[r, x_2] + [y_1, y_2][x_1, x_2]r = 0$$

and hence

$$[y_1, y_2]x_1[r, x_2] = (0)$$

for all $x_1, y_1, x_2, y_2 \in I$ and $r \in \mathcal{R}$. This implies

$$[y_1, y_2]I[r, x_2] = (0).$$

Thus

$$[y_1, y_2]I\mathcal{R}[r, x_2]I = (0).$$

Therefore either $[y_1, y_2] = 0$ or $[r, x_2] = 0$, which gives in each case $[I, \mathcal{R}] = (0)$. This leads to a contradiction that \mathcal{R} is commutative [19].

Next, we suppose that ξ is a non-identity automorphism. Suppose that ξ is a Q -inner automorphism. In this case, there exists an invertible element $p \in Q$ such that $x^{\xi} = pxp^{-1}$ for all $x \in \mathcal{R}$. Then I satisfies

$$(3.2) \quad p[x_1, x_2]p^{-1} \circ p[y_1, y_2]p^{-1} = [p[x_1, x_2]p^{-1}, p[y_1, y_2]p^{-1}]$$

By a theorem of Chuang [7], I and Q satisfy the same generalized polynomial identities. Thus Q satisfies

$$(3.3) \quad p[x_1, x_2]p^{-1} \circ p[y_1, y_2]p^{-1} = [p[x_1, x_2]p^{-1}, p[y_1, y_2]p^{-1}]$$

Thus this is a nontrivial generalized polynomial identity on Q as $p \notin C$. Denote by F the algebraic closure of C if C is infinite and set $F = C$ for C finite. Then $Q \otimes_C F$ is a prime ring with extended centroid F [12, Theorem 3.5]. Clearly $Q \cong Q \otimes_C C \subseteq Q \otimes_C F$. So we may regard Q as a subring $Q \otimes_C F$ and hence (3.3) is also a nontrivial generalized polynomial identity of $Q \otimes_C F$. Let $\mathcal{Q} = Q_{mr}(Q \otimes_C F)$, the maximal right ring of quotients of $Q \otimes_C F$. By [5, Theorem 6.4.4], (3.3) is also a nontrivial generalized polynomial identity on \mathcal{Q} . By Martindale's theorem [17], $\mathcal{Q} \cong \text{End}(\mathcal{V}_{\mathcal{D}})$, where \mathcal{V} is a vector space over a division ring \mathcal{D} and \mathcal{D} is finite dimension over its center F . Recall that F is either algebraically closed or finite. From the finite dimensionality of \mathcal{D} over F , it follows that $\mathcal{D} = F$. Hence $\mathcal{Q} \cong \text{End}(\mathcal{V}_F)$. By Proposition 3.1, we get a contradiction.

We now assume that ξ is a Q -outer automorphism. By Chuang [7, Main Theorem] I and Q satisfies the same polynomial identities and hence \mathcal{R} does as well. Thus,

$$[x_1, x_2]^\xi \circ [y_1, y_2]^\xi = [[x_1, x_2]^\xi, [y_1, y_2]^\xi]$$

for all $x_1, y_1, x_2, y_2 \in \mathcal{R}$. Since x_i, y_i -degree is less than $\text{char}(\mathcal{R})$, therefore by Fact 2.1, \mathcal{R} satisfies

$$[x'_1, x'_2] \circ [y'_1, y'_2] = [[x'_1, x'_2], [y'_1, y'_2]]$$

for all $x'_1, y'_1, x'_2, y'_2 \in \mathcal{R}$. Note that this is a polynomial identities and thus there exists a field \mathbb{F} such that $\mathcal{R} \subseteq M_k(\mathbb{F})$, the ring of $k \times k$ matrices over a field \mathbb{F} , where $k > 1$. Moreover, \mathcal{R} and $M_k(\mathbb{F})$ satisfy the same polynomial identities [15, Lemma 1], that is

$$2[y'_1, y'_2][x'_1, x'_2] = 0$$

for all $x'_1, y'_1, x'_2, y'_2 \in M_k(\mathbb{F})$. Let e_{ij} be a matrix unit with 1 in the (i, j) -entry and zero elsewhere. By choosing $x_1 = e_{12}, x_2 = e_{21}, y_1 = e_{11}, y_2 = e_{12}$, we get $0 = 2[y_1, y_2][x_1, x_2] = 2[e_{11}, e_{12}][e_{12}, e_{21}] = -2e_{12}$, a contradiction and hence proof is completed. \square

As a result above theorem, we can easily get the following corollary

COROLLARY 3.1. *Let \mathcal{R} be a prime ring of characteristic different from 2 and ξ be an automorphism of \mathcal{R} such that $x^\xi \circ y^\xi = [x^\xi, y^\xi]$ for all $x, y \in [\mathcal{R}, \mathcal{R}]$. Then \mathcal{R} is commutative.*

Now, we are ready to prove our principle theorem.

THEOREM 3.2. *Let \mathcal{R} be a prime ring of characteristic different from 2 and ξ be an automorphism of \mathcal{R} such that $(x^n)^\xi \circ (y^n)^\xi = [(x^n)^\xi, (y^n)^\xi]$ for all $x, y \in \mathcal{R}$, where n is a fixed positive integer. Then \mathcal{R} is commutative.*

Proof. We are given that $(x^n)^\xi \circ (y^n)^\xi - [(x^n)^\xi, (y^n)^\xi] = 0$ for all $x, y \in \mathcal{R}$. Let S be the additive subgroup generated by the subset $\{r^n | r \in \mathcal{R}\}$. It is easy to see that $x^\xi \circ y^\xi - [x^\xi, y^\xi] = 0$ for all $x, y \in S$. By main theorem of [7], and since $\text{char}(\mathcal{R}) \neq 2$, we have either S contains a non-central Lie ideal \mathcal{L} of \mathcal{R} or $r^n \in Z(\mathcal{R})$ for all $r \in \mathcal{R}$. It is well known that the latter case forces \mathcal{R} to be commutative. Moreover, by Fact 2.2, there exist I nonzero two-sided ideals of \mathcal{R} such that $0 \neq [I, \mathcal{R}] \subseteq \mathcal{L}$. Therefore $x^\xi \circ y^\xi - [x^\xi, y^\xi] = 0$ for all $x, y \in [I, I]$. Since I and \mathcal{R} satisfy the same differential identities (see [15, Theorem 3]), so we have $x^\xi \circ y^\xi - [x^\xi, y^\xi] = 0$ for all $x, y \in [\mathcal{R}, \mathcal{R}]$. Applying Corollary 3.1, we are done. \square

The following example shows the assumption that \mathcal{R} should necessarily be prime in Theorem 3.1.

EXAMPLE 3.1. Let $\mathcal{S} = \mathcal{M}_2(\mathbb{F})$ denote the ring of 2×2 matrices over a field \mathbb{F} . Let $\mathcal{R} = \mathcal{M}_2(\mathbb{F}) \oplus \mathbb{M}_2(\mathbb{F})$ and $\mathcal{L} = \mathcal{M}_2(\mathbb{F}) \oplus 0$. Then \mathcal{R} is a semiprime ring and \mathcal{L} is a nonzero Lie ideal of \mathcal{R} . We define $\xi : \mathcal{R} \rightarrow \mathcal{R}$ as follows $(x_1, x_2)^\xi = (x_2, x_1)$. It can be easily seen that ξ is an automorphism which satisfying $x^\xi \circ y^\xi = [x^\xi, y^\xi]$ for all $x, y \in \mathcal{L}$.

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