Pre-service teachers’ eliciting student thinking about a long division algorithm: Approximation of teaching via digital simulation

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Abstract

The purpose of this study was to explore the possibility of digital simulation by which pre-service teachers (PSTs) can approximate the core teaching practice of eliciting student thinking. This study examined PSTs’ questions to elicit student thinking, their use of “pause” session and peer feedback, and their reflections on doing a digital simulation. We analyzed a two-hour digital simulation session with 13 PSTs who enrolled in the elementary mathematics methods course. The results showed that PSTs shifted their general questions to more content-specific questions throughout the simulation and made a quick transition to comparing students’ strategies. The number of lead PST-initiated “pause” ranged one to four times for various reasons. Their peer-coaches did not voluntarily “pause” the simulation session but actively shared what they noticed from the student work samples and suggested the next teaching moves. Without utilizing the pause session, the dramatic improvement of questioning was not observed. Even though the PSTs felt overwhelmed when interacting with the student-avatars in real-time, they highlighted the benefits of simulations, appreciated the opportunity to learn the core teaching practice, and viewed this digital simulation as “real” and “authentic” experience. The findings of this study provide implications for re-designing a practice-based teacher education program.
I. Introduction

Over the last few decades, teacher education programs have been criticized for the ineffectiveness in preparing high qualified, competent, and skillful teachers. A division of roles and responsibilities between university coursework (i.e., building theoretical knowledge) and field placement (i.e., building practical skills) might be one account for such ineffectiveness. In addition, an argument has been made that the effect of university coursework has been washed out by their teaching experiences in schools (Zeichner & Tabachnick, 1981). Observing such weaknesses of preparing teachers in teacher preparation programs, Ball and her colleagues (Ball & Cohen, 1999; Ball & Forzani, 2009) call for a practiced-based teacher education program wherein "the work of practitioners" is the center of professional education. One might argue that field placements offer sufficient opportunities for learning practices, but pre-service teachers (PSTs) have opportunities to observe "a limited range of practice" which is varied and selected by an individual cooperating or mentor teacher. Emerging consensus has been made that teacher education programs need to center on teaching and learning practices.

Having a similar vision about a practice-based professional education, Grossman and her colleagues (Grossman et al., 2009) explored how professionals engaged in relational practices and identified three key aspects of professional education in those professions: representations of practice, decompositions of practice, and approximations of practice. Among these three key pedagogies for teacher education, approximations of practice—"opportunities to engage in practices that are more or less proximal to the practices of a profession" (Grossman et al., 2009, p. 203)—are very powerful tools for novices to experience "instructive failure" and experiment with different instructional decisions (Grossman et al., 2009) but are more difficult to be implemented because of challenges in providing authentic and responsive learning spaces (Mikeska, Howell, & Straub, 2017). To approximate teaching practices, researchers have adopted different approaches such as rehearsals (e.g., Ghousseini, 2017), animated classroom stories (e.g., Chazan & Herbst, 2012), videos (e.g., Seidel, Blomberg, & Renkl, 2013), non-digital simulations (e.g., Shaughnessy & Boerst, 2018; Shaughnessy, DeFino, PiaFF, & Blunk, 2020), and digital simulations (e.g., Dieter, Rodriguez, Lignugaris/Kraft, Hynes, & Hughes, 2014) in the nationally leading teacher education programs in the United States of America.

The purpose of this study is to explore the possibility of digital simulation by which PSTs can approximate the core teaching practice of eliciting student thinking. The digital simulation provides opportunities for PSTs to interact with students in a more authentic and safe environment and to engage in "deliberate practice" in a more controlled setting by using the power of pausing, giving immediate feedback, having multiple opportunities to rethink and re-enter to instructional interactions, providing peer-coaching, and finally making a better instructional decision (Dieker et al., 2014). By utilizing a "coaching" model, a teacher educator and other fellow PSTs offer immediate suggestions, comments, and feedback to the PST who is interacting with the virtual students. This study examines the following three research questions: (1) What questions do PSTs ask to elicit student thinking? (2) To what extent do PSTs utilize "pause" session during digital simulation? And what kinds of feedback, suggestions, or comments do other fellow PSTs offer to the lead-PST who interacts with the virtual students during the "pause" session? (3) How do PSTs reflect on the simulation experience? What do PSTs

1) Eliciting and Interpreting Student Thinking (EIST) is one of 19 high-leverage practices (HLPs) that are most likely to lead student learning (see TeachingWorks website at http://www.teachingworks.org for more information).
identify as benefits of doing the digital simulations? The findings of this study provide implications for re-designing a practice-based teacher education program.

II. Theoretical Background

Simulation has been widely used in other fields such as aviation, military, and medical training. In those professions, the success or failure of doing a job is directly related to their own lives or others’ lives. Without harming or risking someone’s lives, simulation allows the practitioners to make errors, to learn from mistakes, to improve performance, and to ultimately master the core skills in a safe environment. In medical training, for example, an interactive simulation has been increasingly used to replace traditional apprenticeship training because it avoids the risk of real patients, reduces undesired interference, controls the complexity of tasks, removes distractions such as patient movement and discomfort, permits the errors of diagnosis and management, deals with various medical conditions, and allows sustained, deliberate, and repeated practices to learn the core skills (Maran & Glavin, 2003). Despite the advancement of using simulation in other fields, simulation has recently emerged in teacher education.

Among many other approaches to approximate the core teaching practices introduced above, a digital simulator developed by the research team at the University of Central Florida is quite noticeable. Initiated by Professors Lisa Dieker, Michael Hynes, and Charles Hughes in 2005, the research team developed the TLE TeachLivETM, a mixed-reality virtual learning environment in which participants (e.g., PSTs) interact with virtual avatars (e.g., student-avatars) in a real-time. Depending on the objectives of simulation, the student-avatars can be customized to typically developing students or non-typically developing students (e.g., students with behavior issues). Unlike other virtual learning environments, the TLE TeachLivETM uses the three critical components of virtual learning environment such as personalized learning, suspension of disbelief, and cyclical procedures (Dieker et al., 2014). As the TLE TeachLivETM lab evolved and became popular, the research team at the University of Central Florida established partnerships with other universities and hosted the annual TLE TeachLivE conference2. Recently, Educational Testing Service (ETS) and University of Central Florida jointly facilitated an NSF-funded conference called Simulations in Teacher Education (SITE) in 2019 where more than 30 teacher educators from 15 universities and two research institutes were invited to present their work using simulations in teacher education, especially for science and mathematics teacher education.

This immersive interactive simulator has been used in educational counseling (e.g., Gonzales, 2011), special education (e.g., Vince Garland, Holden, & Garland, 2016), and content-specific methods courses (e.g., Forsythe & Kay, 2018; Mikeska et al., 2017). More specifically, teacher educators utilized the immersive interactive simulator for parent-teacher conference (e.g., Walker & Legg, 2017), building collaborative relationship for co-teaching (e.g., Reinking, 2017), foundational teaching skills such as delivering specific praises and correcting errors (e.g., Dawson & Lignugaris/Kraft, 2017), questioning strategies for English Learners (e.g., Grissom & Regalla, 2019), planning and teaching a shared reading (e.g., Forsythe & Kay, 2018), beliefs and perceptions of using a mixed reality simulation (e.g., Aguilar & Telese, 2019), eliciting and interpreting student thinking (e.g., Dieker, Hughes, Hynes, & Straub, 2017) and facilitating

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2 Since the first conference in 2013, the TLE TeachLivE hosted the 7th annual conference in 2019. For more information, visit the TLE website at http://teachlive.org.
discussions (e.g., Mikeska et al., 2017).

Researchers have shown the effect of simulation intervention on the growth of teaching performance over multiple sessions (Vince Garland et al., 2016) and the effect of simulation on actual teaching practices (Dawson & Lignugaris/Kraft, 2017; Dieker et al., 2017). For example, Dieker et al. (2017) designed a quasi-experimental study to examine the effects of simulated virtual environment on middle school teachers’ teaching practices in mathematics. In their study, 135 middle school mathematics teachers, recruited from 10 school sites, were randomly assigned to four groups: (1) received lesson plans only; (2) received lesson plans and online professional development (PD); (3) received lesson plans and TeachLivE; and (4) received lesson plans, online PD, and TeachLivE. The participating teachers were observed pre and post in two settings: their real classrooms and their classroom simulators. To analyze the type of eliciting and interpreting student thinking, they collected the data on (1) the type of questions (describe/explain questions, short response questions, and yes/no question) and wait time (greater than or equal to three seconds vs. less than three seconds) and (2) type of feedback (general feedback vs. specific feedback). As a result, teachers who received TeachLivE, asked a significantly higher percentage of describe/explain questions and provided more specific feedback to student-avatars.

It is quite new to use simulation in teacher education. However, the previous studies have shown the promising results that this virtual learning environment can be used effectively to focus on targeted core teaching practices while controlling other factors and complexity of instructional interactions, repeating rehearsal to fix instructional errors without affecting real students, and improving teaching performance through deliberate reflections and feedback.

III. Method

1. Study Context and Participants

To answer the research questions above, this study was conducted in a three-credit elementary mathematics methods course for an undergraduate teacher education program in the United States of America. This undergraduate teacher education program is intended for college juniors who would like to earn their B.A. degree and preliminary credential simultaneously. After completing the lower division pre-credential requirements and getting admission to the credential program, they need to take upper-division subject matter courses (e.g., content courses) and professional education courses (e.g., methods courses and student teaching). As the first sequence of professional education courses, the elementary mathematics methods course is considered as prerequisite to start their two semester-long student teaching assignments. This elementary mathematics methods course is particularly designed to provide opportunities for PSTs to analyze the artifacts of teaching practices (e.g., video of elementary mathematics lessons; transcripts of instructional interactions; student work samples), to model and explain mathematical content, and to approximate core teaching practices around K-8 mathematical topics.

In this study, 13 PSTs participated in a two-hour simulation session. Among them, 12 PSTs were in a multiple-subject credential program (i.e., elementary education) and one PST was pursuing an educational specialist credential (i.e., special education). All but one PST did not have any prior experience of simulations. One PST in the educational specialist credential experienced simulations in one of her special education courses which mainly focuses on classroom management but is not specific to teaching mathematics. During simulation, each team (one PST and one peer-coach PST) has 10 minutes to elicit
student thinking about a long division algorithm.

Among many other core teaching practices, this study focuses on eliciting student thinking because it is important to pose questions that allow students to share their thinking about the specific content, to understand their thinking, to check alternative interpretation of students’ ideas, and to position students as sense-makers (TeachingWorks, 2020). Compared to other core teaching practices, eliciting student thinking can be rehearsed via interacting with one individual student or a small group of students. Because it minimizes the complexity of managing multiple relationships between a whole group of students around the content simultaneously, it is appropriate for PSTs to begin with.

In this study, six lead-PSTs (Andrea, Briana, Christine, Danielle, Emma, and Francesca) interacted with five virtual student-avatars (Dev, Jasmine, Ava, Savannah, and Ethan) and other PSTs provided comments, feedback, and suggestions as peer-coaches. Two teacher educators (TEs)—one is the simulation expert and the other is the content expert—facilitated the two-hour digital simulation session. The first author of this article, as a content expert, facilitated the digital simulation and provided content-specific feedback, whereas the simulation expert explained how the simulation works and what student-avatars can or cannot do.

As shown in Figure 1, the lead-PST was standing in front of a large monitor screen to interact with five student-avatars in real-time. Other PSTs and TEs, sitting behind the lead-PST, provided comments, suggestions, and feedback to the lead-PST. The interactor was not physically present in the simulation lab, but played the role of student-avatars and responded to the lead-PST behind the scene in a

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3) All PST’s names are pseudonyms.
4) The available avatars and classroom settings for the digital simulation at the SIMPACT lab can be found at https://www.csun.edu/node/233136

[Fig 1] SIMPACT digital simulation lab

2. Digital Simulation Lab

The TLE TeachLivETM developed by the University of Central Florida is the virtual classroom simulator. Now, it is called MURSionTM digital simulation. This study used the pre-developed scenario by MURSionTM for one of high-leverage teaching practices (i.e., eliciting student thinking) related to a long division algorithm.

The simulation was launched by providing a brief information about simulation, followed by introducing synopsis, objectives, and materials (student work samples). After a short introduction about the lesson to be simulated, the teacher educators introduced the grounding rules for the simulation such as: (1) The lead PST elicits student thinking about the long division; (2) The lead PST and peer-coach PST can pause the session to discuss teaching moves, questions, or brainstorm ideas; (3) Either peer-coach PST or teacher educators can provide suggestions or immediate feedback during the pause session; (4) The lead PST restarts the session; and (5) Each pair has 10 minutes to elicit student thinking about the long division. After 10-minute simulation session, another pair initiated the next simulation session. During the simulation, the
teacher educators facilitated the simulation, provided feedback and suggestions to the PST who was interacting with the student-avatars if requested.

![Fig 2] Students’ strategies to solve 1080÷15 (MURSION, 2017)

This simulation focuses on alternative solutions for the long division (1080÷15) which is one of several key topics addressed in the course. The avatar profile, synopsis, and scenario were created by MURSION (See Figure 2 for students’ strategies). It says: “They know how to add, subtract, multiply, and divide, but may make some computational errors. You are beginning to work on long division, and you have seen they have a variety of strategies for solving long division problems, but some strategies are cumbersome and inefficient. They have not yet learned the standard algorithm for long division, which they will learn in a later lesson.” Three virtual students (Dev, Jasmine, and Ethan) used repeated subtraction, whereas other two virtual students (Savannah and Ava) used adding up to the dividend. The class discussed two targeted student work samples (Lia and Max) for the long division problem. Both Lia and Max were not present in the simulation session. The five virtual students (Ava, Dev, Ethan, Jasmine, and Savannah) were asked to explain Lia’s and Max’s mathematical thinking.

3. Data Collection and Analysis

A two-hour digital simulation was video-recorded and then transcribed. At the end of simulation, PSTs wrote a short reflection on their experiences. The six lead-PSTs who directly interacted with student-avatars reflected on the most successful aspect of eliciting student thinking, the least successful aspect of eliciting student thinking, benefits of doing simulation, and anything new they learned about teaching from this simulation experience. For other PSTs who provided comments and feedback to the six lead-PSTs, they reflected on what they noticed about the lead-PSTs’ eliciting student thinking, their coaching experience, benefits of doing this simulation, and anything new they learned about teaching from this simulation experience.

As a case study (Stake, 1995), we focused on one specific case of using a digital simulation in teacher education program. In analyzing the type of questions and the type of feedback, Dieker et al. (2017) coded the type of questions into three categories (describe/explain questions, short response questions, and yes/no questions), the length of wait time (greater than or equal to three seconds vs. less than three seconds),
and the type of feedback into two categories (general feedback vs. specific feedback). Instead of quantifying
the type of questions, the use of sufficient wait time, and the type of feedback, we provided an in-depth
narrative analysis of questions and feedback, thick descriptions, and verbatim quotations both to capture
the richness of participants’ experiences and to explore the possibility of employing digital simulation to
approximate the core practice of eliciting student thinking in teacher preparation program.

To establish the trustworthiness of qualitative study (Lincoln & Guba, 1995), we sent the video-recorded
simulation session to the PSTs for member-check (credibility) and provided thick descriptions (transferability).
To exclude any bias about the participating PSTs (confirmability), the second author of this article, who do not have information about each
PST’s mathematical knowledge, performance, and dispositions in the course, watched the recorded
simulation session and analyzed the transcribed data.

IV. Results and Discussion

1. Andrea: Shifting from general to specific questions

Andrea was the first PST who interacted with the five virtual students in the simulation. After briefly
introducing herself to the virtual students, she posed a general question to Ethan such as “What do you think
about Lia’s work sample?” After a little bit pause, Ethan asked back to Andrea “What do you mean what
do I think about it?” and then gave non-mathematical comments that Lia had a nice penmanship and her
writing was super neat. Responding to Ethan’s request for clarifying the question and his non-mathematical
comment, Andrea laughed with embarrassment and paused the session to get feedback from her
peer-coaches and teacher educators. Two suggestions were provided by her peer-coaches: (1) asking Ethan
whether he thinks Lia’s solution is correct or not and
(2) asking Ethan about Lia’s thinking process. The
first suggestion focuses on the correctness of the
answer whereas the second one focuses more on the
process. Andrea accepted the second suggestion and
re-entered the virtual classroom.

Andrea: What process do you think Lia used in her
math sample?
Ethan: Uh… what process… um… I mean, I don’t
know… it’s like… she like… I guess kinda…
un, I don’t know, like… pick the number
sort of like and guess like…what you know
was the closest and then… um, I mean she
got the right answer I think. But, yeah… I
don’t know. I don’t know how to explain it.
It’s kind of hard.
Andrea: Why do you think it’s hard?
Ethan: I mean cause it’s kind of math, I don’t know
what she was like thinking cause I’m not
her, so I don’t know cause she’s kind of, it
looks like, she’s got like 15 times 7 and
stuff and she did like 15 times 8, so you
know I guess she’s probably just… I guess,
and then she subtracted, it looks like… yeah,
but, I—I don’t know.
Andrea: Pause.

At the second trial, Andrea elicited a little bit more
specific responses from Ethan. Ethan expressed
uncertainties and difficulties with explaining someone
else’s thinking, but he pointed out that Lia used the
closest number, got the correct answer, did 15 times 7
and 15 times 8, and subtracted. Even though Ethan’s
explanation was neither coherent nor logically
well-structured, his explanation partially captured what
Lia did. After eliciting Ethan’s initial explanation,
Andrea paused the session to check with the teacher
educators whether she did a better job. As one of the
teacher educators commented that we should figure out more specific questions for Lia’s work sample. Andrea further elaborated her questions such as “why she did 15 times 7 to start with” and “why she did not use 15 times 8.” Compared to her initial questions such as “what do you think about Lia’s work?” or “what process do you think that Lia used?”, Andrea came up with more specific questions by highlighting the mathematical process that Lia used and made a transition from “what” to “why” question. After discussing potential follow-up questions for Ethan with her peer-coaches and teacher educators, Andrea asked whether she can try with another student and then re-started the session.

Instead of probing Ethan’s thinking further, Andrea asked Savannah “what kind of method do you think Lia did in her math problem?” Similar to Ethan’s initial response, Savannah requested to clarify what you mean by method to Andrea. Andrea paused the session again. During the pause, one of her peer-coaches commented not to say “method” but to ask how the problem get solved. Following up her peer-coach’s suggestion, one of the teacher educators commented that it should be even more specific and another peer-coach suggested to ask why she multiplied 15 times 8. Andrea re-entered the virtual classroom.

Andrea: Savannah, why do you think Lia multiplied 15 times 8?
Savannah: Um… 15 times 8… I think… looks like she was um… maybe looking to what equals 108 (one-o-eight) and then…
Andrea: Oh, good!
Savannah: And maybe it was too high, maybe.
Andrea: And you think that’s why she followed by 15 times 7 afterwards?
Savannah: Um, yeah cause that’s not like too much lower, but like just enough. So maybe that’s… yeah, so maybe… She picked too high to start, maybe?
Andrea: Okay, can you tell me why you think she multiplied 72 times 15 after she got the correct answer 72?
Savannah: Um… Why she multiplied 72 times 15?
Um, maybe to see that it did… um, like equals to right answer?
Andrea: Good! Pause.

Andrea’s “why” question finally elicited more specific responses from Savannah. Savannah explained that Lia multiplied 15 x 8 to figure out what equals 108 but it might be too high estimate. Instead of probing further the place value (e.g., what 108 means or what 15 x 8 represents), making connection to the division interpretation (e.g., partitive or measurement), or asking what Lia did next after checking the “high” estimate, Andrea immediately appraised Savannah’s initial explanation and then explained the next step that Lia chose (“That’s why she followed by 15 times 7 afterwards?”). Without further probing the subtraction (1080-1050) and the next estimation (15 x 2), Andrea quickly jumped to the multiplication that Lia used to check her answer and confirmed that Lia got the correct answer.

After Savannah’s explanation, Andrea paused the session again to check with the teacher educators whether she did a better job. Instead of evaluating or complimenting Andrea’s accomplishment, both teacher educators prompted to think about the core teaching practice of this simulation (i.e., elicit student thinking). Andrea responded that she elicited student thinking because she got Savannah’s opinion about why Lia was doing multiple steps for the problem. In discussing the next steps, her peer-coaches suggested to compare between solutions, specifically about similarities and differences. Andrea responded that she did not want to embarrass students by comparing their solutions. It is interesting to observe that Andrea was hesitant with
comparing solutions because of emotions that the virtual students might feel. Again, Andrea was asking whether she can start with another student but the time ran out.

During the simulation, Andrea initially asked general questions but quickly realized that her general questions did not work well to elicit student thinking. After getting feedback from her peer-coaches and teacher educators, Andrea developed more specific question and asked more “why” questions instead of “what” questions. More specifically, Andrea elicited what Lia did (i.e., 15 x 8 and subtraction) from Ethan, but later elicited why Lia did it (i.e., to get the closest estimate to 108 but 15 x 8 was too high estimate) from Savannah. However, Andrea did not further delve into the mathematical ideas in the long division algorithm. On the reflection paper, Andrea identified the most successful experience in eliciting student thinking as hearing students’ opinions, hearing students’ different thought process, and asking specific questions, whereas she identified the least successful experience in eliciting student thinking as asking general questions. Andrea reflected the benefits of simulation as “Different benefits may include practice elicit thinking with students before actually doing this in reality inside the classroom. I learned different ways to ask students questions and how to bounce off different questions.”

2. Briana: Eliciting differences between Lia and Ava

Before entering the virtual classroom, Briana discussed with her peer-coaches whether she needs to ask about Lia’s work sample or compare between strategies. As her peer coaches suggested to focus on similarities and differences, Briana further asked whether she needs to start with similarities or differences. Her peer coaches commented that it depends on students because Lia’s work sample is similar to Jasmine’s but it is different from Ava’s. However, they did not discuss further in what ways Lia’s work sample is similar to Jasmine’s but it is different from Ava’s. After discussing the instructional focus with her peer-coaches, Briana started her simulation session. Connecting is one of the five core practices for leading a whole-group discussion (Smith & Stein, 2018) but the early transition to this practice without having sufficient opportunities to elicit an individual student’s mathematical thinking would make it challenging to lead a more mathematically meaningful discussion. However, it is noticeable that her peer-coaches shared the idea that the purpose of questions depends on students they would like to choose (e.g., choosing Jasmine for similarities and choosing Ava for differences).

Briana began the session by asking Jasmine what “compare” means. After Jasmine defined the meaning, Briana reiterated that “compare” means looking for commonalities and differences. At this point, Briana paused the session to check whether the virtual students can automatically compare and how much time she needs to provide to the students for comparing strategies. Briana explained that she would give five minutes for real kids to compare strategies and one teacher educator commented that she can do the same thing for the virtual students but needs to think about whether five minutes might be too long. After a short discussion about what the virtual students can do, Briana re-started the session by asking Dev to find any similarities or differences between Ava’s work sample and Lia’s work sample. At the beginning of simulation, Briana discussed with her peer-coaches whether she needs to ask similarities or differences and her peer-coaches highlighted that it depends on students because Ava’s work is different from Lia’s and Jasmine’s work is similar to Lia’s.

Briana: Dev, did you see any similarities or differences between the two works?
Dev: Ah, ah, it’s like both, sort of, ah, ah, what sort of estimating. I guess try to get numbers, but Ava did a little bit different I think than Lia did cause she didn’t do, I feel like, Lia did some like subtraction and then estimate, but that’s not what Ava did.

Briana: Okay, can you specifically say what Ava did? Or do you just know that it’s different?

Dev: Um… specific? I just say that she estimated and then she added up what she has estimated.

Briana: Okay, great, so you noticed that Ava’s, she was adding. Can anyone notice anything different in Lia’s? Did she add or, uh, did she add or did she subtract in her problem? Ava?

Dev: I think she did subtraction. Yeah.

Briana: Okay, great! So, can you say that’s difference, Dev?

Dev: Yes. Yes.

After explaining the similarity between Lia’s and Ava’s work samples (i.e., using estimation), Dev pointed out that Lia used subtraction but Ava did a little bit different. As Briana asked a follow-up question to clarify what Ava did differently, Dev elaborated that Ava used addition. Briana elicited that two students used different mathematical operations but did not probe further how the two different approaches led to the long division algorithm, how they are mathematically different, whether both approaches are valid, and whether they are equivalent. Instead, Briana repeated Dev’s explanation and then double-checked the different operations used with Dev. Then, Briana turned to Ava to identify similarity between her own work and Lia’s work.

Ava: Um… similarity with Lia? Um, I mean we both got the same answer.

Briana: Okay, great.

Ava: But I don’t really get the way she went about it cause I estimated higher and then sort of added, but she’s like, I don’t know, I guess lower and subtracted, which is, I don’t know, I guess different than how I see it.

Briana: Great, so you’re gonna go ahead and confirm that you did estimate?

Ava: Yeah.

Briana: And, from your perspective, can I ask you how did you find your starting point to start your estimation? So, why did you set up with…

Ava: I was just like, yeah, I didn’t want to start from too high because okay, if I pick 10, it would be like too much, so then I just cut in half, and kinda like start with 50.

Briana: So you said if you start with 10, it would be too much. Now, is 50 larger number than 10?

Ava: Oh, sorry. I mean 100. And then I cut them in half. Yeah, yeah, my bad.

Briana: No, no. That’s perfectly fine. So, did you use addition in your—the way you solved your problem?

Ava: Yeah, I mean like after estimating with my 50, then I like I have 750, okay, so then I like, again cutting in half, that’s why I have 20, and then I mean, I guess that I added them up together to get 1050, I was only 30 out. So, that was like my two. And I just added my 50 and my 20 and 2. So, I didn’t do any subtraction like, looks like Lia did.

Briana: Great! Now I heard you saying the term cutting it up, can you compare what you use as cutting up to make a different… uh, pause session.
Ava explained that both of them got the same answer. After nominating the similarity, Ava pointed out that she estimated higher and added whereas Lia estimated lower and subtracted. In fact, Ava’s first estimation is 15 x 50, whereas Lia’s first estimation is 15 x 80. However, Briana did not ask follow-up questions about higher or lower estimation. Ava’s comment can be used as a mathematically productive discussable moment. It can address what 15 x 7 actually represents in Lia’s work sample and who actually used higher estimates. Instead, Briana confirmed that Ava used estimation and then focused on how Ava found her starting point for her estimation. Ava explained that she started with 100 but cut it in half and tried 50 as her first estimation. Instead of asking the next steps, Briana asked again whether Ava used addition to solve the problem. Ava provided a full explanation how she got her three partial quotients (15 x 50, 15 x 20, and 15 x 2) and then added them together to get 1080 but Briana paid attention to the term “cutting it up” that Ava used in her explanation. Briana paused the session to search for the term “operation” and then asked Ava which operation she used for cutting it up. As Ava explained that she did not know which operation was used for cutting it up, Briana asked whether “cutting it up” is subtraction or division. At this point, Briana’s instructional focus moved from identifying similarities and differences between Ava’s and Lia’s to deciding whether “cutting it up” is subtraction or division.

Briana: When you cut up the number, subtracting it or dividing it by something? What were you doing?
Ava: I don’t really think of it, like, yeah, I was sort of, I guess adding it up but then like breaking it down like fits like cutting it up … but not actual subtraction or anything.

Briana: Great! We can actually consider that as an actual subtraction because we can look at it as you are subtracting by certain amount, so you can get closer to your goal amount. Or, if you cut it in half we can also look at dividing by half and then you can get closer to your goal amount, which is great as well. So, Jasmine, from our conversation that we are having, what operation do you think we are using during our long division?

Ava explained that it is not actual subtraction, but Briana confirmed that Ava’s method of “cutting it up in half” is subtraction and moved onto Jasmine to ask which operation is used for the long division. Compared to her initial probing questions, Briana asked more closed questions at this point and provided her interpretation about which operation is used for “cutting it up” which deviated from the instructional focus of eliciting student thinking about the long division. One of the teacher educators paused the session because the time ran out. Briana’s conclusion was neither mathematically accurate nor efficient.

During the simulation, Briana was mainly searching for similarities and differences between Ava’s and Lia’s work samples. After Dev pointed out the difference that Ava used addition and Lia used subtraction, Briana turned her attention to search for similarities between the two work samples. Ava identified the similarity as the same answer, but pointed out that she estimated higher and added whereas Lia estimated lower and subtracted. Without further probing the accurate mathematical meaning, Briana mainly focused on the term “cutting in half” in Ava’s explanation and concluded that it is subtraction. The key mathematical differences identified were not further explored and the key mathematical points were diverted into less important mathematical ideas for the long division. Unlike Andrea who asked more general questions at
the beginning and changed from “what” to “why” question, Briana surfaced similarities and differences but did not probe how such differences matter; focused on the operations used; and changed the type of question from open-ended question to closed question. On the reflection paper, Briana identified the most successful experience in eliciting student thinking as asking to find similarities and differences, whereas she identified the least successful experience in eliciting student thinking as asking broad questions. Briana reflected the benefits of simulation as “The benefits include teaching in a live setting while others can help you when you pause. It helps you see different perspectives. I learned that five minutes is too long of a time to compare.”

3. Christine: Eliciting differences between Lia and Jasmine

Before entering the virtual classroom, Christine discussed what she needs to do with her peer-coaches, especially for selecting students. As Smith and Stein (2018) argued, selecting is one of the key practices of orchestrating a whole group discussion. Per Christine’s request, her peer coaches asked back what Christine wanted to talk about and then discussed that Dev started with a higher estimation but it was not clear where 270 and 300 come from. At this point, one of the teacher educators commented that we did not know much about how Jasmine got the answer. Christine started the simulation session by asking Jasmine to explain how she solved the division problem. Both Andrea and Briana made their own choices about whom they would like to interact with, but Christine chose Jasmine based on the teacher educator’s suggestion. She expressed a lot of anxiety and showed lack of confidence with doing a simulation. As one of her peer-coaches commented that it is not real, Christine responded that it is real and then entered to the virtual classroom.

Christine: So, can everybody take a look at Jasmine’s work? Jasmine, can you please explain to us how you solved your division problem?

Jasmine: Um, well, I didn’t really show it there, but I guess it was kind like Ava really, at first I thought maybe 100 but then I knew that that was too high so then I was trying to think about what to do next, but I thought that 50 was kind of low, so then I was like, well, I can try 70 but which I did and which I mean kind of work but I guess it could not work. But, yeah, and I was lucky that 70 was right because 100 is too high. I could be done with 50 though too, I guess.

Christine: Pause.

Jasmine explained that she did something similar to Ava, but Christine did not probe her thinking further. In the previous discussion, Briana and her peer-coached agreed that Lia’s solution is different from Jasmine’s but it is similar to Ava’s. At this point, it would be interesting to probe further why Jasmine thinks that her work is similar to Ava. After Jasmine explained her solution, Christine paused the session and checked whether she can ask what Jasmine would have done if 70 did not work. After confirming with her peer-coaches and teacher educators, Christine re-entered the virtual classroom.

Christine: So, Jasmine, what would you have done if 70 didn’t work out?

Jasmine: Um, well I guess if I haven’t done 70, you mean like it’s too high again sort like 100 was?

Christine: Yes, if it’s too high or too low?

Jasmine: Um, well then, I guess if 70 was too high then I would have went lower again until
like I get the closest if I could get 1080 (ten eighty), and then I would just from whatever the difference was, then I would try to get the closest to that keeping the closest until I could get 0.

Christine: So can you take a look at Lia’s work? And pau.

Without further elaborating, Christine asked what Jasmine would have done if 70 does not work. Christine paused the session again to confirm whether she can ask similarities and then resumed the session.

Christine: So when we look at Lia’s work, Jasmine, do you think that the way she showed her work explains what you wanna—what you just explained to us?

Jasmine: Um, I mean I guess kind of she was, you know, she did like, um, like smaller numbers, and then she was trying to look to get 108 but I was trying to do 1080 (one thousand eighty) that was a little bit different, um, she did a little bit different, but I guess it’s kind of same idea.

Christine: Pause.

Before asking similarities between Jasmine’s work and Lia’s work, Christine asked Jasmine to explain Lia’s work. Jasmine pointed out that Lia did smaller numbers to get 108 but she was trying to get 1080. After one exchange with Jasmine, Christine paused the session again to ask whether she can move onto another student. One of the teacher educators asked the whole-class whether anyone would like to ask more questions to Jasmine. Other peer-coaches suggested to ask about the place value concept by pointing out that Lia actually multiplied by 70 and then ask the actual difference between “multiplied by 7” and “multiplied by 70.” Instead of moving onto another student, Christine was staying with Jasmine to find the difference between 15 x 7 in Lia’s work and 15 x 70 in Jasmine’s work.

Christine: So, Jasmine, you explained that when you looked at Lia’s work, you noticed that it’s a little bit different than yours. She multiplied 15 times 7 and you multiplied 15 times 70. Do you know what’s the difference?

Jasmine: Um, you mean, like, I mean literally the difference you know like, obviously, mine is higher number, she was just like not using, I guess the, the place holder with the zero I guess or something. But I was doing all of them cause it seems like easier to me that way.

Christine: (silent) Okay, pau.

After eliciting Jasmine explanation about the difference between 15 times 7 and 15 times 70, Christine paused the session again. Her peer coaches complimented that Christine did an awesome job with eliciting Jasmine’s thinking. Without complimenting or evaluating her accomplishment, one of the teacher educators checked what we knew about Jasmine’s thinking with the whole group. One of peer coaches responded that Jasmine understood zero as a value but it does not necessarily mean that she knows the difference between 7 and 70. Another teacher educator commented that you can ask what 105 means—which it represents 105 ones or 105 tens. At this moment, the time ran out.

Among six PSTs whom interacted with the virtual students in the simulation session, Christine had the shortest exchanges with the virtual student each time. After eliciting Jasmine’s explanation, Christine paused the session to discuss the next teaching moves with her peer coaches. Toward the end of her simulation
session, Jasmine was able to elicit more specific responses from Jasmine about the difference between her work and Lia’s work: (1) Lia tried to get 108 but Jasmine tried to get 1080; and (2) Lia multiplied 15 by 7, but Jasmine multiplied 15 by 70. Considering that the PSTs discussed that Lia’s work was similar to Jasmine in the previous simulation session, it is interesting to elicit the difference between Lia’s work and Jasmine’s work. On the reflection paper, Christine identified the most successful experience in eliciting student thinking as hearing from Jasmine that her place value was different than Lia’s, whereas she identified the least successful experience in eliciting student thinking as not being able to elaborate more on place value. Christine reflected the benefits of simulation as “I believe this is a great way for future teachers to get practice because I believe this is more productive that most work or reading we do because we get to see and experience a “real” set of students.”

4. Danielle: Convincing Lia’s as an easier method

In contrast to Christine who consulted with her peer-coaches and teacher educators about what she needs to do, Danielle immediately started the simulation session without any hesitation. Danielle pointed out that Dev did something interesting, started off 15 times 90, and then asked him for explaining his thinking.

Dev: Ah, yeah, I-L... started with 90 but it was, it was too high, yeah, so that’s where 1350 was, so I wanted to know how much higher it was, so th-then I subtracted the 180 [sic 1080] that’s where I got 270, which is close to 300 that’s why I was like, well, I just took like 20 off from the 90. That’s how I got 70.

Danielle: Good work. And then how did you use that to solve the rest of problem?

Dev: On-once I subtracted cause 70 times 15 was 1050 (ten fifty) and I have only 30 left and I knew 15 times 2 was 30 and then I added up 70 to my 2 to get 72, yeah.

Among student work samples, Dev used the most complicate process to solve the division problem. He first used the higher estimation but subtracted 1080 from it to figure out how much higher the estimation is. Instead of probing his work further or orienting students each other, Danielle asked Dev to explain Lia’s solution because Lia did similar thing to Dev.

Danielle: Good work. Could you now take a look at Lia’s work? And Lia did something somewhat similar to you. Can you try to explain what Lia did offside here? She multiplied 15 by 8 and then 15 by 7?

Dev: I-I think she was estimating but tried to get 108 (one-o-eight) and I was trying to get 1080 (ten eighty) so she went too high with times by 8 and she went to 7 which was 105 (one-on-five) and she subtracted that.

Danielle: What was, so you said she was trying to get 108 (one-o-eight) and you were trying to get 1080 (ten eighty). What’s the difference?

Dev: Ah, well, she—she was just getting not doing the smaller number, I was doing the—the whole, what do you call that? Ah, di-di—dividend.

Danielle: Okay. And, would you think it was easier for her to start with a small number in a placement of bigger number? Or would you think that it’s easier to do a bigger number in the whole dividend?

Dev: Um, I just didn’t think to do 108 (one-o-eight). I guess I could do. That might be easier to start. It just didn’t occur to me.

Danielle: So, now you’re comparing your work to Lia’s work, who do you think has an easier method?
Danielle: Uh... her-her's looks neeter. And she did, cause I got 270 which wasn't closer to 300 then I have to kinds of guess. Hers seems to be a little bit closer maybe. And not as many steps.

In this exchange, Danielle first addressed that Lia did something similar to Dev and pointed out that Lia multiplied 15 x 8 and then multiplied 15 x 7. Instead of explaining the similarity with using higher estimate, Dev pointed out the difference between Lia's (trying to get 108) and hers (trying to get 1080). In the previous simulation session with Christine, Jasmine also pointed out that Lia was trying to figure out 108 but she was trying to figure out 1080. Instead of probing the equivalence between these two, Danielle asked Dev which method is easier and then further asked him whether he would like to try Lia's method. Dev responded that he got the correct answer but he can try Lia's method next time. Danielle turned to the rest of students and asked whether they would like to try Lia's method or they are more comfortable with their own methods. Savannah explained that she started with low numbers and had more steps, so her method was not so good and had a mistake for adding. Danielle asked again whether Lia has a good method to try with and Savannah said that Lia got the right answer so it looks good. After getting confirmation from Savannah that she can try Lia's method next time, Danielle asked other students whether they would like to try Lia's method. Both Jasmine and Ava preferred their methods. Danielle turned to Ethan and asked whether he liked his method or want to try something like Lia's method.

Danielle: What about you, Ethan? You also got the right number, but you tried something a little different. Do you like your method? Or would you want to try something like Lia's method?

Ethan: Yeah, I mean I guess I can try like Lia's maybe it is a little bit easier cause it's not like so many big numbers and stuffs, but I think I got lucky because I pick 60 which was like, you know pretty easy. 900 and I just had my 180, so like this time it was kind of easy but maybe if it's different number be harder, so Lia's might be a bit easier.

Danielle: So, you still had to use the number in tens place when you were working on the second part of your problem. Would you wanna maybe try a little problem on the side like Dev and Lia did to help you find bigger number so you wouldn't have to put two numbers in tens place, only one number in the tens place and one number in ones place?

Dev: Oh, man, I didn't even really think about that. I mean I guess it doesn't seem to be bad, but I just added 60 and 12 but I do see what you mean like they didn't have to do that. So, but, did Ava kinda have to do that too? She had 50 and 20, you know, and then her 2s? So, it's kind of same thing.

Danielle: Yes, but Ava said that she is more comfortable with the way she did. Are you also more comfortable with the way you did? Or, would you wanna try something different next time?

After hearing back from Jasmine and Ava that they were comfortable with their own methods, Danielle asked Dev whether he would like to try Lia's methods. As Ethan explained his own method, Danielle pointed out that Dev's method has the tens place in the second part of the problem so he still needs to add two numbers in tens place. As Dev pointed out that Ava did the same thing, Danielle responded that Ava was
more comfortable with her own method but asked Dev whether he would like to try Lia’s method. Danielle paused the session because she does not know what to do from here. One of the teacher educators asked about the purpose of asking questions and what kind of conclusions she would like to make at the end of her discussion. Danielle responded that she would like to know whether the students would like to try and accept Lia’s method before trying all different ways to solve the problem because she thought that Lia’s method is the standard way of doing the long division problem.

During the simulation, Danielle started with Dev because he did something interesting and pointed out the similarities between Dev’s and Lia’s. After Dev explained his own method, Danielle shifted her focus on discussing whether Lia’s method is an easier method and whether other students also would like to try Lia’s method. If the virtual students responded that they felt comfortable with their own methods, Danielle moved on. If not, Danielle further asked whether they would like to try Lia’s method. Whereas Brian pointed out the similarities and differences between students’ work samples, Danielle focused on whether Lia’s method is an easier method and tried to convince to use this method for other virtual students. Among the six FSTs who interacted with the virtual students, Danielle utilized the pause least (paused only once at the end of her simulation session). For the most successful experience in eliciting student thinking, Danielle mentioned that Francesca had a good method in having students do a think-pair-share and compare their work to Lia’s. As the least successful experience in eliciting student thinking, Danielle mentioned that the broad questions did not get very good answer because students did not really understand. On the reflection paper, Danielle identified the benefits of simulation as “The simulation is very similar to a real classroom. This simulation helped me realize that more specific questions elicit better answers.”

5. Emma: Recording the quotient on the answer spot

Emma began the simulation by asking Ethan to look at Max’s work sample and where he found the answer in Max’s work. Ethan responded that he found the answer on the top and Emma asked a follow-up question what the symbol is called. As Ethan did not recall the name, Emma asked Savannah what the bar is called. As Savannah also did not find the name, Emma asked Dev what the bar is called. Instead of naming the bar, Dev answered that the number 72 is quotient. At this point, Emma mainly focused on where the answer was recorded on Max’s work and searched for the mathematical term, quotient. Afterwards, Emma asked other students whether they remember what the dividend is and what the divisor is.

Emma: So, we have a quotient. Does anybody remember what the dividend is? Maybe Jasmine?

Jasmine: I think the dividend is the number inside the little… I don’t know that thing is called but like 1080 (ten-eighty)?

Emma: Yeah, that thing, that thing. Awesome! So, okay, what’s the divisor? Ava?

Ava: The divisor is 15. Like the number in front.

Emma: Cool, Ava. What’s the name of that symbol?

Ava: Yeah, I have no idea.

Emma: Okay, cool. Pause session.

Responding to the request of identifying the dividend and its meaning, Jasmine identified the location of dividend without fully explaining what it means. Emma complemented Jasmine’s explanation and then asked Ava about the divisor. Again, Ava identified the location of divisor without fully explaining its meaning. In these exchanges, Emma used some vague terms such as “that thing” and “that symbol.” Emma paused
the session to get some ideas about what the symbol is called in the long division problem but did not get the name from her peer-coaches. Emma re-entered the virtual classroom. After naming and identifying the key elements of the division problem (quotient, dividend, and divisor), Emma asked how many students solved a long division problem before. As Ava responded that no one solved it before, Emma complimented that they got a really awesome solution without solving the long division problem before. Emma asked Ethan about similarities between Ethan’s work and Max’s work. It is interesting to observe that Emma asked similarities between Ethan’s and Max’ work sample whereas Briana asked both similarities and differences between Ava’s and Lia’s work sample.

Emma: Awesome. Okay. Ethan, I wanna compare your work with Max’s, do you see similarity and where do you guys try to add to get to your solution?

Ethan: Um, okay, Max, what was he doing, um… I mean I guess yeah, kinda, he just doesn’t show on the side, he just like, I guess kind of shows it, like, you know, it kinda of actually, a little bit reminds me more of Ava’s but… Because he got those numbers on the side that he added up, and that’s kind of what Ava did, she didn’t circle around them.

Emma: So, all these numbers on the side, are you talking about that the 50, the 10, the 10, and the 2?

Ethan: Yeah.

Emma: Okay. And how are these numbers somehow equals to your 60 and your 12 that were also circled on the side?

Ethan: Oh, yeah. Oh, man, I didn’t even, that’s so cool. I didn’t even see it. Yeah.

Emma: Okay, so.

Ethan: I just guessed higher.

Emma: You did. You guess higher. Max guesses lower and adds it up. Do you think that you prefer that or do you think that yours is easier?

Ethan: Um… I felt mine kinda easier, but I think like, if Max is here, he probably says that his was easier maybe.

Emma: Yeah, probably. That’s what Max did it. But, do you think the big difference is it or do you think both your methods are going to get the accurate result?

Instead of explaining the similarities between his work and Max’s work, Ethan first explained that Max’s work reminded of Ava’s work. Ava had three partial quotients (15x50; 15x20; 15x2) and Max had four partial quotients (15x50; 15x10; 15x10; 15x2). In addition, both started with the estimation of 15x50. Instead of further probing the similarities between Ava’s work and Max’s work, Emma asked the equivalence between Ethan’s two partial quotients (15x50; 15x12) and Max’s four partial quotients (15x50; 15x10; 15x10; 15x2). As Ethan explained that he estimated higher, Emma added that Max guessed lower and asked which method is easier. Ethan responded that his method is easier for him but Max might say that his method might be easier. Emma did not conclude the easiest method but asked the big differences between two methods and whether both methods get the accurate result. Ethan explained that both got the same answer. Emma asked Ethan where he recorded the answer and concluded that he should record the answer on the top in the future. At the beginning of the simulation, Emma asked Ethan whether he can find answer in Max’s work sample.

Again, Emma asked Savannah what she should have recorded on the top in the long division problem. Emma also commented that Savannah basically had the
same set-up like Ethan and Max, but pointed out that there is a little bit problem in her solution. Savannah responded that she missed 10 when she was adding. Pointing out that Savannah made one mathematical error, Emma asked Savannah to slow down when doing the addition. Emma explained that Ethan, Savannah, and Dev had one similarity in the answer. Ethan responded that the quotient was not recorded on the top and asked whether it is wrong if the answer is not recorded on the top. Emma responded that the answer should be recorded on the top and it would be awesome if you circle the answer. Emma mentioned that they had something very special and asked what it is. Savannah explained that Lia checked the answer to make sure whether she is correct. Emma asked students whether it is something that we should be all doing and Savannah responded that she realized that she was not right. Emma pointed out that there was an easier way to solve this problem. Emma called Dev and he would be able to realize that he can work with some easier factors to work in the future and Dev responded that it was confusing when he was estimating. At this point, time ran out.

During the simulation, Emma focused on searching terms in division (e.g., quotient, dividend, and divisor), finding out prior experience of solving the long division problem, emphasizing the importance of recording the quotient on the answer spot, commenting the importance of checking the answer, and finding an easier way to solve the problem. Emma paused only once to search the term for the symbol used in the long division. Emma was the only PST who noticed the equivalence between partial quotients (Max’s 50, 10, 10, and 2 vs. Ethan’s 60 and 12) but quickly moved onto who used a higher estimate (Ethan used a higher estimate of 60 than Max’s 50). For the most successful experience in eliciting student thinking, Emma identified that Jasmine and Ava learned a bit about vocabulary. Emma reflected the benefits of simulation as “I enjoyed every minute of it. This was a really enjoyable experience and I feel that I learned a great deal. This is something I would be very interested in trying again.”

6. Francesca: Finding similar thinking process to Lia

Before the simulation, one of the teacher educators asked the purpose of asking questions. Francesca responded that she would like to focus on reviewing and getting input on new things that they learned. Another teacher educator commented that they can think about who have similar thinking process to Lia and Max. Francesca was asking whether the virtual students can work in groups and how to group them. Francesca started the session by asking students to do think-pair-share and see any similarities between their own work and Lia’s work sample. After a short partner work, Francesca asked Savannah and Dev to share their thoughts with the whole group. Dev explained that Lia’s work sample was similar to Jasmine’s, Ethan’s, and his work. Francesca asked follow-up questions to elicit the reason.

Francesca: Okay, how so, how would you say that Lia’s work is similar to those students?
Dev: Because she, um, like picked the number and multiplied it by estimating, and then subtracted. Even though she was subtracting from the 108 (one-o-eight), all of us are doing 1080 (ten-eighty) but, it was still very similar.
Francesca: Okay. Savannah, did you have any similarities as well?
Savannah: Like, in my own work?
Francesca: We’re looking at Lia’s and comparing it to all of your guys’ work.
Savannah: No, I agree with Dev that made most sense cause I kinds of thought that Ava
and I, sort of, did it similar but I kinda seems like that Dev, Ava, and Jasmine did their kinds of similar way and theirs kind of remind me of Lia’s.

Francesca: I’m gonna go ahead to the next group, so Ava, Jasmine, and Ethan, would any of you would like to share with us? Which student is the most similar to Lia’s?

After eliciting why Dev nominated that Lia’s work was similar to Jasmine’s, Ethan’s, and his work, Francesca did not probe his thinking further. In fact, Dev pointed out that they used the repeated subtraction for the long division problem but Lia used 108 instead of 1080. Instead of probing Dev further, Francesca moved onto Savannah. When Savannah explained that her work was similar to Ava, Francesca did not further probe the meaning but moved onto another group of Ava, Jasmine, and Ethan. Ava agreed on Dev and Savannah that Lia’s work is similar to Jasmine’s, Ethan’s, and Dev’s.

Francesca: Okay, Why do you think that those are the closest or most similar?
Ava: Um, it was just like, I don’t know, but the way they set it up with, like timesing and numbers and stuffs, doing multiplication, so it looks literally like the closest, and I think was like, I mean, if I have to guess, it’s like, that’s the closest I’ve seen so. Those are same work.

Francesca: Okay, thank you.

Compared to Dev, Ava provided a vague, general, and unclear justification using terms such as timesing and numbers and stuffs. Similar to the previous exchange, Francesca did not ask follow-up questions further but complimented Ava’s contribution. Next, Francesca chose Jasmine to ask a question.

Francesca: Besides your own work, which student’s work made the most sense to you and why?
Jasmine: Um… Oh… Um… (whispering) which student made the most sense to me? (to the teacher) I mean it’s kinds of different than mine, but I would say um, Ava’s makes a lot of sense and didn’t look like very many steps. So, I kinds of like that one. And she’s my friend, so that’s probably why she was picked.
Ava: That’s why you should pick each other, yeah.
Francesca: Pause.

For Ava’s non-mathematical comment, Francesca paused the session. Because Francesca had only one more minute left for her simulation, she quickly re-entered the classroom and asked Ethan, Dev, and Ava what is one new thing they learned about solving the long division. Ethan said that he should have recorded the quotient on the top. Otherwise, it is not the right answer. Dev repeated that he did not put his quotient on the top and did not need to do the whole number similar to what Lia did with 108. Both Ethan’s and Dev’s responses were related to the conclusion that Emma made in the previous simulation session. Francesca just accepted their responses. Lastly, Francesca asked one thing you found difficult while performing long division. The time just ran out.

During the simulation, Francesca mainly focused on hearing who has similar thinking process to Lia, whose work makes the most sense, and one new thing students learned about solving the long division problem. Similar to Danielle and Emma, Francesca paused the session only once. On the reflection paper, Francesca identified the most successful experience in eliciting student thinking as asking more specific
questions, whereas she identified the least successful experience in eliciting student thinking as asking wrong questions or not phrasing them correctly. Francesca identified the benefits of simulation as "The benefits of doing this simulation are getting a chance to practice in a classroom setting with students and being able to get support and feedback from my classmates. I learned that it is very important to be prepared and ask open-ended questions to get students thinking."

V. Conclusion and Implication

The purpose of this study is to explore the possibility of digital simulation by which PSTs can approximate the core teaching practice of eliciting student thinking. More specifically, this study explored the following three research questions: (1) What questions do PSTs ask to elicit student thinking? (2) To what extent do PSTs utilize "pause" session during digital simulation? And what kinds of feedback, suggestions, and comments do other fellow PSTs offer to the PSTs who interact with the virtual students during the "pause" session? (3) How do PSTs reflect on the simulation experience? What do PSTs identify as benefits of doing the digital simulations? Whereas the previous result section provides the detailed analysis of individual case of each simulation session, this section starts with a concise but cross-case analysis across six simulation sessions to set the basis for implications to the teacher education.

The first PST in the simulation started with general questions (e.g., What do you think about Lia's work sample?) but the last PST in simulation asked more content-specific questions using turn-and-talk and revocing talk moves (e.g., So, you are saying that Lia was most like Jasmine? How would you say that Lia’s work is similar to those students?) toward the end of the simulation. After the first simulation session led by Andrea, the PSTs quickly realized that they need to ask more specific questions in order to elicit student thinking. However, each PST had his or her own pattern of asking questions. For example, the first PST (Andrea) asked a series of questions about methods (e.g., What do you think? What kind of methods do you think that Lia did in her math problem?); the second PST (Briana) focused on comparing between strategies and finding out which operation was used (e.g., Did she add or subtract? Is cutting it up subtraction or division?); the third PST (Christine) focused on asking for explaining how to solve the problem; the fourth PST (Danielle) focused on convincing Lia’s method because she considered it as an easier method; the fifth PST (Emma) focused on mathematical vocabulary (e.g., quotient, dividend, and divisor), checking the correctness of the answer, and writing the quotient on the answer spot; and the last PST (Francesca) focused on similar thinking process and new thing learned about the long division. The number of student-avatars selected, the duration of interaction with one student-avatar’s idea, and the length of exchanges with one student-avatar also varied by PSTs.

Regarding the first research question, it is interesting to observe that PSTs quickly made a transition to connecting between students’ strategies. After Andrea elicited an explanation about how Lia solved the problem, Briana compared Lia’s with Ava’s, Christine compared Lia’s with Jasmine’s, Danielle compared Lia’s with Dev’s, and Emma compared Max’s with Ethan’s. The last PST in the simulation session, Francesca, did not choose a particular student’s work to compare with Lia but asked the virtual students to find out whose work is similar to Lia. The virtual students agreed that Lia’s work is similar to Jasmine, Ethan, and Dev, but it is different from Ava and Savannah. As Smith and Stein (2018) argue, connecting is one of the five practices of orchestrating a
whole-group discussion. However, the early transition to connecting without fully eliciting individual student thinking might make it more challenging for PSTs to meaningfully connect between students' strategies. During the simulation session, several key mathematical ideas were emerged in the process of discussing similarities and differences between students' strategies: (1) the first estimation chosen by the virtual students (e.g., Lia's 15×8; Ava's 15×30; Jasmine's 15×70; Dev's 15×90) and whose estimation is higher; (2) different operations used for the long division problem (e.g., repeated subtraction vs. adding up to the dividend); (3) difference between 1080 and 108; (4) difference between 15×70 and 15×7; and (5) equivalence between different partial quotients (e.g., Ethan's 60 and 12 vs. Max's 50, 10, 10, and 2 vs. Ava's 50, 20, and 2). However, PSTs explored such key mathematical ideas on the surface level without further challenging students or elaborating those ideas in depth.

To elicit student thinking, it is important to emphasize that PSTs need to diagnose mathematical meaning of individual student's thinking in-depth and interpret how the student uses a particular strategy first, instead of jumping into comparing and contrasting various strategies from the very beginning of instruction. In this process of eliciting student thinking, the instructional decision made by an individual PST reflects their own mathematical knowledge. Often, PSTs made mathematically incorrect conclusions or missed the mathematically productive discussable moments. This is consistent with the finding that instructional decisions are based on content knowledge, discourse, and behavior compliance (Dieker et al., 2017).

Considering that connecting is the most challenging practice for leading a productive whole-group discussion even for in-service teachers (Pang, 2016), it is more effective for PSTs to learn what strategies they need to choose, compare, and contrast for making mathematically meaningful connections and how the particular strategy chosen can be connected to the mathematical point of the lesson or the lesson objectives in teacher education program.

Regarding the second research question, the number of pause and the reason for pause varied. The number of lead PST-initiated "pause" ranged one to four times but they mainly paused the session when they had difficulty in handling students' non-mathematical comments, asking for clarifying their questions, searching for accurate mathematical vocabulary, confirming whether their questions work, or discussing the next teaching moves. The peer-coaches did not voluntarily "pause" the simulation sessions but actively shared what they noticed from the student work samples and suggested the next teaching moves. Without utilizing the pause session, however, the dramatic improvement of questioning was not observed.

The "pause" session is one of the tools for the digital simulation used in this study, but has many implications for teacher education program. PSTs who just started to learn teaching, might feel overwhelmed to manage multiple relationships between students around the content simultaneously (Ball and Cohen, 1999) and find it difficult to make the best instructional decisions on the spot. It is possible that PSTs might insufficiently elicit student thinking, make incorrect mathematical conclusions, or miss the mathematical point of discussion. Given that instructional failure harms students significantly in a real setting, the use of pause session can improve instructional decisions based on comments provided by peer-coaches and teacher educators while maintaining the complexity of teaching practices. Because making a decision in the pedagogical dilemmas is especially challenging for PSTs, it is important for teacher educators to slow down the instructional interactions by using the "pause" session in approximating teaching practices (Schutz, Grossman, & Shaughnessy, 2018). The "pause" session can be effectively used in other settings such
as video-club and rehearsals.

Regarding the third research question, the PSTs mentioned that they felt intimidating and overwhelmed to interact with the student-avatars in front of their classmates and professors. However, all PSTs—both the lead PSTs who directly interacted with the student-avatars and the peer-coaches who gave comments to the lead PSTs—highlighted the benefits of simulations including the power of "pause" session; getting supports from others; knowing where students stand and struggle; understanding student thinking; bouncing off different questions; the importance of asking more specific and open-ended questions to elicit better responses; and knowing different ways to solve the problem. Even though they realized that this is the virtual setting, all PSTs viewed this digital simulation as "real" and "authentic" experience.

It is often assumed that PSTs are well-prepared for teaching by learning theoretical knowledge in the university coursework and practical knowledge in the field placement during their teacher education program, but the gap between the university coursework and field experience is often reported (Pang, Kim, & Choi, 2009). Recently, the efforts have been made to offer elementary mathematics methods courses in K-8 classrooms, but still many methods courses are only offered in the university with limited opportunities to engage in the "problem of enactment" (Schutz et al., 2018, p. 62). Without much preparation about learning the core teaching practices, PSTs often feel frustrated or overwhelmed with teaching a whole-class during their student teaching experience. Mentor teachers might provide supports, feedback, and comments on the overall teaching performance, but might vary in terms of the depth of content-specific pedagogies and knowledge. Considering that PSTs who experienced a digital simulation reflect positively, it is important to provide a safe environment for them and an opportunity to learn a core teaching practice by experiencing instructional failure in a simulated setting but making a better instructional decision in a real context.

Although this is not directly related to the research questions of this study, a noteworthy remark is that the PSTs had tendencies to conclude the discussion by asking the most preferred, comfortable, and easier method to solve the long division problem after comparing students' various strategies. In some cases, PSTs concluded that the standard algorithm with the least number of steps involved is the easiest method to solve the problem, which did not fully capture the rich mathematical ideas emerged in the whole-group discussion. In other cases, they concluded the discussion by surveying personal preference about the method but did not address conceptual knowledge behind the long division or procedural fluency (e.g., flexibility, appropriateness, efficiency, or accuracy). Besides the five practices of orchestrating a whole group discussion such as anticipating, monitoring, selecting, sequencing, and connecting (Smith & Stein, 2018), concluding the discussion by highlighting a mathematical point needs to be further considered.

Despite the affordances of using a digital simulation to approximate the core practice of eliciting student thinking, some challenges still remain. First, it is quite expensive to design a new scenario, consult with the simulation interactor, and run simulations multiple times. Because of the cost, only six PSTs played a role of teachers in the simulation, whereas other PSTs served as peer-coaches. If the budget allows, this study could get more benefits from doing after-action-review (AAR) session, tagging the video, and doing multiple post-simulation sessions. Second, it is possible to communicate with the simulation interactor during the simulation session, but controlling the reactions of virtual students in a real time can be somewhat challenging for teacher educators.

Lastly, this study did not aim to show the effect of
this particular mathematics methods course (specifically using a digital simulation) on PSTs’ performance on eliciting student thinking. Rather, this study shows that a digital simulation serves as a great tool to identify the pattern of questioning, teaching moves, pedagogical practices (e.g., selecting and connecting), instructional decisions and its rationale, and any shifts of instructional focus. Even though this study was conducted in the United States of America, this innovative approach to approximate the core teaching practices via an immersive interactive digital simulation can be easily adopted in Korea. To run the simulation, the minimal equipment such as a large monitor screen and web camera are needed for the users (i.e., teacher educators).

Over the past twenty years, research on teacher education in Korea has quantitatively increased, especially for preparing pre-service teachers and in-service teachers (Pang et al., 2019; Shin, 2020). However, except some studies such as video-case analysis (Pang et al., 2009), microteaching (Shim, 2018), and task dialogue activity (Kim & Lee, 2017), the pedagogy of teacher education and available pedagogical tools that teacher educators can effectively use are not well known yet. Given this, a digital simulation can provide more authentic and realistic experience for PSTs where each student’s response was deliberately designed by the simulation expert and content-expert teacher educator. The affordances of using a digital simulation to approximate a core teaching practice in this study sets forth an example of practice-based teacher education program in Korea.

References


