

# Impact of Correlation on Superposition Coding in NOMA for Interactive Mobile Users in 5G System: Achievable Sum Rate Perspective

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## *Abstract*

*The fifth generation (5G) mobile communication has been more commercialized worldwide. One of the promising 5G technologies is non-orthogonal multiple access (NOMA). We present the achievable sum rate of non-orthogonal multiple access (NOMA) with correlated superposition coding (SC). Then this paper investigates the impacts of correlation on the achievable sum rate of correlated SC NOMA. It is shown that the achievable sum rate of correlated SC NOMA is greater than that of standard independent SC NOMA, for the most of the values of the power allocation factor over the meaningful range of the user fairness. In result, correlated SC could be a promising scheme for NOMA.*

**Keywords:** *Non-orthogonal multiple access, Superposition coding, Successive interference cancellation, Power allocation, Correlation coefficient.*

## 1. Introduction

In the fifth generation (5G) and beyond mobile networks, the users have demanded the faster networks, compared to existing orthogonal multiple access (OMA) [1-2]. One of the state-of-the-art techniques for 5G and beyond mobile radio access networks is non-orthogonal multiple access (NOMA) [3-6], which the standard bodies have been considering as a promising multiple access in the cellular networks. In NOMA, the superposition principle in the power-domain is exploited to increase the system capacity. The 5G interactive mobile users result in the correlated information sources. Recently, correlated superposition coding (SC) NOMA not only achieves the near-perfect successive interference cancellation (SIC) bit-error rate (BER) performance for the stronger channel user, but also mitigates the performance degradation for the weaker channel user [7]. For correlated SC NOMA, this paper investigates the impacts of the correlation on the achievable sum rate of NOMA.

### 1.1 Our Contribution Summary

Our new contributions are summarized as follows; first, we calculate the achievable sum rate of correlated SC NOMA. Second, based on the calculation, we depict the results to various fading channel gains. Third, one of the most important contributions of this paper is that the superiority of correlated SC NOMA over

independent SC NOMA increases as the channel gain difference decreases.

## 2. System and Channel Model

Assume that the constant total transmitted power is  $P$ , and the powers of the user-1 signal  $s_1$  and the user-2 signal  $s_2$  are normalized as  $\mathbb{E}[|s_1|^2] = \mathbb{E}[|s_2|^2] = 1$ , with the correlation coefficient  $\rho_{1,2} = \mathbb{E}[s_1 s_2^*]$ . The power allocation factor is  $\alpha$ , with  $0 \leq \alpha \leq 1$ . Then the superimposed signal is expressed by

$$x = \sqrt{\alpha P_A} s_1 + \sqrt{(1-\alpha) P_A} s_2. \quad (1)$$

where the total allocated power  $P_A$  is given by [7]

$$P_A = \frac{P}{1 + 2 \operatorname{Re}\{\rho_{1,2}\} \sqrt{\alpha} \sqrt{1-\alpha}}. \quad (2)$$

Note that for  $\rho_{1,2} = 0.0$ , i.e., standard independent SC,  $P_A = P$ . Before SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and user-2 are represented as

$$\begin{aligned} r_1 &= |h_1| \sqrt{\alpha P_A} s_1 + (|h_1| \sqrt{(1-\alpha) P_A} s_2 + n_1) \\ r_2 &= |h_2| \sqrt{(1-\alpha) P_A} s_2 + (|h_2| \sqrt{\alpha P_A} s_1 + n_2) \end{aligned} \quad (3)$$

where the channel gains are  $h_1$  and  $h_2$ , with  $|h_1| > |h_2|$ ,  $n_1$  and  $n_2 \sim \mathcal{N}(0, N_0/2)$  are additive white Gaussian noise (AWGN) and  $N_0$  is one-sided power spectral density.

## 3. Achievable Sum Rate

The correlation coefficient of correlated SC NOMA [7] is calculated by

$$\rho_{1,2} = \frac{1}{\sqrt{2}} \simeq 0.707. \quad (4)$$

The achievable rates to the first user and second users are given by, for  $i = 1, 2$ ,

$$\begin{aligned} R_i^{(\text{correlated SC NOMA})} &= \max_{p_{B_i}(b_i)} H(r_i) - H(r_i | b_i) \\ &= - \int_{-\infty}^{\infty} p_{R_i}(r_i) \log_2 p_{R_i}(r_i) dr_i + \sum_{b_i=0}^1 \int_{-\infty}^{\infty} p_{R_i|B_i}(r_i | b_i) p_{B_i}(b_i) \log_2 p_{R_i|B_i}(r_i | b_i) dr_i \end{aligned} \quad (5)$$

where

$$\begin{aligned}
 p_{R_i|}(r_i) &= \frac{1}{4\sqrt{2\pi N_0/2}} e^{-\frac{(r_i - |h_i|\sqrt{\alpha P_A} - |h_i|\sqrt{2(1-\alpha)P_A})^2}{2N_0/2}} \\
 &+ \frac{1}{4\sqrt{2\pi N_0/2}} e^{-\frac{(r_i - |h_i|\sqrt{\alpha P_A})^2}{2N_0/2}} \\
 &+ \frac{1}{4\sqrt{2\pi N_0/2}} e^{-\frac{(r_i + |h_i|\sqrt{\alpha P_A})^2}{2N_0/2}} \\
 &+ \frac{1}{4\sqrt{2\pi N_0/2}} e^{-\frac{(r_i + |h_i|\sqrt{\alpha P_A} + |h_i|\sqrt{2(1-\alpha)P_A})^2}{2N_0/2}},
 \end{aligned} \tag{6}$$

$$\begin{aligned}
 p_{R_i|B_1}(r_i | b_1) &= \frac{1}{2\sqrt{2\pi N_0/2}} e^{-\frac{(r_i - (-1)^{b_1}(|h_1|\sqrt{\alpha P_A} + |h_1|\sqrt{2(1-\alpha)P_A}))^2}{2N_0/2}} \\
 &+ \frac{1}{2\sqrt{2\pi N_0/2}} e^{-\frac{(r_i - (-1)^{b_1}|h_1|\sqrt{\alpha P_A})^2}{2N_0/2}},
 \end{aligned} \tag{7}$$

and

$$\begin{aligned}
 p_{R_2|B_2}(r_2 | b_2) &= \frac{1}{2\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_2|\sqrt{\alpha P_A} - b_2|h_2|\sqrt{2(1-\alpha)P_A})^2}{2N_0/2}} \\
 &+ \frac{1}{2\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 + |h_2|\sqrt{\alpha P_A} + b_2|h_2|\sqrt{2(1-\alpha)P_A})^2}{2N_0/2}}.
 \end{aligned} \tag{8}$$

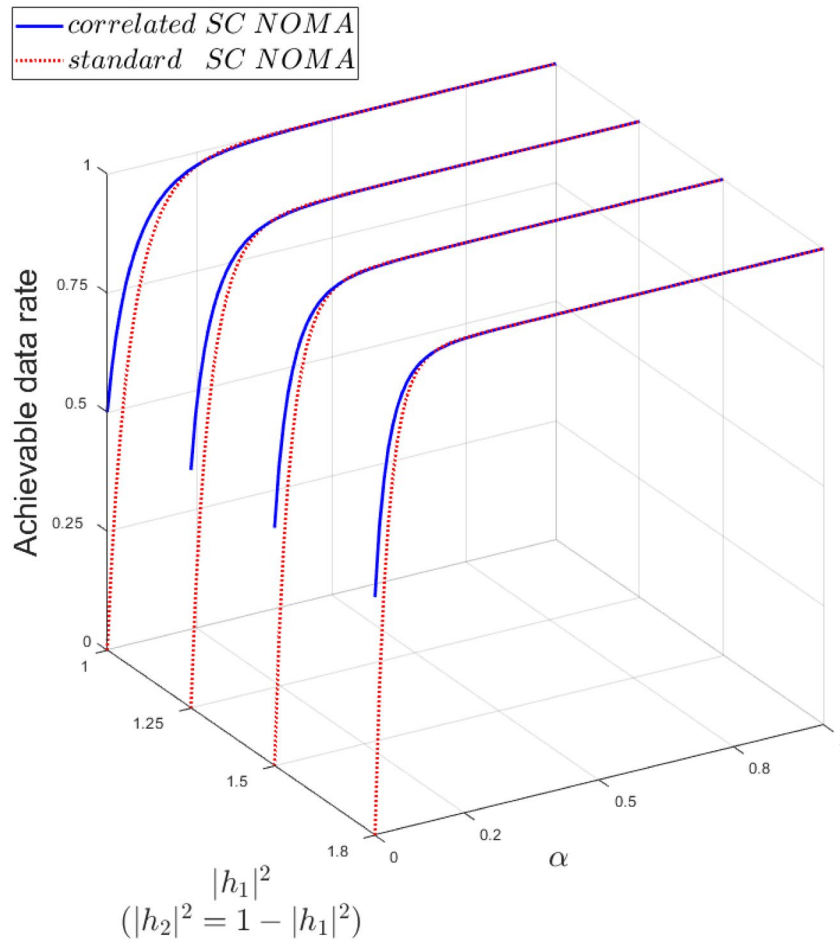
Then the achievable sum rate  $R_{sum}^{(correlated\ SC\ NOMA)}$  is given by

$$R_{sum}^{(correlated\ SC\ NOMA)} = R_1^{(correlated\ SC\ NOMA)} + R_2^{(correlated\ SC\ NOMA)}. \tag{9}$$

## 4. Results and Discussions

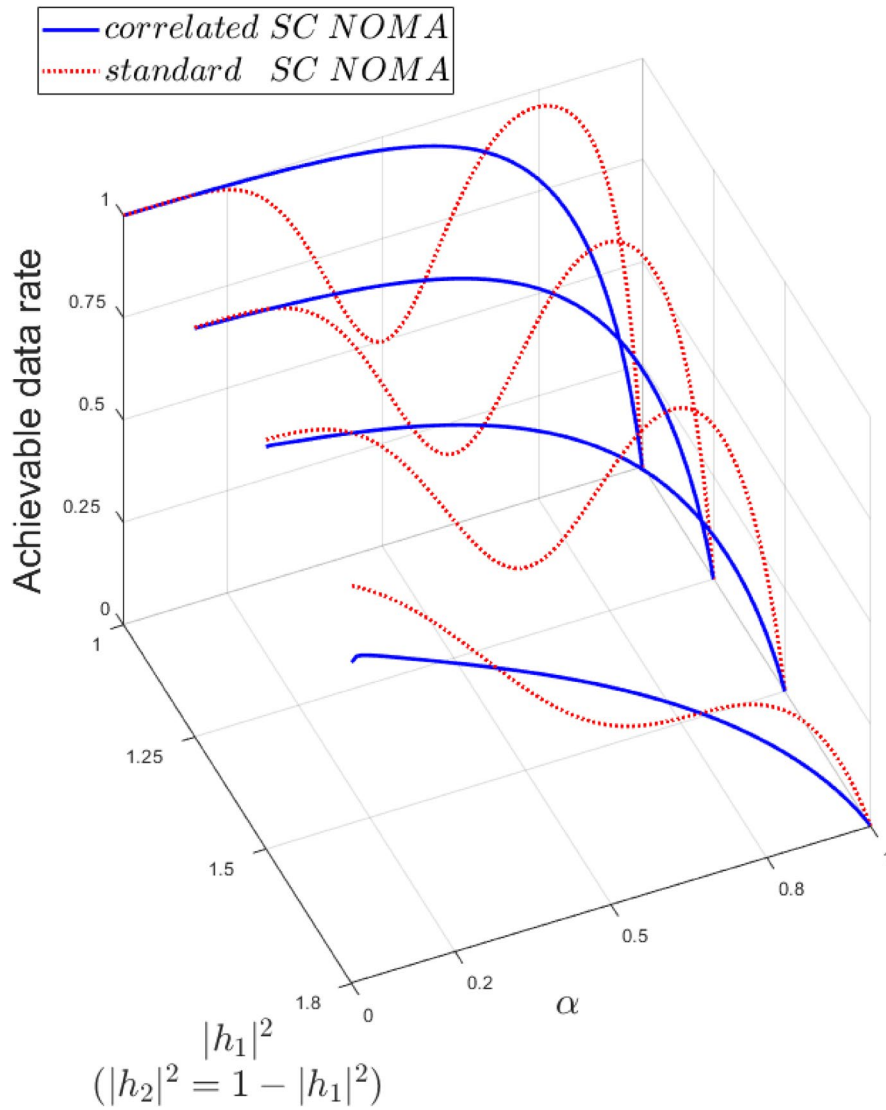
Assume the constant total transmitted signal power to noise power ratio (SNR) is  $P/N_0 = 15$ . In NOMA, the performance greatly depends on the channel gains  $|h_1|$  and  $|h_2|$ . Then, we consider the various channel gains, i.e.,  $|h_1|^2 = 1.0, 1.25, 1.5$ , and  $1.8$ , with  $1 = \frac{|h_1|^2 + |h_2|^2}{2}$ , which guarantees the unit average instantaneous channel gain of the energy conservation of the given system. In conventional standard NOMA, the achievable rates to the first user and the second user are given in [8].

As shown in Fig. 1, for the first user, the achievable data rate for correlated SC NOMA is greater than that for standard SC NOMA, for all the values of the power allocation factor,  $0 \leq \alpha \leq 1$ , and for all the values of the channel gains  $|h_1|^2 = 1.0, 1.25, 1.5$ , and  $1.8$  and  $|h_2|^2 = 1.0, 0.75, 0.5$ , and  $0.2$ .



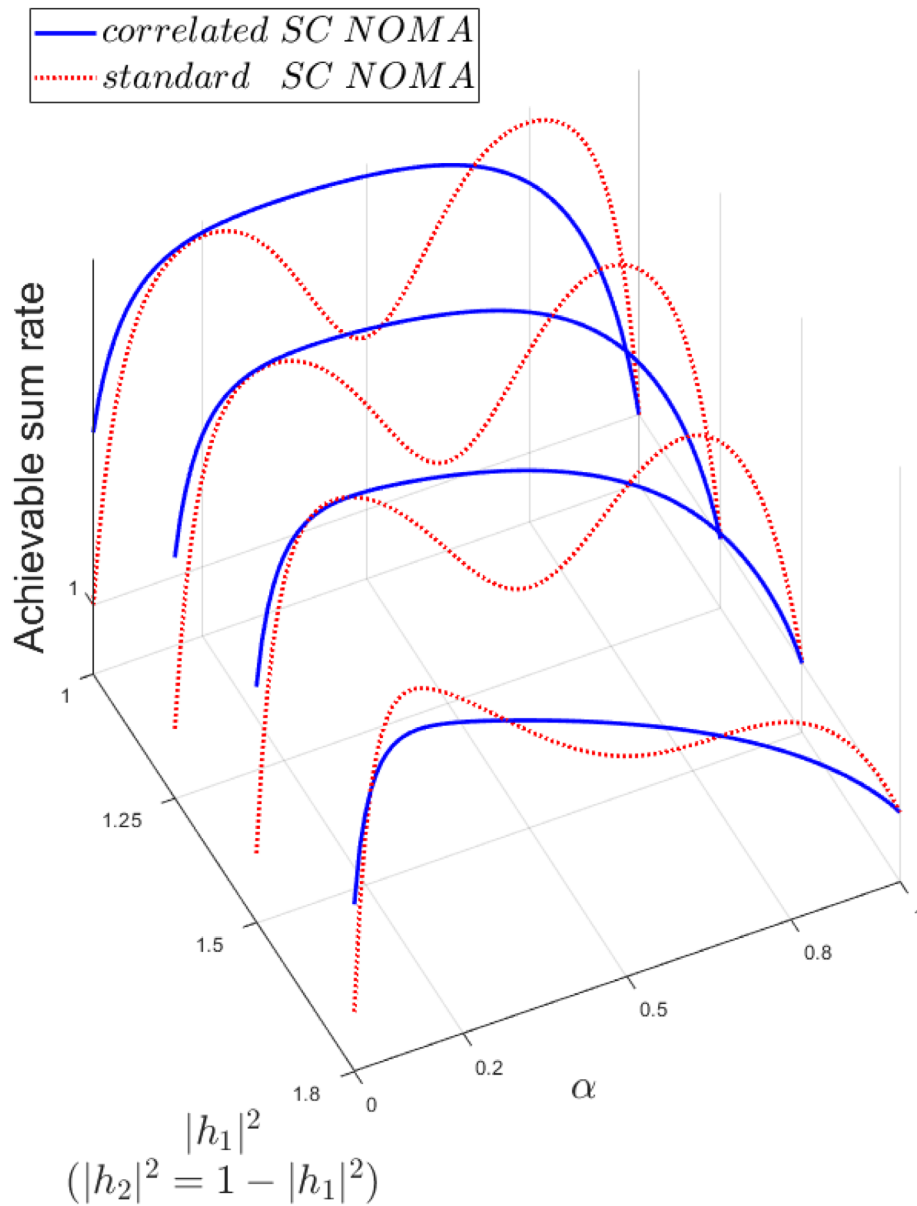
**Figure 1. Comparison of achievable data rates for standard SC NOMA, and correlated SC NOMA, for first user.**

However, for the second user, the achievable data rate for correlated SC NOMA is greater than or less than that for standard SC NOMA, dependent on the power allocation factor, as shown in Fig. 2.



**Figure 2. Comparison of achievable data rates for standard SC NOMA, and correlated SC NOMA, for second user.**

Since in NOMA, the two users share the channel resources, the performances of the two users should be considered together. Therefore, we depict the achievable sum rates of the both users, in Fig. 3.



**Figure 3. Comparison of achievable sum rates for standard SC NOMA, and correlated SC NOMA.**

We observe that the superiority of the correlated SC NOMA is dependent on the channel gain difference. Specifically, although the maximum of the achievable sum rate for standard SC NOMA is greater than that for correlated SC NOMA, the achievable sum rate of correlated SC NOMA is greater than that of standard independent SC NOMA, for the most of the values of the power allocation factor over the meaningful range of the user fairness,  $\alpha < 0.5$ , (i.e., the more power to the weaker channel user, the less power to the stronger channel user). When the difference of the channel gains becomes smaller, i.e.,  $(|h_1|^2, |h_2|^2) = (1.0, 1.0), (1.25, 0.75)$ , the efficiency of correlated SC is more significant, as shown in Fig. 3.

## 5. Conclusion

We presented the achievable sum rate of NOMA with correlated SC. Then this paper investigated the impacts of correlation on the achievable sum rate of NOMA. It was shown that the achievable sum rate of correlated SC NOMA is greater than that of standard independent SC NOMA, for the most of the values of the power allocation factor over the meaningful range of the user fairness. As a consequence, correlated SC could be a promising scheme for NOMA.

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