Current Optics and Photonics
Vol. 4, No. 4, August 2020, pp. 267-272

ISSN: 2508-7266(Print) / ISSN: 2508-7274(Online)
DOI: https://doi.org/10.3807/COPP.2020.4.4.267

Propagation Dynamics of a Finite-energy Airy Beam with Sinusoidal Phase in Optical Lattice

Xiaoyuan Huang, Manna Chen, Geng Zhang, Ye Liu, and Hongcheng Wang*

School of Electrical Engineering and Intelligentization, Dongguan University of Technology, Dongguan 523808, China

(Received April 10, 2020 : revised June 4, 2020 : accepted June 10, 2020)

The propagation of a truncated Airy beam with spatial phase modulation (SPM) is investigated in Kerr nonlinearity with an optical lattice. Before the truncated Airy beam enters the optical lattice, a sinusoidal phase is introduced on the wave-front of the beam. The effect of the spatial phase modulation and optical lattice on propagation behavior is analyzed by direct numerical simulation. It is found that the propagation direction of a truncated Airy beam can be effectively controlled by adjusting the values of phase shift. The effects of optical amplitude, truncation factor, spatial modulation frequency, lattice period and lattice depth on the propagation are discussed in detail. By choosing a high modulation depth, the finite-energy Airy beam can be deflected with a large deflection angle in an optical lattice.

Keywords : Optical propagation, Truncated Airy beam, Phase modulation, Optical lattice
OCIS codes : (000.4430) Numerical approximation and analysis; (190.3270) Kerr effect; (190.6135) Spatial solitons

I. INTRODUCTION

Recently, self-accelerating Airy beams have received a great deal of attention since they were first theoretically and experimentally demonstrated in 2007 [1, 2]. This concept and the finite-energy solution to the paraxial wave equation were transferred from the solution of the free-particle Schrödinger equation within the context of quantum mechanics [3]. Because the perfectly diffractionless Airy beams have infinite energy, they cannot be realized in experiment. To make them realizable in optics, an exponentially decaying factor was introduced to the ideal Airy beam. These truncated Airy beams can retain their unique properties of nondiffraction, self-acceleration, and self-healing over long distances. Now spatially truncated Airy beams have found applications in creating self-bending plasma channels [4], particle micromanipulation [5], and ultrafast self-accelerating pulse generation [6], etc. Self-accelerating Airy beams have also been widely investigated in nonlinear media, such as the nonlinear generation of Airy beams [7], spatial Airy solitons [8, 9], as well as spatiotemporal Airy light bullets [10]. Later, the evolutions of Airy-Gaussian beams with different intensity and phase profiles were also investigated in the nonlinear Kerr medium [11-13]. In the nonlinear regime, some interesting phenomena have been found in the interactions of truncated Airy beams. The interactions of two Airy beams and nonlinear accelerating beams in Kerr and saturable nonlinear media made it possible to form bound and unbound soliton pairs, as well as single solitons [14, 15]. In 2015 and 2016, Shen and his coworkers revealed a controllable manipulation of anomalous interactions between Airy beams in nonlocal nonlinear media, and nematic liquid crystals, [16, 17] respectively. The interaction of two Airy-Gaussian beams can lead to the formation of single breathers and breather pairs [18]. Recently, the mutual interaction of Airy beams were also investigated in photorefractive media [19] and in the fractional nonlinear Schrödinger equation [20]. In nonlocal nonlinear media, the interactions between a truncated Airy beam and a soliton beam can not only

*Corresponding author: wanghc@dgut.edu.cn, ORCID 0000-0001-6842-4326
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produce new optical beams but also generate and control breather solitons [21].

On the other hand, phase distribution has an important effect on the beam propagation and optical manipulation. For example, phase engineering of an axicon's phase transmission function can result in an exponential growth of an on-axis intensity of a diffraction-free or Bessel beam [22]. Careful design of the initial phase can be used to create self-bending wave packets propagating along arbitrary prescribed convex trajectories [23]. In optical Kerr nonlinearity, an initial sinusoidal phase can be used to steer the propagation direction of a Gaussian beam [24]. The basic idea is to introduce a sinusoidal phase on the wavefront before the beam enters the nonlinear Kerr medium. This method was used to control the optical propagation of Gaussian beams or solitons in a photovoltaic crystal and optical lattices [25, 26]. Recently, the propagation of a finite-energy Airy beam with spatial phase modulation was investigated in an optical Kerr medium. It was found that optical deflection, optical splitting, and periodical oscillation can be realized by choosing proper modulation parameters [27]. In a photonic lattice, the presence of periodically varying refractive index in the transverse spatial dimension can also affect the propagation and localization of optical beams in nonlinear media. Then questions naturally arise: What will propagation dynamics be when a truncated Airy beam with SPM propagates in Kerr nonlinearity with a lattice potential? Can the spatial phase modulation steer the propagation effectively in an optical lattice? In this paper, we investigate the propagation of a finite-energy Airy beam in Kerr nonlinearity with a periodical lattice. The influences of optical field amplitude, modulation amplitude, spatial frequency and phase shift on the optical steering are discussed in detail.

II. THEORY

To illustrate the propagation dynamics, we consider a one-dimensional truncated Airy beam propagating in Kerr nonlinearity with a periodical lattice potential. In the paraxial approximation, the optical propagation can be described by the following equation:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+|u|^2 u+V(x)u=0\,, \tag{1}$$

where u is the normalized optical field envelope, z and x are the normalized coordinates of the longitudinal direction and the transverse direction, respectively. $V(x)=V_0\cos^2(x/T)$ describes the transverse profile of the optical lattice potential, V_0 describes the modulation depth, and T is the period of the lattice.

The optical field of a transverse self-accelerating Airy beam with finite energy can written as

$$u(x,0)=Ai(x)\exp(ax)\,, \tag{2}$$

where a is a positive decay factor. The introduction of this decay factor ensures the physical realization condition of finite energy. To investigate the propagation dynamics of the truncated Airy beam with phase modulation in optical lattice, a programmable spatial phase-only modulator is inserted before the nonlinear Kerr medium with optical lattice on the optical path. The sinusoidal phase can be generated by computer generated holograms [28]. When the truncated Airy beam passes through a thin sinusoidal phase grating located in the (x, y) plane with grating lines parallel to the y-axis, the optical field can be expressed as

$$u(x,0)=r\cdot Ai(x)\exp(ax)\exp[i\phi_0\sin(2\pi px+\delta)]\,, \tag{3}$$

where r is the amplitude of optical filed. ϕ_0, p, and δ represent the modulation amplitude, modulation frequency, and phase shift of the sinusoidal phase, respectively. By using the standard Bessel-function to expand the modulation term, Eq. (3) can be rewritten as

$$u(x,0)=r\cdot Ai(x)\exp(ax)\sum_{m=-\infty}^{+\infty}J_m(\phi_0)\exp[im\sin(2\pi px+\delta)]\,. \tag{4}$$

According to Eq. (4), the input beam is divided into several subbeams which propagate at different angles by the sinusoidal phase modulation. The initial amplitude of the mth subbeam is determined by $J_m(\phi_0)$. High modulation depth will lead to some energy creeping back into the central subbeam, while low modulation depth will shed some of the power from the central subbeam. In Ref. [25], modulation depth ϕ_0 is chosen as $\phi_0 = 2.405$, the first zero-point of the zero-order Bessel function. Due to the self-accelerating property of Airy beam and the presence of the optical lattice, the steering of a truncated Airy beam will be difficult. To steer the propagation of finite-energy Airy beam effectively, we hereafter choose $\phi_0 = 5.502$, the second zero-point of J_0. In this case, phase modulation not only leaves nearly no power in the central portion of the beam but also makes the optical steering more effective. Besides, the modulation frequency p, which determines incident angle of each subbeam, also has great impact on the propagation dynamics. High values of the modulation frequency p lead to large angles of the subbeams. However, extra-large values of p will violate the paraxial approximation while the optimum effect of beam steering cannot be obtained for extra low values of p. The appropriate range of modulation frequency p is $0.1 \le p \le 0.3$. This parameter range can ensure no violation of the paraxial approximation as well as effective control of the optical beam.

III. RESULTS

By using split step Fourier method, Eq. (1) can be simulated for the propagation dynamics of the phase-modulated Airy beam in an optical lattice. Under the combined effects of Kerr nonlinearity, self-deflection effect of the Airy beam, spatial phase modulation, and the localization of the optical lattice, the optical propagation of a truncated Airy beam can be effectively controlled by choosing different values of optical parameters.

3.1. Optical Beam Deflection

In this section, we discuss how to steer the deflection of the finite-energy Airy beam in an optical lattice by changing the values of δ. For the facility of demonstration, we assume that the lattice parameter is set as $T = 1$, and $V_0 = 2$, and the decay factor $a = 0.2$. Figure 1 shows the evolutions of finite-energy Airy beams with sinusoidal phase when $p = 0.1$. As demonstrated in Ref. [24], the sinusoidal phase modulation brings a force which can make the beam deflect during the propagation. For a truncated Airy beam, the main lobe owns a majority part of the energy, while only a little energy is left in the side lobes. Under the effect of the optical lattice, the phase-modulated Airy beams with different parameters present different propagation dynamics. In the top row, the phase shift $\delta = 0$. One can see that the spatial phase modulation makes the truncated Airy beam deflect to the positive direction of the x axis. In Fig. 1(a), $r = 1$. In this case, most energy remains in the main lobe of the truncated Airy beam, and the main lobe is deflected to the positive direction of the x axis. After short distance

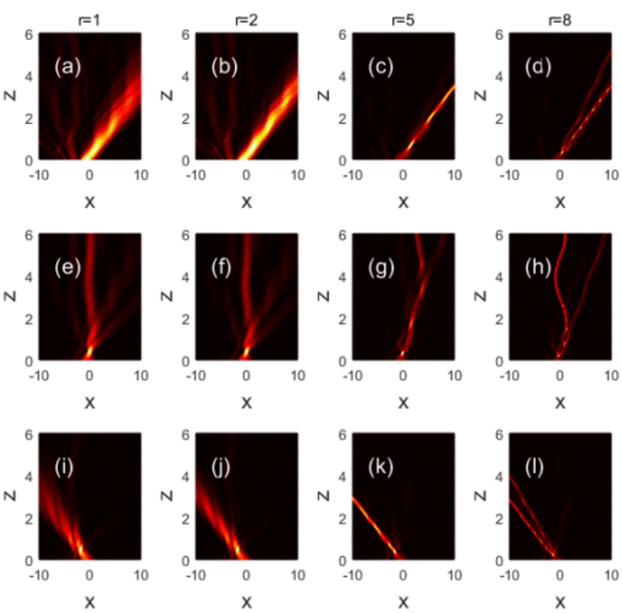

FIG. 1. Deflection of a spatial-phase-modulated Airy beam in an optical lattice when $p = 0.1$, $a = 0.2$, and $V_0 = 2$. The value of phase shift δ is 0 (top row), $\pi/2$ (middle row), and π (bottom row), respectively.

propagation, some energy departs from the main lobe and propagates in the optical lattice. Besides, the energy in the side lobes is caught by the waveguide effect and thus the side lobes propagate along the optical lattice soon after they come into the optical lattice. As the optical amplitude r increases, the deflection effect caused by the spatial phase modulation is getting stronger, and relatively low energy is caught by the optical lattice. As shown in Fig. 1(b), most energy in the main lobe keeps the deflection in the deflection direction when $r = 2$. If we increase the optical amplitude r further, e.g., $r = 5$ and 8, the spatial phase modulation and the self-deflection property of the Airy beam dominate the propagation dynamics, as shown in Figs. 1(c) and 1(d). When $r = 5$, as shown in Fig. 1(c), the beam width of the main lobe first becomes narrow and the optical field amplitude increases dramatically. Due to the strong self-focusing and the waveguide effect of the optical lattice, a part of relatively high energy is departed from the main lobe and a soliton-like beam comes into birth. Both these beams propagate as soliton-like breathers. When $r = 8$ [Fig. 1(d)], the main lobe is divided to two soliton-like beams soon after it enters the optical lattice. In this case, the first side lobe also oscillates and propagates as a snake shape under the confinement effect of the optical lattice. When $\delta = \pi/2$, a phase-modulated Gaussian beam propagates along the z axis in a Kerr nonlinear medium with an optical lattice [26]. Different from the counterpart of a Gaussian beam, the finite-energy Airy beam still deflects under the spatial phase modulation during the propagation because of the self-accelerating property of the Airy beam. Figures 1(e)-1(h) give the propagation dynamics when $\delta = \pi/2$. When r has a low value (e.g., $r = 1$, 2), the self-accelerating is relatively low, and most of the energy is caught by the optical lattice. Therefore, N-soliton-like propagation forms [Figs. 1(e) and 1(f)]. If we increase the value of r, the self-accelerating force increases. When $r = 5$, as shown in Fig. 1(g), a part of energy deflects at first. After a long-distance soliton-like deflection, the beam reflects under the effect of the optical lattice and the nonlinearity. If we increase r further to be 8, as shown in Fig. 1(h), the self-accelerating momentum of the truncated Airy beam is large enough so that, a part of the energy can be free of the confinement of the central lattice and the new birth beam can keep the propagation at a small angle. Most energy propagates as an s-shape in the central lattice.

In the case $\delta = \pi$, the optical beam is deflected to the negative direction of the x axis [Figs. 1(i)-1(l)]. As shown in Figs. 1(i)-1(j), when the intensity is relatively low, interactions are observed between the main lobe and the side lobes. Some energy is transferred to the side lobes. Each side lobe propagates separately in a waveguide of the lattice but vanishes in the background quickly under the effect of diffraction. When $r = 5$, the self-deflection produced by spatial phase modulation is strong enough to overcome the self-accelerating and the lattice confinement. As a result, a soliton-like beam comes into birth and it self-deflects to the negative direction of the x axis [Fig. 1(k)]. When r is increased further to be 8, as shown in Fig. 1(l), two beams are shed from the main lobe. Both of them can propagate as a soliton-like beam at a certain angle. Besides, the first side lobe also has enough large energy and it swings periodically in the lattice.

3.2. Effect of Modulation Frequency *p* on the Steering

To study the propagation dynamics at different spatial modulation frequency p, Fig. 2 is drawn to show the effect of p on the propagations of finite-energy Airy beams at $r = 2$, 5, and 8 when $a = 0.2$ and $\delta = 0$. As we demonstrate in Sec. 3.1, the finite-energy Airy beam can be effectively controlled by the spatial phase modulation in Kerr nonlinearity with optical lattice [see that in Figs. 1(a)-1(d)]. When we change $p = 0.2$, each subbeam has a larger incident angle compared with the case of $p = 0.1$. Therefore, a larger deflection angle can be reached when $p = 0.2$, as shown in Figs. 2(a)-2(c). From Fig. 2(a), one can see that the main lobe deflects to the positive direction of the x axis though it diffracts during the propagation when $r = 2$. In the meantime, a small part of the energy remains in the side lobes. They are caught in the waveguides and vanish in the background quickly. When $r = 5$, most energy can propagate as a soliton and it deflects with a large angle [Fig. 2(b)]. If we increase the value of r to be 8, a new soliton-like beam comes into birth from the main lobe. Both of these two beams can propagate as a soliton but the deflection angle is different [Fig. 2(c)]. That is to say, the spatial phase modulation can be used to steer the propagation direction to some extent when $p = 0.2$. If we further increase the value of p to 0.3, it is difficult to control the beam by sinusoidal phase modulation. As shown in Figs. 2(d)-2(f), the propagation becomes more complex as the modulation frequency p increases because the amount of the exciting subbeams increases at the same time. Therefore, we can conclude that it is difficult to control the propagation of the truncated Airy beam at a too high value of p, while the benefits of the phase modulation can be maximized at a too low value of p. For effective control of the beam propagation direction, the optimal value of p is about 0.1.

3.3. Effect of Truncated Coefficient *a* on the Steering

To understand the optical steering in optical lattice better, we investigate the propagation dynamics of a finite-energy Airy beam with different values of truncation coefficient a. When $p = 0.1$, $\delta = 0$, $r = 3$, we can plot Fig. 3 to show the propagation dynamics. When a has a relatively low value, e.g., $a = 0.1$ in Fig. 3(a) and $a = 0.2$ in Fig. 3(b), most energy remains in the main lobe and the beam width increases during the deflection. At the same time, the side lobes have considerable energy. They are trapped in the waveguides of the lattice potential quickly once they enter the optical lattice. The evolutions of the side lobes are always accompanied by strong diffraction because the intensity of each side lobe is not strong enough to form a soliton in the lattice.

Compared to that in the main lobe, the energy remaining in the side lobes becomes relatively low as the truncation coefficient increases. For example, most energy can be steered when $a = 0.3$, as shown in Fig. 3(c). When the coefficient is high enough, e.g., $a = 0.6$, the side lobes disappear gradually and the steering is similar to that of Gaussian beams, as shown in Fig. 3(d). Therefore, we can conclude from Fig. 3 that as the truncated coefficient increases, the steering of finite-energy Airy beam becomes easier and more effective.

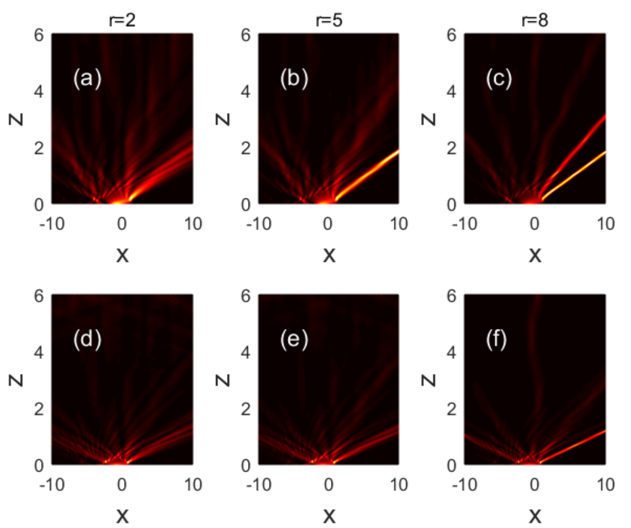

FIG. 2. The effect of p on the optical steering when $a = 0.2$, and $V_0 = 2$, and $\delta = 0$. The value of p is 0.2 (top row) and 0.3 (bottom row), respectively.

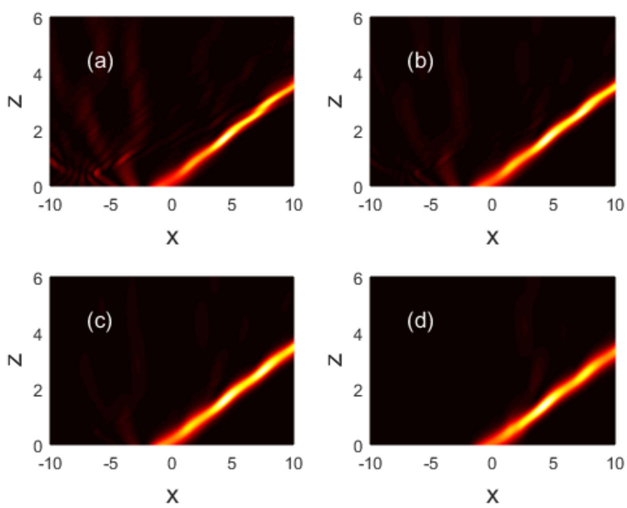

FIG. 3. Propagation of a phase-modulated finite-energy Airy beam when $a = 0.1$ (a), 0.2 (b), 0.3 (c), and 0.6 (d). Other parameters are $r = 3$, $p = 0.1$, $\delta = 0$.

3.4. Effect of the Optical Lattice on the Steering

In this section, we discuss the effect of the lattice parameter on the optical steering. First, we discuss the effect of the modulation depth V_0 on the optical steering. Figure 4 shows the propagation dynamics of a finite-energy Airy beam with sinusoidal phase at different V_0 when $r = 5$, $p = 0.1$, $\delta = 0$ and $T = 1$. When $V_0 = 1$, the modulation depth is relatively low, the deflection induced by spatial phase modulation dominates the propagation and it can overcome the confinement effect of the optical lattice potential. Therefore, most energy deflects with a large deflection angle. The beam energy remaining in the side lobe is almost negligible, as shown in Fig. 4(a). If we increase the value of V_0 to be 2, a little energy begins to depart from the main lobe and propagate in the waveguides induced by the optical lattice, as shown in Fig. 4(b). If V_0 achieves a large value, the confinement effect of the lattice plays an important role in the propagation. When $V_0 = 5$, a considerable amount of energy is confined to the central waveguide, although most of the energy still keeps deflection, as shown in Fig. 4(c). When V_0 gets an extra-large value, e.g., $V_0 = 10$, the optical lattice forms a series of deep potential wells which forms a competition relation with the phase modulation. As shown in Fig. 4(d), a large amount of energy is caught in the central waveguide and the adjacent waveguide. Only a small part of energy can be steered to the positive direction of the x axis by spatial phase modulation. Therefore, we can conclude that the optical lattice is a barrier to the deflection. If the lattice depth is extra high, the restraint of the potential wells is hard to conquer and the steering becomes difficult.

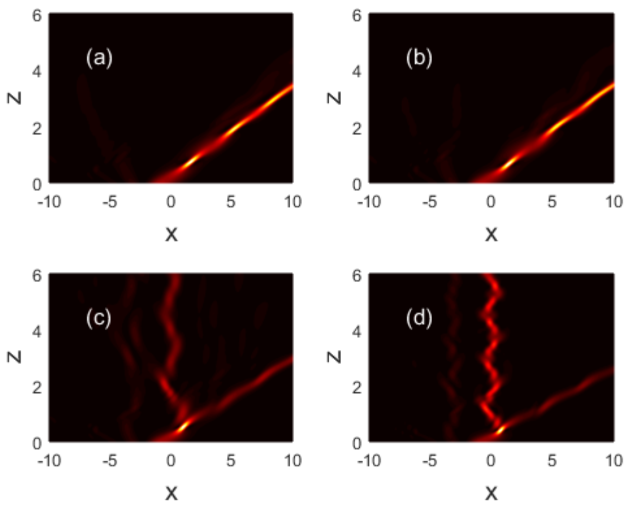

FIG. 4. Propagation dynamics of a finite-energy Airy beam with spatial phase modulation at several values of V_0 when $r = 3$, $p = 0.1$, $\delta = 0$ and $T = 1$: (a) $V_0 = 1$, (b) $V_0 = 2$, (c) $V_0 = 5$, and (d) $V_0 = 10$.

Next, we discuss the effect of the lattice period on the steering. Figure 5 shows the optical steering by spatial phase modulation in optical lattice with several values of period T when $r = 5$, $p = 0.1$, $\delta = 0$ and $V_0 = 2$. For low values of period T, the deflected beam will encounter lots of potential barriers. From Fig. 5(a), which shows the propagation at $T = 0.5$, some energy is caught by the potential and propagates in the lattice because of the confinement effect of the lattice. As the lattice period increases, the waveguide width increases and thus the potential barrier becomes flat when the modulation depth V_0 is the same. In Fig. 5(b), $T = 2$, most energy can be deflected to the same direction though the deflected beam propagates as a breather. When $T = 10$, less energy is lost in the side lobes and the main beam encounters less resistance during the deflection [Fig. 5(c)].

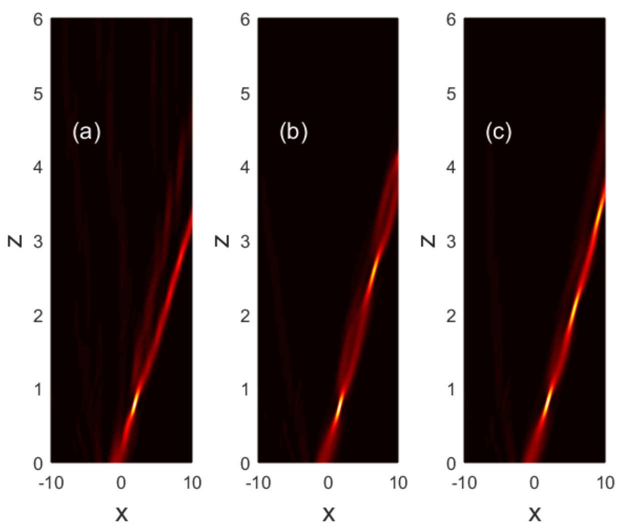

FIG. 5. Propagation of a phase-modulated finite-energy Airy beam in optical lattice when $T = 0.5$ (a), 2 (b), and 10 (c). Other parameters are $r = 5$, $p = 0.1$, $\delta = 0$ and $V_0 = 2$.

IV. CONCLUSION

In conclusion, we have numerically investigated the propagation dynamics of a truncated Airy beam with sinusoidal phase in nonlinear Kerr media with optical lattice. The effects of optical field amplitude, modulation amplitude, spatial frequency and phase shift on the optical steering are discussed in detail. For arbitrary values of optical field amplitude, the phase modulation can make the truncated finite-energy Airy beam deflect as a soliton. Spatial modulation frequency, which affects the amount of the exciting subbeams, has great influence on the evolution. Though larger deflection angle can be reached at larger value p, the optical steering will be difficult and the propagation dynamics becomes complex. When the modulation depth is 5.502, the optimal spatial frequency is 0.1. In this case, the deflection angle can be effectively controlled by changing the phase shift δ. The truncation coefficient a determines the relative power of the side lobes to the main lobe in the truncated Airy beam. As the truncation coefficient a increases, the steering of finite-energy Airy beam becomes easier and more effective.

The effect of lattice depth on the steering has also been discussed in detail. The lattice can be seen as a barrier to restrain the deflection of the optical beam. In a lattice with high modulation depth, the restrain force becomes large. The steering of the Airy beam will become difficult in the case of extra high lattice depth. In addition, the control of optical beam becomes easier as the lattice period increases. The presented spatial phase modulation can provide an effective method to control the propagation of Airy beams, and the propagation properties may have important applications in optical switches, optical logic gates and optical waveguides.

ACKNOWLEDGMENT

This study was supported by National Natural Science Foundation of China (Grant Nos. 11947122, 11874111), Guangdong Science and Technology Planning Program (Grant No. 2017A010102019) and Dongguan Social Science and Technology Development Project (Grant No. 2019507 140172).

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