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Original Article

Acceleration method of fission source convergence based on RMC code

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ABSTRACT

To improve the efficiency of MC criticality calculation, an acceleration method of fission source convergence which gives an improved initial fission source is proposed. In this method, the MC global homogenization is carried out to obtain the macroscopic cross section of each material mesh, and then the nonlinear iterative solution of the SP3 equations is used to determine the fission source distribution. The calculated fission source is very close to the real fission source, which describes its space and energy distribution. This method is an automatic computation process and is tested by the C5G7 benchmark, the results show that this acceleration method is helpful to reduce the inactive cycles and overall running time.

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1. Introduction

The Monte Carlo method (MC) is widely studied because of its high fidelity. When running the MC criticality calculation, the inactive cycles should be carried out first to obtain the converged fission source. The calculation of inactive cycles does not contribute directly to the final results, and it consumes a large amount of computing time, especially for systems with high dominance ratio [1]. Therefore, the acceleration of fission source convergence is very important for the efficiency of the MC criticality calculation.

A variety of fission source convergence acceleration methods have been developed for the MC criticality calculation, e.g. CMFD method [2], response matrix method [3], sourcerer method [4], superhistory method [5], source bias method [6], batch size method [7]. To further improve the acceleration of fission source convergence in the MC criticality calculation, we suggest a new acceleration method based on the simplified spherical harmonics (SP3) method [8,9].

In this method, the MC global homogenization [10] is first carried out to determine the multi-group cross sections. To ensure that the cross sections of each mesh are under the same confidence interval, the fixed-source calculation is performed with a uniform external source. To restrain excessive fission for supercritical models, no fission neutrons are produced during the neutron transport process. The absence of fission neutrons is compensated by the external source in the Watt fission spectrum distribution.

The cross sections from the MC global homogenization are then used to solve the SP3 equations. To improve the numerical stability and computation efficiency, the SP3 equations are solved by an author-developed nonlinear iterative method [11]. The calculated global flux has high accuracy, which has a small deviation from the flux directly simulated by RMC code [12]. The flux obtained by solving the SP3 equations can be used for variance reduction [13] and acceleration of fission source convergence.

The flux is used to accelerate the fission source convergence. The fission source distribution is calculated from the flux distribution, and then the biased Maxwell energy spectrum is used to sample fission neutrons in each energy interval. The sampled fission source is taken as the initial fission source of the MC criticality calculation. The above calculation steps are designed as an automatic calculation process, which is developed in RMC code. Results tested on the C5G7 benchmark [14] show that this method can greatly reduce the number of inactive cycles required for converged fission source, and also reduce the overall calculation time.

In this paper, Section 2.1 introduces the global homogenization method based on RMC code, with its main characteristics. The SP3 theory is given in Section 2.2, with its algorithm and solution.

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Section 2.3 describes the process of using global flux to determine the fission source. Followed by the numerical calculation and analysis in Section 3. Finally comes the summary in Section 4.

2. The acceleration method

The acceleration method of fission source convergence is designed as an automatic process and consists of three steps. First, we perform MC global homogenization for multi-group cross sections. Then we solve the SP3 equations for global flux. Finally, we use the global flux to determine the fission source and sample the initial fission source. These three steps are described in the following three sections separately.

2.1. MC global homogenization

We have developed a code named NLSP3 [11] for solving global SP3 equations based on RMC, and want to use it for acceleration of fission source and global variance reduction. To solve the SP3 equations, the multi-group cross sections are required, so the multi-group cross sections should be worked out first. In this paper, the MC global homogenization based on RMC code is developed to calculate the multi-group cross sections. The MC global homogenization is selected for the following reasons:

(1) No leakage correction [15] and resonance effect treatment [16] are needed during the MC global homogenization.
(2) Compared with the traditional assembly homogenization, the MC global homogenization has higher accuracy.
(3) The MC global homogenization can be used to determine the cross sections of all the material meshes simultaneously.
(4) The MC global homogenization is suitable for the SP3 equations and is helpful for an automatic calculation process.

To ensure the cross sections of all the material meshes from the MC global homogenization are at the same confidence level, the following techniques are used.

(1) The MC Homogenization is carried out during a fixed-source calculation.

The MC criticality calculation simulates the real neutron transport process in the system, and the energy spectrum is converged after inactive cycles. However, the neutron histories are not distributed equally over the system during the criticality calculation, which leads to different confidence intervals for the cross sections obtained from the MC global homogenization. For example, there are many neutron histories in the reactor core while few neutron histories in the reactor periphery or shielding layer, resulting in high accuracy in the reactor core, and low accuracy in the reactor periphery and shielding layer.

(2) A uniform external source is given for the fixed-source calculation.

It is very important to ensure that all the cross sections have the same accuracy for the SP3 calculation, so the fixed-source calculation is selected for the MC global homogenization. By setting a uniform external source, the fixed-source calculation ensures all the material meshes have the same number of neutron histories. Uniform external source means the source particles are uniformly distributed in space, and the dimension of the external source is determined by the dimension of the calculated models. Therefore, the cross sections obtained from the fixed-source calculation are under the same confidence interval.

(3) No fission neutrons are produced during the neutron transport process.

Neutrons are absorbed during the transport process and are released in a fission reaction. For the supercritical models, to restrain excessive fission, no fission neutrons are produced during the MC global homogenization. Only various reaction probabilities during the neutron histories are recorded.

(4) Watt fission spectrum and corrections are used to approximate the critical spectrum.

The energy spectrum is very important for the MC global homogenization. However, we do not know the critical spectrum before the fixed-source calculation. So the critical spectrum can only be treated by approximate methods.

No fission neutrons are produced during the neutron transport. To compensate this processing, the external source of the fixed-source calculation is distributed according to the Watt fission spectrum [17]. We have to state that using Watt fission spectrum is an approximation, and it may not be applicable to all reactor problems.

$$p(E) = Ce^{-E/a}\sinh\sqrt{bE} \tag{1}$$

where, C, a and b are coefficients, normally equal to 0.453,0.965 MeV and 2.29 MeV^{-1}, respectively.

To make the simulated energy spectrum more and more asymptotic to the critical spectrum, we reduce the cutoff weight limit for the neutron transport to achieve more scattering reactions, and the reaction rate is tallied directly according to the number of simulated reactions.

For the SP3 global calculation, total cross section $\Sigma_{t,g}$, absorption cross section $\Sigma_{a,g}$, fission production cross section $\nu\Sigma_{f,g}$, scattering matrix $\Sigma_{s,g'\to g}$, and fission spectrum η_g are needed. Therefore, these five quantities are mainly recorded during the MC global homogenization.

We choose the traditional volume weight method during the MC global homogenization. So the calculated cross section is defined

$$\overline{\Sigma}^{i}_{x,g} \equiv \frac{\int_{V_i}\Sigma_{x,g}\phi_g(r)dV}{\int_{V_i}\phi_g(r)dV} \qquad x = a,f,s,\cdots \qquad g = 1,\cdots,G \tag{2}$$

where, r means a point in space; $\phi_{A,g}(r)$ is the flux in the calculated region; The subscript i in V_i represents different areas of space.

RMC code chooses the track length to calculate the neutron flux and various reaction rates [18]. The definition of volume weight group flux in RMC code is

$$\phi_g = \frac{\int_{E_g}^{E_{g-1}} dE\int_V dV\sum_{i=1}^{N} W\cdot TL^i_V(E)}{V\sum_{i=1}^{N} W^i_0} \tag{3}$$

And the definition of reaction rates in RMC code is

$$\Sigma_g \phi_g = \frac{\int_{E_g}^{E_{g-1}} dE \int_V dV \sum_{i=1}^{N} W \cdot TL_V^i(E) \cdot \Sigma(r,E)}{V \sum_{i=1}^{N} W_0^i} \tag{4}$$

Therefore the calculation formula of the homogenization cross section is

$$\Sigma_g = \frac{\int_{E_g}^{E_{g-1}} dE \int_V dV \sum_{i=1}^{N} W \cdot TL_V^i(E) \cdot \Sigma(r,E)}{\int_{E_g}^{E_{g-1}} dE \int_V dV \sum_{i=1}^{N} W \cdot TL_V^i(E)} \tag{5}$$

where E_g and E_{g-1} are the upper boundary and the lower boundary of an energy interval; $W.TL^i_V$ is the product of weight and track length of the *ith* neutron at energy E and volume V; W is the neutron's weight; N the total number of neutrons.

For the fission spectrum, we calculate it by counting the fission neutrons' energy.

$$\chi(E) = p(E) \tag{6}$$

where, $p(E)$ is the probability of neutron production in each energy interval.

For the scattering matrix, the neutron energy before and after scattering is recorded to obtain the scattering probability,

$$P_{g\to g'} = \frac{\int_{E_{g'}}^{E_{g'-1}} dE' \int_{E_g}^{E_{g-1}} dE \int_V \phi(r,E)\Sigma_s(r,E\to E')dV}{\int_{E_g}^{E_{g-1}} dE \int_V \phi(r,E)\Sigma_s(r,E)dV} \tag{7}$$

The inter-group transfer cross section is the product of the scattering fraction and scattering cross section

$$\Sigma_{s,g\to g'} = P_{g\to g'} \cdot \Sigma_{s,g} \tag{8}$$

Through the above MC global homogenization, the cross sections of all the material meshes are worked out simultaneously, which will be used in the subsequent global SP3 calculation. The calculation flow chart of the MC global homogenization is shown in Fig. 1.

2.2. Solution of the SP3 equations

With the homogenization cross sections, RMC code has the global SP3 equations and then obtains the global flux by solving the SP3 equations. The SP3 equations are selected for the following reasons [19],

(1) Solution of SP3 equations have higher accuracy than that of diffusion equations;
(2) Solving SP3 equations takes about same computing time as diffusion equations;
(3) Compared to the memory usage and the calculation quantity of SN equations, the SP3 equations can more easily be applied to larger models;
(4) SP3 equations are helpful for an automatic calculation process.

SPN theory is the simplified spherical harmonics method for

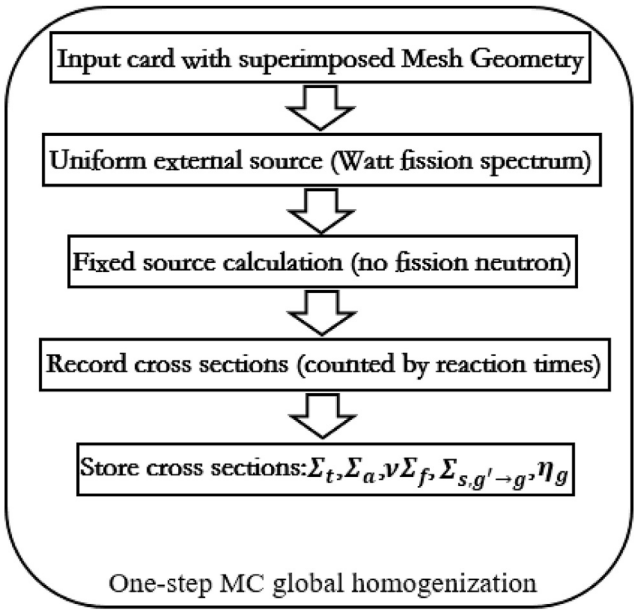

Fig. 1. RMC calculation flowchart for MC global homogenization.

solving the neutron transport equation

$$\begin{aligned} &-\frac{n(n-1)}{(2n+1)(2n-1)\Sigma_t}\nabla^2\phi_{n-2}(r) - \frac{2n^2+2n-1}{(2n+3)(2n-1)\Sigma_t}\nabla^2\phi_n(r) \\ &-\frac{(n+1)(n+2)}{(2n+1)(2n+3)\Sigma_t}\nabla^2\phi_{n+2}(r) + \Sigma_t\phi_n(r) \\ &= S\delta_{n0} \end{aligned} \tag{9}$$

where, $\delta_{n,0}$ is the Dirichlet function; and S is the source term; n is equal to $0, 1, \cdots, N$.

The SP3 equation is obtained when N is equal to three. After mathematical derivation, a similar form to the diffusion equation is obtained [19].

$$-D^k_{0,g}\nabla^2\phi^k_{0,g}(r) + \Sigma^k_{r0,g}\phi^k_{0,g}(r) - 2\Sigma^k_{r0,g}\phi^k_{2,g}(r) = S^k_{0,g}(r) \tag{10}$$

$$-D^k_{2,g}\nabla^2\phi^k_{2,g}(r) + \Sigma^k_{r2,g}\phi^k_{0,g}(r) - \frac{2}{5}\Sigma^k_{r0,g}\phi^k_{0,g}(r) = -\frac{2}{5}S^k_{0,g}(r) \tag{11}$$

Where k means the number of nodes; g means the energy; $\phi^k_{0,g}$ is the zero-order scalar flux at k node with energy g; $\phi^k_{2,g}$ is the second-order scalar flux at k node with energy g.

$$D^k_{0,g} = \frac{1}{3\Sigma^k_{t,g}},\quad D^k_{2,g} = \frac{9}{35\Sigma^k_{t,g}},\quad \Sigma^k_{r0,g} = \Sigma^k_{t,g} - \Sigma^k_{s,gg'},\quad \Sigma^k_{r2,g} = \frac{9}{5}\Sigma^k_{t,g} - \frac{4}{5}\Sigma^k_{s,gg'} \tag{12}$$

The source term expression

$$\begin{aligned} S^k_{0,g}(r) &= \Sigma^G_{\substack{g'=1\\ g'\neq g}} \Sigma^k_{0,g'g}\left[\phi^k_{0,g}(r) - 2\phi^k_{2,g}(r)\right] \\ &+ \frac{1}{K_{eff}}\chi^k_g \Sigma^G_{g'=1}\nu\Sigma^k_{f,g'}\left[\phi^k_{0,g'}(r) - 2\phi^k_{2,g'}(r)\right] \end{aligned} \tag{13}$$

For boundary conditions, we have the neutron flux and current under the Marshak boundary condition

$$J_0^{\pm}(r) = \frac{1}{4}\phi_0^M(r) \pm \frac{1}{2}\vec{\mathbf{n}} \cdot J_0^M(r) - \frac{3}{16}\phi_2^M(r) \tag{14}$$

$$J_2^{\pm}(r) = \frac{21}{80}\phi_2^M(r) \pm \frac{1}{2}\vec{\mathbf{n}} \cdot J_2^M(r) - \frac{3}{80}\phi_0^M(r) \tag{15}$$

where, the superscript "*M*" represents the quantities at the node boundary.

Some traditional nodal method can be used to solve the SP3 equations, such as the nonlinear iterative method [20]. The conventional nonlinear iterative method is based on the nodal method and nonlinear iterative strategy, which is characterized by the use of the CMFD and couple corrective formula. The traditional coupling corrective formula is

$$\begin{aligned} J_{gu+}^k = &- D_{gu+}^{k,FDM}\left(f_{gu-}^{k+1}\overline{\phi}_g^{k+1} - f_{gu+}^k\overline{\phi}_g^k\right) \\ &- D_{gu+}^{k,NOD}\left(f_{gu-}^{k+1}\overline{\phi}_g^{k+1} + f_{gu+}^k\overline{\phi}_g^k\right) \end{aligned} \tag{16}$$

Where J_{gu+}^k is the partial current at the boundary of the k node with energy g; $D_{gu+}^{k,FDM}$ is the pseudo diffusion coefficient; f_{gu-}^{k+1} is the discontinuity factor; $\overline{\phi}_g^k$ is the scalar flux.

Solving the SP3 equations with the conventional nonlinear iterative method, the numerical instability problem is found. To ensure the numerical stability, a new coupling corrective formula is put forward by the authors

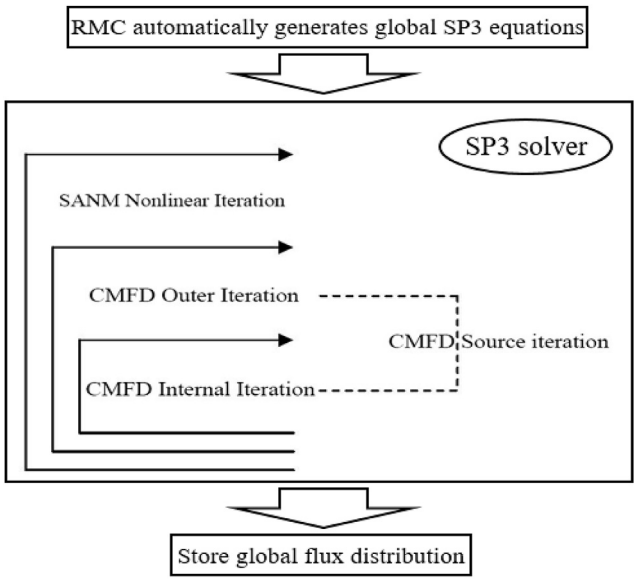

Fig. 2. RMC calculation flowchart for solving the SP3 equations.

$$\begin{aligned} J_{0,gu+}^k = &-D_{1,gu+}^{k,FDM}\left(f_{0,gu-}^{k+1}\overline{\phi}_{0,g}^{k+1} - f_{0,gu+}^k\overline{\phi}_{0,g}^k\right) - D_{1,gu+}^{k,NOD}\left(f_{0,gu-}^{k+1}\overline{\phi}_{0,g}^{k+1} + f_{0,gu+}^k\overline{\phi}_{0,g}^k\right) \\ &-D_{3,gu+}^{k,FDM}\left(f_{2,gu-}^{k+1}\overline{\phi}_{2,g}^{k+1} - f_{2,gu+}^k\overline{\phi}_{2,g}^k\right) - D_{3,gu+}^{k,NOD}\left(f_{2,gu-}^{k+1}\overline{\phi}_{2,g}^{k+1} + f_{2,gu+}^k\overline{\phi}_{2,g}^k\right) \end{aligned} \tag{17}$$

$$\begin{aligned} J_{2,gu+}^k = &-D_{2,gu+}^{k,FDM}\left(f_{2,gu-}^{k+1}\overline{\phi}_{2,g}^{k+1} - f_{2,gu+}^k\overline{\phi}_{2,g}^k\right) - D_{2,gu+}^{k,NOD}\left(f_{2,gu-}^{k+1}\overline{\phi}_{2,g}^{k+1} + f_{2,gu+}^k\overline{\phi}_{2,g}^k\right) \\ &- D_{4,gu+}^{k,FDM}\left(f_{0,gu-}^{k+1}\overline{\phi}_{0,g}^{k+1} - f_{0,gu+}^k\overline{\phi}_{0,g}^k\right) - D_{4,gu+}^{k,NOD}\left(f_{0,gu-}^{k+1}\overline{\phi}_{0,g}^{k+1} + f_{0,gu+}^k\overline{\phi}_{0,g}^k\right) \end{aligned} \tag{18}$$

Where the subscript 0 and 2 in $J_{n,gu+}^k$, $D_{n,gu+}^{k,FDM}$, $f_{n,gu-}^{k+1}$, $\overline{\phi}_{n,g}^{k+1}$ means zero-order and second-order quantities, respectively.

The new coupling corrective relationship focuses on the angular coupling. It seems a bit more complicated than the conventional one. Except for the calculation at the vacuum boundary, the coupling corrective coefficients have many ways to calculate, which brings in more margin to deal with the numerical instability problem.

With the global SP3 equations, RMC code solves the SP3 equations by the new nonlinear iterative method and the semi-analytical nodal method (SANM). The calculation flowchart is shown in Fig. 2.

2.3. Sampling of initial fission source

After solving the global SP3 equations, we have the global flux, which is then used to determine the fission source and accelerate the fission source convergence. According to the relation between neutron flux and fission reaction rate, the fission rate in each fissile region and each energy interval is

$$R_{f,g,m} = \phi_{g,m} \cdot \nu\Sigma_{f,g,m} \tag{19}$$

where $\nu\Sigma_{f,g,m}$ is the fission production cross section; $\phi_{g,m}$ is the group flux; $R_{f,g,m}$ is the fission source rate; g represents the energy interval and m represents the material mesh.

After the fission source rate in each fissile region and each energy interval is determined, the normalization is carried out

$$\overline{R_{f,g,m}} = \frac{R_{f,g,m}}{\sum\limits_{g,m} R_{f,g,m}} \tag{20}$$

Then the number of source neutrons to be uniformly sampled in each fissile region and each energy interval is calculated

$$n_{g,m} = \overline{R_{f,g,m}} \cdot N \tag{21}$$

where N represents the number of neutrons in each MC cycle.

With the number of fission source in each fissile region and each energy interval, the initial source is sampled. To get a more reasonable distribution of energy spectrum in each energy interval, Maxwell spectrum is used to sample the energy of the fission

source

$$p(E) = C\sqrt{E}\cdot e^{-E/a} \tag{22}$$

where a is an adjustable coefficient, normally 1.2895 MeV.

Reject sampling is used to determine the energy. For an energy interval $[E_g, E_{g+1}]$, an energy is sampled according to the Maxwell spectrum, which is accepted if it is in the energy interval, and re-sampled if it is not in the energy interval.

To improve the efficiency of the Maxwell sample, the Maxwell spectrum is biased according to the boundaries of each energy interval. Therefore, the value of a is calculated

$$a = \frac{E_g + E_{g+1}}{2} \tag{23}$$

With the calculated fission source distribution and the above sampling techniques, the initial fission source of the MC inactive cycle is obtained. Then the obtained initial fission source is used to accelerate the fission source convergence. The overall calculation flowchart is shown in Fig. 3.

3. Numerical calculation and analysis

The acceleration method is tested on the C5G7 benchmark, which is a quarter of a reactor. The whole reactor is built, and its axial geometry is shown in Fig. 4 [14].

RMC code reads the input card and builds superimposed mesh geometry according to the configuration of assemblies. The C5G7 whole reactor has 4 × 4 layout of assemblies, with the outer water layer. Therefore, the MC global homogenization of the C5G7 whole reactor is divided into 6 × 6 material meshes, and we chose the energy boundaries arbitrarily with four energy intervals, [0, 6.25E-07], [6.25E-07, 5.53E-03], [5.53E-03, 8.21E-01], [8.21E-01, 20] MeV.

For the source strength, the number of source neutrons was determined by our experience arbitrarily. Fewer source neutrons may lead to insufficient accuracy of the cross sections, while more source neutrons increase the calculation quantity but improve the results little. So we empirically think 10,000 source neutrons are good for a material mesh. That means to save computing time and guarantee certain calculation accuracy, we would like to give 10,000 source neutrons per each material mesh. Therefore, each material mesh has 10,000 external source neutrons, and the fixed-source calculation simulates a total of 360,000 source neutrons.

With the cross sections of 6 × 6 material meshes, RMC transmits the cross sections to the module of the SP3 solver and begins to solve the SP3 equations. For the SP3 solver, the nodal method is used, and each material mesh is divided into 4 × 4 nodes. Therefore, the SP3 solver deals with a total of 24 × 24 nodes. For the SP3 calculation, the convergence criteria for inner iteration is 0.001, the convergence criteria for outer iteration is 0.001, and the convergence criteria for keff iteration is 0.0001.

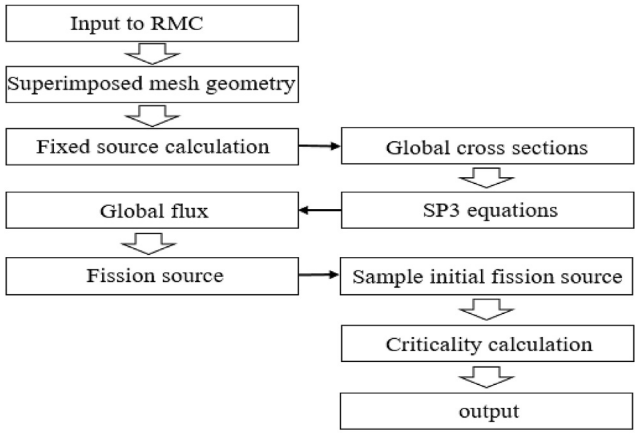

Fig. 3. Overall calculation flowchart of the acceleration method.

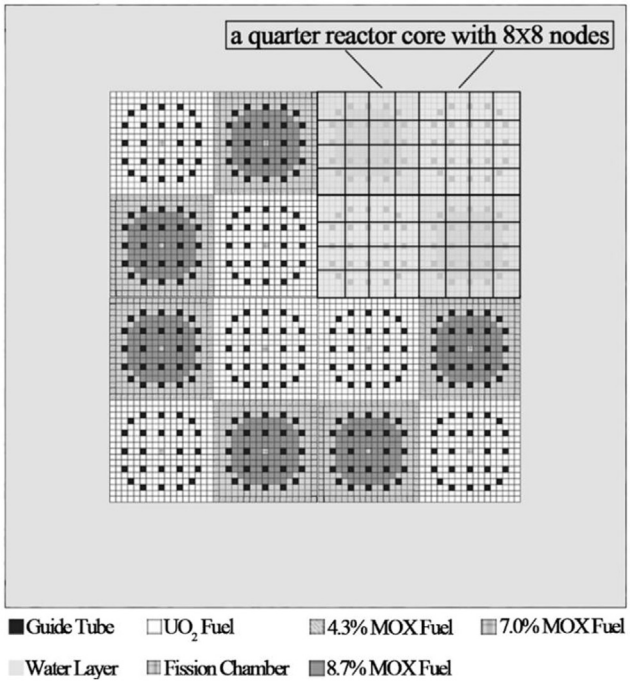

Fig. 4. Axial geometry of the C5G7 whole reactor.

All the calculation are performed with 36 cores, and the total time for the MC global homogenization and the SP3 solver is 0.43 min. After these, we have the global flux of the four energy groups, as shown in Fig. 5.

To verify the accuracy of the solution of the SP3 equations, RMC code performs a direct simulation to obtain the reference flux distribution. There are a total of 1000 cycles, and 50 inactive cycles, each cycle has 5,000,000 neutrons. The parallel calculation time of direct simulation is 1624.36 min, and the variance of direct simulation is less than 0.1%. The summary of calculation parameters and calculation times for direct simulation and Homogenization-SP3 calculation is given in Table 1.

The relative difference of total flux between the RMC direct simulation and the Homogenization-SP3 calculation in the fissile regions (one-quarter of the reactor core, shown in Fig. 4) are given in Table 2. The relative differences are calculated by Equation (24)

$$d = \frac{|\phi_{direct} - \phi_{SP3}|}{\phi_{direct}} \tag{24}$$

Where ϕ_{direct} is the total flux from RMC direct simulation; ϕ_{SP3} is the total flux from Homogenization-SP3 calculation.

The comparison of the four group flux in fissile regions between the RMC direct simulation and the Homogenization–SP3 calculation is shown in Table 3.

As can be seen from Table 2, the total flux distribution of the direct simulation is slightly different (<4%) from that of the Homogenization-SP3 calculation, so the flux distribution from the Homogenization-SP3 calculation can ensure the spatial distribution of the fission source. It can be seen from Table 3 that the flux deviations in each energy group between the direct simulation and

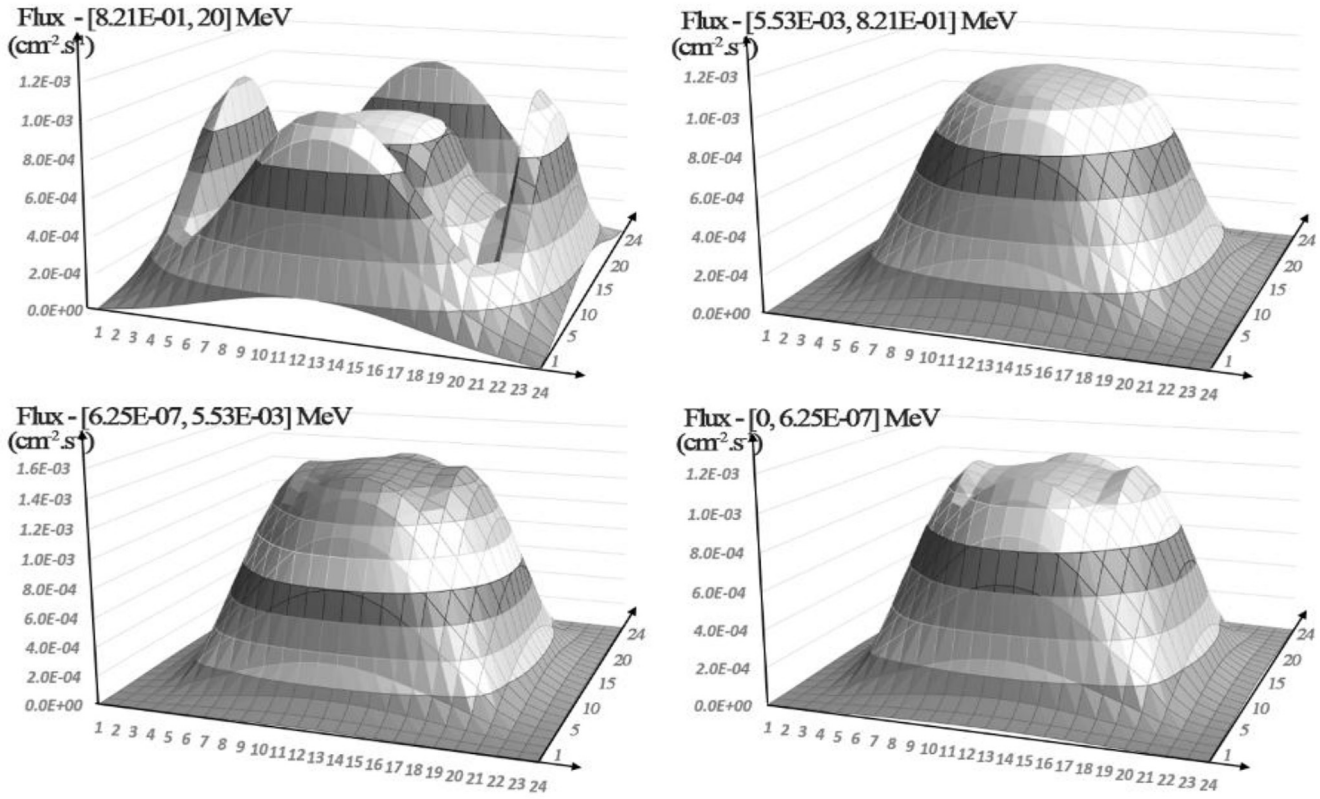

Fig. 5. Flux distribution from the solution of the SP3 equations.

Table 1
The summary of calculation parameters and calculation times.

	Direct simulation	Homogenization-SP3
Calculation mode	Criticality	Fixed source
Number of Neutrons	5,000,000 50 1000	360,000
Calculation time (mins)	1624.36	0.43

Table 2
The relative difference of the total flux in fissile regions.

Relative Difference (8 × 8 nodes)							
0.66%	1.03%	0.66%	0.34%	0.73%	0.37%	0.37%	0.84%
1.19%	0.96%	1.32%	2.23%	2.83%	2.62%	2.36%	0.55%
1.76%	1.54%	1.88%	2.86%	3.49%	3.24%	2.89%	0.82%
2.38%	1.92%	2.31%	3.55%	4.34%	3.93%	3.50%	1.53%
2.50%	2.08%	2.44%	3.71%	3.94%	3.63%	3.25%	1.46%
1.44%	1.16%	1.54%	2.68%	2.93%	2.91%	2.60%	0.66%
1.14%	0.87%	1.24%	2.41%	2.66%	2.67%	2.31%	0.36%
1.43%	0.99%	1.38%	2.71%	2.95%	2.71%	2.38%	0.58%

Table 3
The relative difference of the grouped flux in fissile regions.

Energy Interval (MeV)	Average Relative Difference
[0, 6.25E-07]	3.63%
[6.25E-07, 5.53E-03]	3.85%
[5.53E-03, 8.21E-01]	3.04%
[8.21E-01, 20]	5.37%

the Homogenization-SP3 calculation are also acceptable (With the approximations during global homogenization, the calculation accuracy of SP3 is limited. We are still trying to improve the calculation accuracy, but currently we can only achieve <4%. Therefore, we think <4% is accepted now.), so the calculated flux in each energy group can also ensure the distribution of fission source in terms of energy. By comparing calculation time, the time of the direct simulation is 3777 times that of the Homogenization-SP3 calculation. Therefore, the homogenization-SP3 framework can be used to accelerate fission source convergence.

To measure the effectiveness of the Homogenization-SP3 calculation on accelerating fission source convergence, the convergence rate of the fission source is judged by Shannon entropy, and the calculation of Shannon entropy is done by using the same number of neutrons per cycle. The convergence rate for an initial point source, an initial uniform source, and an initial Homogenization-SP3 fission source are compared. The comparison is shown in Fig. 6. The calculation times needed to get converged fission source with the three initial fission sources are compared in Table 4.

It can be seen in Fig. 6that it is unreasonable to use an initial point source or an initial uniform source as the initial fission source. In that case, it takes more inactive cycles to achieve fission source convergence. It takes only three cycles for the improved initial source, while takes 14 cycles for the initial uniform source and 19 cycles for the initial point source. Therefore, the Homogenization-SP3 method can effectively accelerate the convergence of the fission source, reduce the number of inactive cycles. Besides, as shown in Table 4, the time of the improved initial source method to achieve convergence is much smaller than that of the initial point source and the initial uniform source, so the Homogenization-SP3 method can reduce the overall calculation time, and improve the

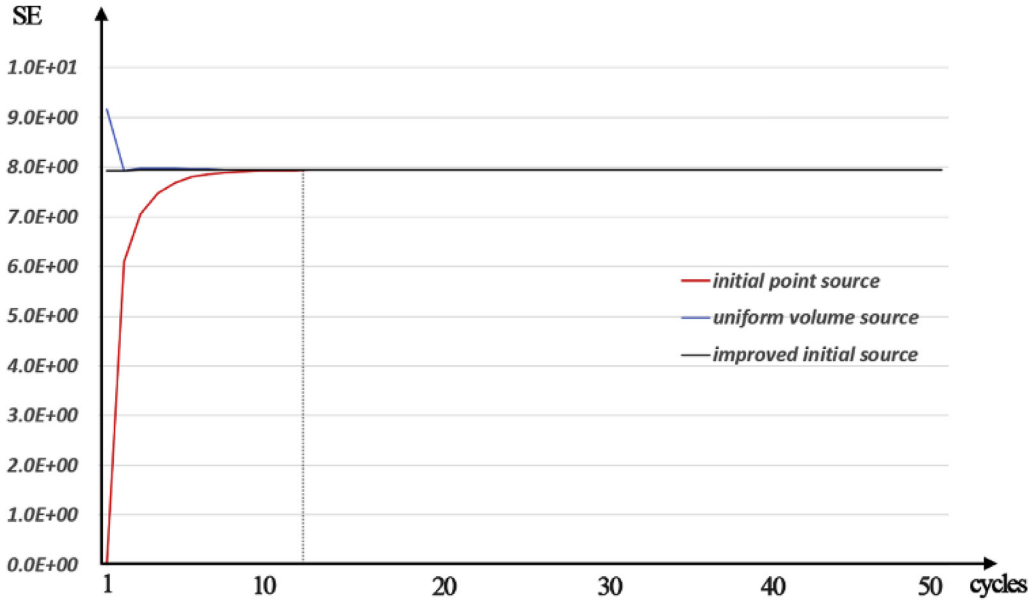

Fig. 6. Shannon entropy for the three initial fission sources.

Table 4
Calculation times of convergence for the three initial fission sources.

Initial fission source type	Time (mins)
Initial point source	24.1400
Initial uniform volume source	18.6246
Improved initial source (The time for MC homogenization and SP3 solution is included)	3.7219

efficiency of the MC criticality calculation.

Shown in Fig. 7, a PWR assembly [6] is also calculated. This PWR assembly has a total of $17 \times 17 = 289$ cells, including 264 fuel rods and 25 control rod guide thimbles. This model is divided into $17 \times 17 = 289$ material meshes, and it is surrounded by vacuum boundary condition.

Compared to the C5G7 benchmark in Fig. 4, the PWR assembly needs to deal with more material meshes for global homogenized cross sections. For the C5G7 benchmark, only $6 \times 6 = 36$ material meshes are dealt with during the homogenization process, then each material mesh is subdivided into $4 \times 4 = 16$ nodes for global SP3 calculation. While for the PWR assembly, there are $17 \times 17 = 289$ material meshes needed to be dealt with during the global homogenization process, and each material mesh is only subdivided into one node for the global SP3 calculation. The extra time introduced for this acceleration method is basically used to calculate the global homogenized cross sections, so the more nodes subdivided per material mesh, the better the acceleration. Therefore, using 17×17 material meshes in PWR assembly is to shown the acceleration effect on the worst case.

For the Homogenization-SP3 simulation, the homogenized cross sections of the 289 material meshes are calculated by the fix-source

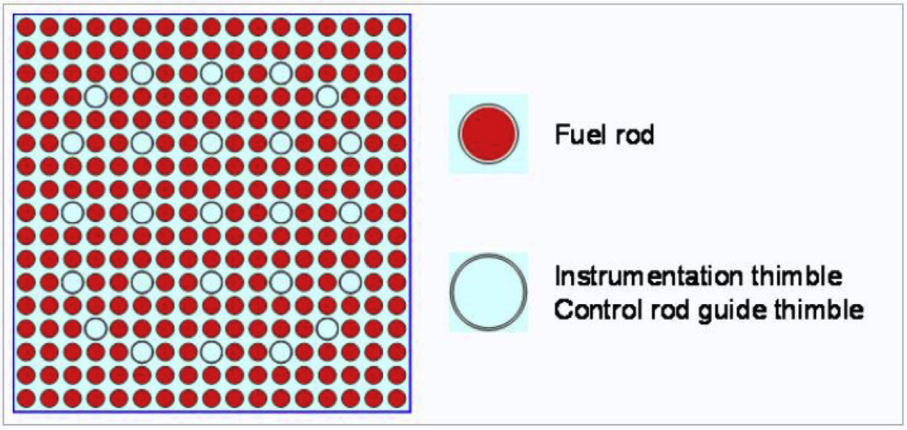

Fig. 7. The structure of the PWR assembly.

Table 5
Times required for convergence with the three initial fission sources for the PWR assembly.

Initial fission source type	Time (mins)
Initial point source	4.2491
Initial uniform volume source	3.1366
Improved initial source (The time for MC homogenization and SP3 solution is included)	2.4910

calculation, with 10,000 source neutrons per material mesh. Then the global SP3 equations are solved to determine the fission source. With the distribution of fission source, criticality calculation is perform to determine the time required for the convergence of Shannon entropy with the calculated fission source. For comparison, the times needed for convergence of Shannon entropy when using the initial point source and initial uniform volume source are also calculated. All the calculations are performed with a 36-core parallel computer, and there are 10,000,000 neutrons per cycle when calculating the Shannon entropy. Table 5 gives the times for the three calculations to achieve fission source convergence.

Can be seen from Table 5, even tested on the worst case, the time of the improved initial source method to achieve convergence is also smaller than that of the initial point source and the initial uniform source, so the Homogenization-SP3 method can universally reduce the overall calculation time, and universally improve the efficiency of the MC criticality calculation.

4. Conclusion

To improve the efficiency of the Monte Carlo Criticality calculation and reduce the number of inactive cycles, an acceleration method for fission source convergence is proposed. Based on the MC global homogenization and solution of the SP3 equations, this method gives improved initial fission source, so that the Monte Carlo criticality calculation has good fission source distribution in terms of space and energy since the first cycle.

The acceleration method is designed as an automatic calculation process, and it is developed in RMC code. Tested on the C5G7 benchmark, the method achieves 6.5 times acceleration compared with the initial point fission source, and 5 times acceleration compared with the initial uniform fission source. Therefore, this method can efficiently accelerate the convergence of the fission source, reduce the number of inactive cycles, and improve the efficiency of the Monte Carlo criticality calculation.

In fact, this hybrid Monte Carlo calculation framework was initially developed for variance reduction, and it is still under development. Compared with SN method or MC method, SP3 method requires less computation and memory, and is more suitable for large scale modes. So its advantages mainly lies in the SP3 method and its potential also lies in the future development of SP3 theory. In the future, we will focus on constructing more accurate results for both variance reduction and fission source convergence, and extending this calculation framework to more processors.

Declaration of competing interest

The authors declared that they have no conflicts of interest to this work.

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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