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Dynamic Response Analysis of Open Section Structures with Warping Restraint Conditions and Impact Load Durations

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Abstract

The response analysis of frame structure with open section beams considering warping conditions and short duration load have been performed. When a beam of frame structure is subjected under torsional moment, the cross section will deform a warping as well as twist. For some thin-walled sections warping will be large, and accompanying warping restraint will induce axial and shear stresses and reduce the twist of beam which stiffens the beam in torsion. Because of impact or blast loads, the wave propagation effects become increasingly important as load duration decreases. This paper presents that a warping restraint in finite element model effects the behavior of beam deformation, dynamic mode shape and response analysis. The computer modelling of frame is discussed in linear beam element model and linear thin shell element model, also presents a correlation between computer predicted and actual experimental results for static deflection, natural frequencies and mode shapes of frame. A method to estimate the number of normal modes that are important is discussed.

Keywords: *wave propagation effect, impact load, warping restraint factor, modal analysis, natural frequency.*

1. INTRODUCTION

A structure welded in sub-frames with open section beams are mostly used in automotive frames demanding high strength and stiffness. So it is great important to establish the secure frame having high torsional strength as well as bending strength in the viewpoint of safety and cost advantages having thin and light frame materials. Generally in Bernoulli-Euler beam theory or Timoshenko beam theory, it is assumed that shear center axis and torsional center axis are identical. When a beam is subjected to a torsional moment and the cross section is not symmetrical, the cross section will deform out of plane, which is called warping, as well as twist.[1-4]. For the modal and stress analysis of frames which open section cross members, the compatibility of any warping displacements in the cross member and rate of twist in the side member has to be ensured at the joints. Even when warping free sections are used, any non-zero twisting strain of the beam to which they are joined at a node has to be taken account. The additional displacements arising from the deformation of the cross section can be added to the displacements assumed for open sections and lateral bi-moment added to internal loads. The load-displacements for torsion in open sections are more complex in applying to all open section. In this case, the implicit Hilber-Hughes method and the explicit central difference method produce quite different relative period errors.

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In this paper a simple beam finite element model with different warping restraint factors are discussed comparing test results. Through use of structural dynamic behaviour comparing thin shell finite element and beam element model, comprehensive and detailed structural description of technique are described. Also presented is a correlation between computer codes predicted results and actual test results for static results, natural frequencies and normal modes of frame.

2. Dynamic Modal Analysis

2.1 Modal Analysis

We make use of a mixed mode of solution for the analysis of structural systems. A direct stiffness matrix approach is employed for the solution of the system joint displacements. The system matrix equation can be written as,

$$F = [(K) - \omega^2 (M)]X \quad \text{or} \quad F = [D]\,X \tag{1}$$

where, F: column matrix of external forces applied to the joints of the structure, K: square static stiffness matrix, M: diagonal mass matrix, ω: harmonic frequency of the excitation force, X: column matrix of displacements occurring at the joints of the structures, D: dynamic stiffness matrix.

To formulate the dynamic stiffness for each span which is made up of number of span segments and lumped masses, a recurrence matrix or transfer matrix approach is used. The recurrence matrix method is described by the following matrix equation,

$$Z_{i+1} = [U]_i \, Z_i \tag{2}$$

where, Z_{i+1}, Z_i: state vectors at stations $i+1$ and i along the span beams. Stations are designated at the segment ends. $[U]_i$ is the transfer matrix which relates the state vector at station $i+1$ to that at station i as a function of excitation frequency of the structure.

As many times as the number of segments existing for a given span, equation (3) can be recognized to yield a dynamic stiffness matrix for the entire span.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{3}$$

2.2 Warping Restraint

When a beam is subjected to a torsional moment and the cross section is not circular, the cross section will deform out of plane, which is called warp, as well as twist. In Saint-Venant torsion[1] this warping is free to occur and thus the equation for torsion can be written as,

$$\theta(x) = \frac{Tx}{GK} \qquad \text{or} \qquad \frac{d\theta}{dx} = \frac{T}{GK} \tag{4}$$

where, θ: angle of twist, T: applied torque, x: distance along the beam, K: torsional constant, G: shear modulus.

For the compact solid type cross sections such as rectangles, the amount of warping is small and it has little effect on the torsional behavior. However, for some thin-walled cross sections such as channels and wide flange sections the warping will be quiet large. If the warping is not restrained, then the equation (1) are still valid. Even though in the real world structures, the warping will normally be restrained to some degree since the end of the beam will be connected to other beams or other structural points. This restraint will induce axial and shear stresses in the beam and reduce the amount of twist of the beam which in effect stiffens the beam in torsion. When the effects of warping restraint are included, the equation for torsion is,

$$E\Gamma \frac{d^3\theta}{dx^3} - GK\frac{d\theta}{dx} = -T \tag{4}$$

where, E is modulus of elasticity, Γ is warping constant. And the general solution of this equation is,

$$\theta(x) = \frac{Tx}{GK} + A_1 + A_2 \sin h\beta x + A_3 \cos h\beta x \tag{5}$$

where, $\beta^2 = \frac{GK}{E\Gamma}$ or $\frac{1}{2(1+\gamma)}\frac{K}{\Gamma}$. Applying boundary conditions, and assuming that the angle of twist is zero at $x = 0$ and that the warping is zero at both ends, we get the angle of twist at $x = L$,

$$\theta(L) = \frac{TL}{G}\frac{(\beta L - 2\tan h\frac{\beta L}{2})}{\beta LK} \tag{6}$$

If we duplicate this result with equation (1) by replacing the torsional constant K by an effective torsional constant K_e, we get the following equation for the effective torsional constant,

$$K_e = K\frac{\beta L}{(\beta L - 2\tan h\frac{\beta L}{2})} \tag{7}$$

When $\beta L > 100$ in equation (7), the effective torsional constant is essentially the same as the nominal torsional constant. From this point, the warping is insignificant when $\Gamma < \frac{KL^2}{1\times 10^5}$. By defining a warping restraint factor, we can control the amount of torsional stiffening that occurs as the actual torsional constant used will be an interpolated value between nominal and the effective torsional constant,

$$K_e = fK + (1-f)K \ , \quad \text{where } 0 < f < 1.0 \tag{8}$$

2.3 Modal Test

To determine the dynamic characteristics of frame, frequency response was measured under free-free supporting condition. Each natural frequency represents specific patterns and deformation patterns of frame, which is called mode shape, is determined by adjusting oscillator frequency with natural frequency of frame. This deformed data is read by A-D converter and mode shapes are plotted. The body mounting points were chosen as excitation points and impacted in vertical and horizontal directions. The frame is free-free condition and measured at the fifty-one frame points.

3. Dynamic Response and Modal Analysis

For comparing warping restraint effect, automotive frame was modelled using finite element method. In beam element model, cross member and side rail were connected node to node linear beam elements and cross member and side rail were connected rigid element model. Also frame was modelled with linear thin shell element model as shown in figure 1.

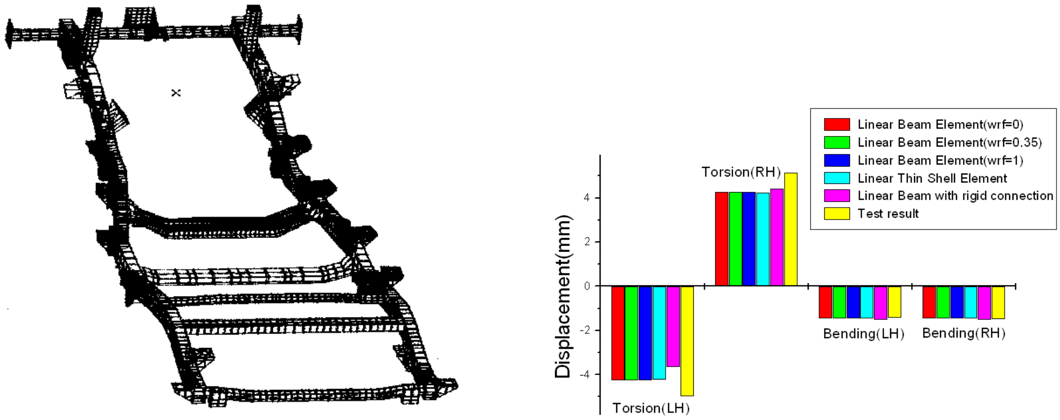
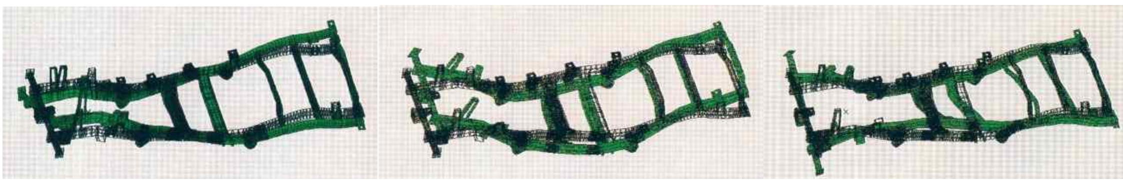
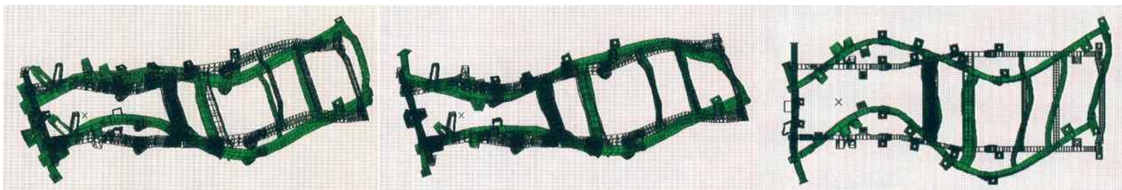


Figure 1. Finite element model of automotive frame and displacement results

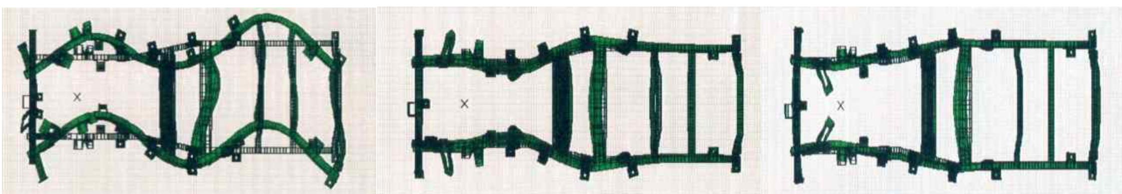
A static test was performed in which the frame was clamped at all four mounting bracket points, and loads were imposed to cause the frame twist and bend in both the vertical and horizontal planes. By predicting with computer analysis and test result, warping restraint effect was examined to torsion and bending deformation of frame in figure 1. In a short term, it can be confirmed that linear beam element model is more effective that linear thin shell element model to measure the deformation of frame. The result of beam element model with warping restraint factor is 0.35 is more opened to the test result than that of beam element model with warping restraint factor is 1 in bending test. The close correlation in bending results than in torsion results explains axial stress due to torsion is of a greater magnitude than that due to bending. For further investigation we may compare the correlation with modal test.



(a) 1st torsion vertical mode (b) 1st bending vertical mode (c) 2nd torsion vertical mode



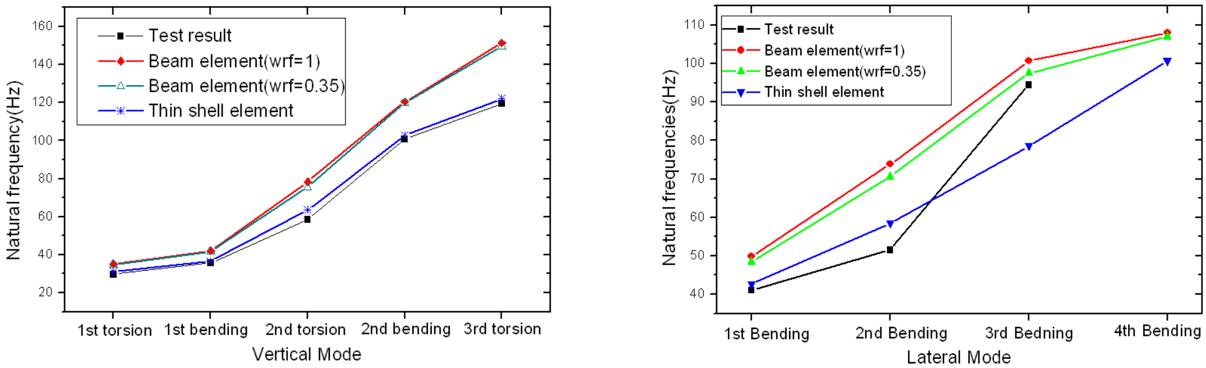
(d) 2nd bending vertical mode (e) 3rd torsion vertical mode (f) 1st lateral bending mode



(g) 2nd lateral bending mode (h) 6th lateral bending mode (i) 7th lateral bending mode

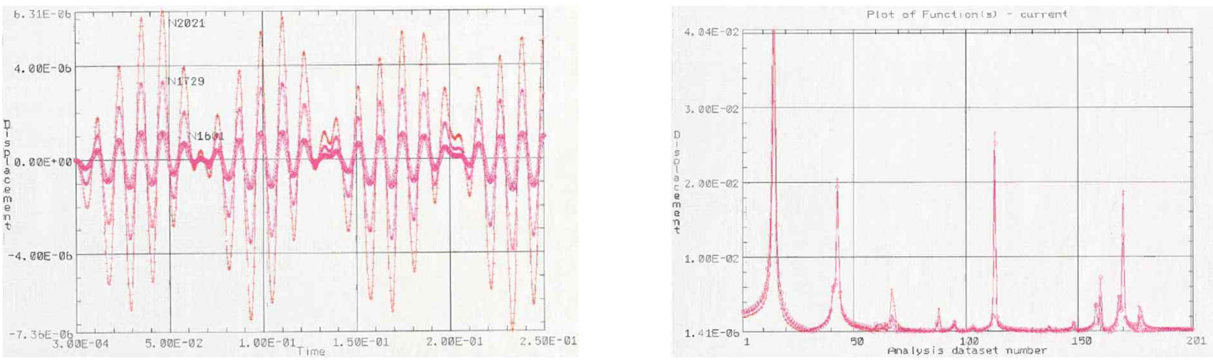
Figure 2. Mode shape of automotive frame

Figure 2 shows the computer analysis results of dynamic modal analysis. The peak deflections predicted by the analysis model compares within the standard engineering accuracy. A comparison of the analysis predicted to the actual (experimentally obtained) natural frequencies of the frame is given in figure 3. The results indicated that natural frequencies correlation for modes are well accuracy of engineering standard error. However higher the modes, higher the correlation error. The third torsion bending shows correlation of approximately 21%. A possible explanation for the lesser degree of correlation is found in studying the internal loading of this mode. For a static load, while bending are still small, high peaks of shear forces can be seen. Thus, the difference static response and dynamic response are the wave propagation effects.



(a) natural frequency of vertical mode **(b) natural frequency of lateral mode**

Figure 3. Comparison of natural frequencies with three models and test



(a) displacement response under bending moment and shear force **(b) frequency response under impact loads**

Figure 4. Dynamic response of open section beam structure

It is most easily visualized as the bending of a beam with the frame side-rails acting as the flanges and the cross-member acting as the web. For bending of a beam the web acts as a shear panel in plane loading in transmitting the bending load from one flange to the other. For the frame, this in plane loading is taken axially through the cross-members at six discrete locations rather than across a continuous plate. High localized loading occurs in cross-member to side-rail connection, and for the thin walled cross sections this loading can result in significant local distortion in the members. A beam representation of the member does not account for these added deformation, and model predicts this natural frequency higher than actual found in the test, as seen in figure 3 (a). This frame joint deformation, joint flexibilities (or joint stiffness) and joint slippage can significantly affect mode dynamic behaviour of frame mode shape. A number of structural analysis computer codes are available to model frame elements, and the results are similar tendencies.

The dynamic response under short impact load are represented in figure 4. It is seen that displacemnt waves move inward from the supports and interact in the middle of the beam after a while. The more normal modes are used, the better wave propagation is described. The type of dynamic deflections can be described with just a few normal modes, bending moments need more and shear forces need the largest number of modes to be described. When the duration of the load decreases, the more modes are excited. So the number of normal modes depends on the duration of the load relative to the first normal frequency.

4. Conclusions

In this study a method for efficient and accurate analysis of frame has been established. The warping conditions of beam elements was discussed. The beam elements method analysis was well matched with experimental results in the frame displacement test. In the displacement test results, the linear beam element results were well matched than thin shell element model with experimental result. However it is not valid to model beam segments with the same warping restraint. In a long beam, the warping restraint in the middle segment might actually be zero. The displacement and frequency response under impact load, the beam elements method are more predictable than the thin shell elements method. It is seen that the number of normal modes reflects the frequency response of the frame under impact load.

But the number of natural frequency of the thin shell element model is well matched with the test result. It can be the influence of joint flexibilities of the frame, so it is more desirable to use fine thin shell element model in modal test. In dynamic modal analysis the analysis predicted results well matched with test results. The frame was excited vertically and the resulting vertical motion measured at a point near the front of the frame on the left side-rail. The number of normal modes needed to describe the response in the elastic case can only be an indication of the number of modes to be described in the plastic cases. Because the plastic hinge of frame structure might develop, and displacemnts waves could be partially absorbed by it. Also the choosing the mesh and time step to describe these modes is a decisive point in simulations of structures loaded by impact loads.

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