

# An Alternative State Estimation Filtering Algorithm for Temporarily Uncertain Continuous Time System

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## Abstract

An alternative state estimation filtering algorithm is designed for continuous time systems with noises as well as control input. Two kinds of estimation filters, which have different measurement memory structures, are operated selectively in order to use both filters effectively as needed. Firstly, the estimation filter with infinite memory structure is operated for a certain continuous time system. Secondly, the estimation filter with finite memory structure is operated for temporarily uncertain continuous time system. That is, depending on the presence of uncertainty, one of infinite memory structure and finite memory structure filtered estimates is operated selectively to obtain the valid estimate. A couple of test variables and declaration rule are developed to detect uncertainty presence or uncertainty absence, to operate the suitable one from two kinds of filtered estimates, and to obtain ultimately the valid filtered estimate. Through computer simulations for a continuous time aircraft engine system with different measurement memory lengths and temporary model uncertainties, the proposed state estimation filtering algorithm can work well in temporarily uncertain as well as certain continuous time systems. Moreover, the proposed state estimation filtering algorithm shows remarkable superiority to the infinite memory structure filtering when temporary uncertainties occur in succession.

## Keywords

Declaration Rule, State Estimation Filter, Temporarily Uncertain System, Test Variable

## 1. Introduction

There have been many types of filters in diverse fields such as science, engineering, business, and mathematics [1-4]. Among them, the state estimation filter is the extract dynamic system's state values from full or partial measurements with noises. In particular, in real-time control problems for various dynamic systems, reliable state estimation has played an important role in obtaining accurate and safe control. Dynamic systems are represented accurately through state-space models on a long-time scale. However, in dynamic systems, there can be unpredictable changes such as abrupt jumps in frequency, phase, and velocity. These are called temporary uncertainties because they affect usually for a very short interval. Typical temporary uncertainties include model uncertainty, abnormal signal such as unknown inputs and sensor faults, and incomplete measurement information, etc. In estimation filtering for dynamic systems, the state estimation filter is required to be robust for diminishing effects of these temporary uncertainties.

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Manuscript received May 28, 2018; first revision December 11, 2018; accepted July 20, 2019.

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There are two types of state estimation filtering algorithms depending on the measurement-processing manner. The first is the infinite memory structure filter which utilizes all past measurements for the state estimation and thus is called the IMS filter. As a representative reference for the IMS filter, the Kalman filter has been generally chosen and applied successfully in diverse research fields such as control engineering and signal processing [5-8]. The other type is the finite memory structure filter which utilizes finite number of measurements on the most recent window and thus is called the FMS filter. The FMS filter has been designed for both discrete-time systems [9-11] and continuous time systems [13,14]. The FMS filter developed as an alternative to the IMS filter with drawbacks in some situations has worked well for diverse engineering applications as shown in [15-18]. The FMS filter is known to be more robust than the IMS filter when applied to temporarily uncertain systems.

Meanwhile, it is well known that the state estimation filter can provide better noise reduction with increasing the measurement memory length. Thus, the noise reduction of the state estimation filter can be greatly affected by the measurement memory length. However, even if the state estimation filter can have greater noise reduction with increasing the measurement memory length, the traceability of the state estimate for the actual state deteriorates in proportion to the measurement memory length, which may degrade the estimation filtering performance. This demonstrates the tradeoff of two types of state estimation filters, the IMS and FMS filters, between noise reduction and traceability of the filtered state estimate. According to this observation, the estimation error of the state estimation filter with long measurement memory length, that is, the IMS filter in [5-8], can be smaller than that of the state estimation filter with short measurement memory length, that is, the FMS filter in [9-14], on the uncertainty interval. This indicates the superiority of the state estimation filter with long measurement memory length in terms of the noise reduction ability. On the other hand, the convergence of the estimation error for the state estimation filter with long measurement memory length can be much slower than that of the state estimation filter with short measurement memory length, which demonstrates the superiority of the state estimation filter with short measurement memory length in terms of traceability. Hence, applying these two types of state estimation filters in parallel to the temporarily uncertain system might be very useful. That is, although the robustness is not considered during the filter design process, the FMS filter can be superior to the IMS filter if it is applied to temporarily uncertain systems. On the other hand, the FMS filter can be comparable or inferior to the FMS filter when applied to certain systems. To verify the above observation, the selective filtering mechanism with FMS and IMS filters was developed in [15] and the alternative selective filtering with two FMS filters using different measurement window lengths was developed in [18] for discrete time systems. Because continuous time dynamic systems are considered for practical engineering problems in many kinds of dynamic process plants, the filtering algorithm for the state estimation might be also required to cover both certain and temporarily uncertain continuous time systems. In addition, the control input term has not been considered in existing work. In actuality, the state-space model with control input can be often used for various control engineering problems.

Therefore, in this paper, an alternative state estimation filtering algorithm is designed for continuous time systems with noises as well as control input. Two types of estimation filters, IMS filter and FMS filter, with different memory structures are operated selectively to make the best use of both filters. These two estimation filters are operated for a certain continuous time system and a temporarily uncertain continuous time system, respectively. Then, one of the IMS and FMS filtered estimates is operated selectively to acquire a valid estimate depending on the presence of uncertainty. A couple of test variables

and the declaration rule are defined to detect the uncertainty presence, to operate the suitable one from the IMS and FMS filters, and acquire ultimately a valid filtered state estimate. Computer simulations for a continuous time F404 engine system with different measurement memory lengths and temporary model uncertainties show that the proposed state estimation filtering algorithm can work well in a temporarily uncertain continuous time system as well as a certain continuous time system. In particular, the proposed state estimation filtering can significantly outperform the IMS filter when temporary uncertainties occur in succession.

The current paper is structured as follows. Two types of state estimation filters with different memory structures are described in Section 2. A state estimation filtering algorithm for a continuous time system is designed in Section 3. Computer simulations using the aircraft engine system are performed in Section 4. Then, Section 5 concludes this paper.

## 2. Two Kinds of State Estimation Filters with Different Memory Structure

A continuous time system can be modeled by a following standard state-space form with noises as well as input terms:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Gw(t), \\ z(t) &= Cx(t) + v(t),\end{aligned}\tag{1}$$

where several variables are defined as follows.  $x(t) \in R^n$  is unknown state variable to be estimated and  $z(t) \in R^q$  is a measurement variable. The state  $x(t_0)$  at the initial time  $t_0$  of system is assumed to be a random variable with a mean  $\bar{x}(t_0)$  and its covariance  $P(t_0)$ . The system noise  $w(t) \in R^p$  and the measurement noise  $v(t) \in R^q$  are zero-mean white Gaussian random processes with covariances  $Q$  and  $R$ , respectively.

The Kalman filter, well known IMS filter, gives the following minimum variance state estimate  $\hat{x}_{ims}(t)$  of the current state  $x(t)$  [5,6]:

$$\dot{\hat{x}}_{ims}(t) = A\hat{x}_{ims}(t) + P(t)C^T R^{-1}[z(t) - C\hat{x}_{ims}(t)] + Bu(t),\tag{2}$$

where the estimation error covariance  $P(t)$  of the state estimate  $\hat{x}_{ims}(t)$  can be represented by the following Riccati equation:

$$\dot{P}(t) = AP(t) + P(t)A^T + GQG^T - P(t)C^T R^{-1}CP(t)\tag{3}$$

with initial state value  $\hat{x}_{ims}(t_0) = \bar{x}(t_0)$  and its covariance value  $P(t_0)$ . As shown in [5,6], the Kalman filter, known as a representative IMS filter, has been chosen generally and thus applied widely in diverse research areas such as control engineering and signal processing. However, due to its recursive formulation as well as infinite memory structure utilizing all past measurements processed with the same weighting, the estimation error tends to accumulate over time in the IMS filtering operation. In severe

cases, even divergence phenomenon can be shown.

Thus, in order to resolve the drawback of the IMS filter's error accumulation, the FMS filter [9-14] was designed by optimizing a performance criterion. Since only finite measurements are processing on the most recent window, the FMS filter can be derived from the IMS filter on the window  $[t-T, t]$  as following procedure. Usually, an information filter form can be used when the error covariance  $P(t_0)$  in (3) is very large but not infinite. For the information filter form, the new variable is defined as  $\Omega(t) \equiv P^{-1}(t)$  when the  $P(t)$  is nonsingular. Then, using the newly defined variable  $\Omega(t)$ , the estimation error covariance  $P(t)$  of the IMS filter can be expressed by

$$\dot{\Omega}(t) = -\Omega(t)A - A^T\Omega(t) - \Omega(t)GQG^T\Omega(t) + C^TR^{-1}C, \quad (4)$$

where  $\Omega(t_0) = P^{-1}(t_0)$ . Hence, the information form for the standard IMS filter (2) is represented by

$$\dot{x}(t) = [A - \Omega^{-1}(t)C^TR^{-1}C]\hat{x}(t) + \Omega^{-1}(t)C^TR^{-1}z(t) + Bu(t). \quad (5)$$

With applying the moving window strategy to (4) on  $[t-T, t]$ , the estimation covariance of the FMS filter can be time-invariant as follows:

$$\begin{aligned} \frac{d}{d\sigma}\Omega(\sigma) &= -\Omega(\sigma)A - A^T\Omega(\sigma) - \Omega(\sigma)GQG^T\Omega(\sigma) + C^TR^{-1}C, \\ \Omega(\sigma) &= 0, \quad 0 \leq \sigma \leq T. \end{aligned}$$

Then, with the definition  $\hat{\eta}(\cdot) \equiv \Omega(\sigma)\hat{x}_{fms}(\cdot)$ , the FMS filter  $\hat{x}_{fms}(t)$  is formulated for the system state  $x(t)$  as follows [13,14]:

$$\hat{x}_{fms}(t) = \Omega^{-1}(T)\hat{\eta}(T), \quad (6)$$

where the intermediate estimate  $\hat{\eta}(\sigma)$  is defined by

$$\begin{aligned} \frac{d}{d\sigma}\hat{\eta}(\sigma) &= -[A^{-T} + \Omega(\sigma)GQG^T]\hat{\eta}(\sigma) + C^TR^{-1}z(\sigma + t - T) + Bu(\sigma + t - T), \\ \hat{\eta}(0) &= 0, \quad 0 \leq \sigma \leq T. \end{aligned} \quad (7)$$

### 3. An Alternative State Estimation Filter Using Both FMS and IMS Filters

In general, the dynamic system is represented in a state-space model. Although dynamic systems are represented accurately through state-space models on a long-time scale, there can unpredictable changes such as abrupt jumps in frequency, phase, and velocity. These are called temporary uncertainties because they affect usually for a very short interval. Representative temporary uncertainties include model uncertainty, abnormal signals such as unknown inputs and sensor faults, and incomplete measurement information.

It is well known that there can be a trade-off between two filtering performance indices, estimation error and estimation speed, when these two types of filters with different memory structures are operated for temporarily uncertain systems. In most cases, the IMS filter can be superior to the FMS filter in terms of the estimation error for the certain system with no temporary uncertainty. However, the FMS filter can be superior to the IMS filter while modeling uncertainty exists. Moreover, the FMS filter can show the much faster convergence of the estimation error than the IMS filter after uncertainty has completely disappeared. This means the FMS filter can be remarkable in terms of estimation speed. These observations show that the FMS filter will be superior to the IMS filter when they are operated for temporarily uncertain systems, although the robustness is not considered during the design process of the FMS filter. This observation can be explained from the fact that the memory length for past measurements might have a considerable effect on the estimation error and estimation speed of the state estimation filter. The state estimation filter can lead to smaller estimation error as the measurement memory length becomes longer and thus the estimation filtering performance can be improved. On the other hand, the error convergence time of the filtered estimate becomes long as the measurement memory length becomes shorter, and thus the estimation speed is degraded.

One of FMS and IMS filtered estimates for continuous time systems is operated selectively to incorporate temporarily uncertain system as well as certain system. The valid estimate  $\hat{x}(t)$  is obtained from one of two filtered estimates, the IMS filter  $\hat{x}_{ims}(t)$  and the FMS filter  $\hat{x}_{fms}(t)$ , is selected to be the valid estimate  $\hat{x}(t)$  depending on the presence of uncertainty as follows:

$$\hat{x}(t) = \begin{cases} \hat{x}_{fms}(t) & \text{if temporarily uncertain system,} \\ \hat{x}_{ims}(t) & \text{if certain system.} \end{cases}$$

A couple of test variables and declaration rule are required to detect presence of uncertainty, to perform the suitable one from IMS and FMS filters, and obtain ultimately the valid filtered state estimate. Usually there is no uncertainty in the beginning of system operation. So, the IMS filter  $\hat{x}_{ims}(t)$  runs at the beginning of operation. Then, the uncertainty appears from the nominal system, which means the uncertainty presence. For the uncertainty presence declaration, a test variable  $\tau_{ims}(t)$  using the estimation error  $\hat{x}(t) - \hat{x}_{ims}(t)$  of IMS filtered estimate is defined by

$$\tau_{ims}(t) = [x(t) - \hat{x}_{ims}(t)]^T P^{-1}(t) [x(t) - \hat{x}_{ims}(t)] \quad (8)$$

where  $P(t)$  is the estimation error covariance of  $\hat{x}(t) - \hat{x}_{ims}(t)$  and obtained from (2). Then, after some time, the uncertainty disappears, which means the uncertainty absence. For the uncertainty absence declaration, a test variable  $\tau_{fms}(t)$  using the estimation error  $\hat{x}(t) - \hat{x}_{fms}(t)$  of FMS filtered estimate is defined by

$$\tau_{fms}(t) = [x(t) - \hat{x}_{fms}(t)]^T \Omega(T) [x(t) - \hat{x}_{fms}(t)] \quad (9)$$

where  $\Omega(T)$  is the estimation error covariance of  $\hat{x}(t) - \hat{x}_{fms}(t)$  and obtained from (6).

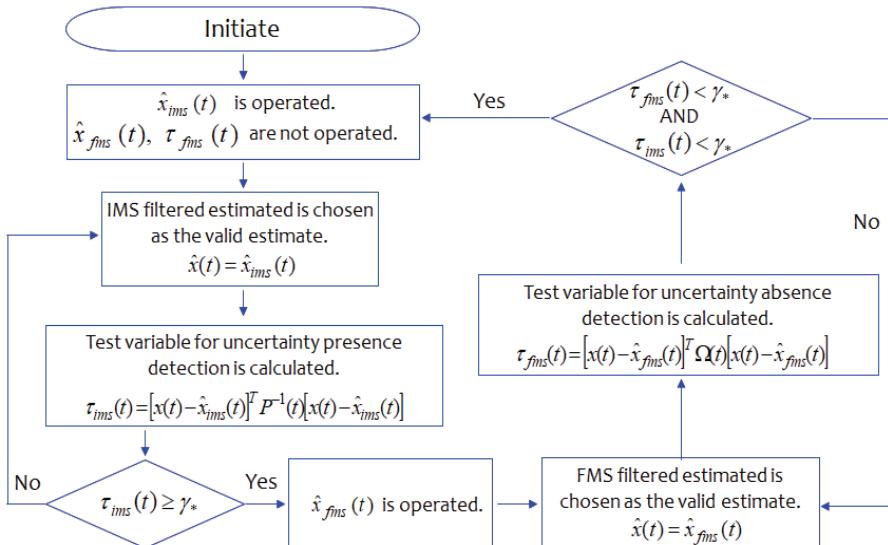
A couple of test variables  $\tau_{ims}(t)$  (8) and  $\tau_{fms}(t)$  (9) have the chi-squared distribution. The chi-square test statistic is known as one of feature selection methods for the machine learning [18,19]. As shown in (9), a chi-square distribution is developed from the difference between state and its filtered estimate and then compared with a precomputed threshold to declare uncertainty presence or not. These two types of test variables work differently depending on uncertainty presence or not. Firstly, the IMS test variable, denoted by  $\tau_{ims}(t)$ , will increase from the chi-squared distribution if an uncertainty appears. Secondly, the FMS test variable, denoted by  $\tau_{fms}(t)$ , will decrease from the chi-squared distribution if an uncertainty disappears. Then, through comparing the test variable  $\tau_{ims}(t)$  or  $\tau_{fms}(t)$  to a threshold value  $\gamma$ , uncertainty presence or uncertainty absence can be declared. The threshold value can be determined from the following probability of false alarm (PFA) of the test variables:

$$PFA = 1 - P_{\chi^2}(\gamma_*) = 1 - \frac{1}{2.5066} \int_0^{\gamma_*} \varepsilon^{-1/2} e^{-\varepsilon/2} d\varepsilon,$$

with the threshold value  $\gamma_*$ . Therefore, if  $\tau_{ims}(t) \geq \gamma_*$  which means that the uncertainty occurs, the FMS filtered estimate  $\hat{x}_{fms}(t)$  is to be the valid estimate  $\hat{x}(t)$ . If  $\tau_{fms}(t) < \gamma_*$  and  $\tau_{ims}(t) < \gamma_*$  which mean that the uncertainty disappears, the IMS filtered estimate  $\hat{x}_{ims}(t)$  is to be the valid estimate  $\hat{x}(t)$ . These are summarized by the following declaration rule:

$$\hat{x}(t) = \begin{cases} \hat{x}_{ims}(t) & \text{if } \tau_{ims}(t) < \gamma_* \text{ AND } \tau_{fms}(t) < \gamma_* \\ \hat{x}_{fms}(t) & \text{if } \tau_{ims}(t) \geq \gamma_* \end{cases}.$$

The overall operation process after the proposed continuous time state estimation filtering algorithm startup is shown in Fig. 1.



**Fig. 1.** Operation process after continuous time system startup.

## 4. Simulations

Simulations with MATLAB software are performed for a continuous time F404 engine model which has been known as a reliable and high-performance engine [20], in order to validate the state estimation filtering algorithm proposed in this paper and to compare with the existing IMS filter. A model uncertainty is considered as a temporary uncertainty. The continuous time F404 engine model without model uncertainty is represented by

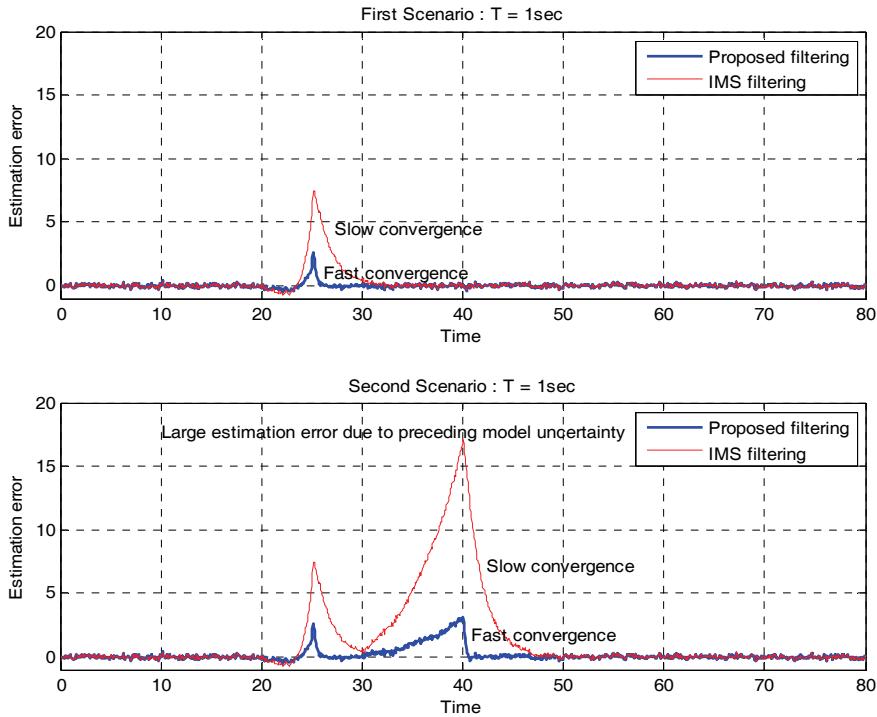
$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643 & -0.4 & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}x(t) + \begin{bmatrix} 0.4182 & 5.2026 \\ 0.3901 & -0.1245 \\ 0.5186 & 0.0237 \end{bmatrix}u(t) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}w(t), \\ z(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}x(t) + v(t).\end{aligned}\quad (10)$$

Two kinds of filters (2) and (6) are designed using state space models (10) for F404 engine, which means the certain system. When temporary uncertainties are considered, actual continuous time state-space models become temporarily uncertain systems as  $\bar{A} = A + \Delta A$  where  $\Delta A = \delta(t) \cdot I_{3 \times 3}$  for the F404 engine system. Simulation parameters, such as system and measurement noise covariances, measurement memory length, model uncertainty, are shown in Table 1. Even if IMS and FMS filters (2) and (6) are developed for the state space model (10) with the system matrix  $A$ , these two filters actually work on the temporarily uncertain system with the uncertain system matrix  $\bar{A}$ . For the convenience of computer simulations, the discretized F404 engine model of [11] is applied with 20 discrete samples taken as 1 second.

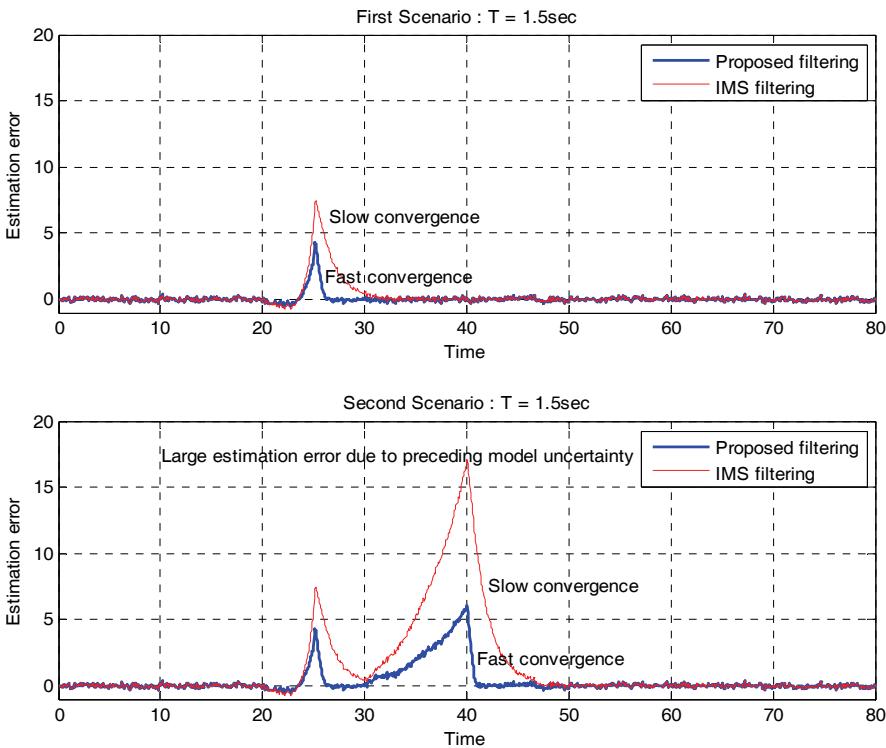
**Table 1.** Simulation parameters and scenarios

Parameter	F404 engine system
Noise covariances	$Q = 0.1^2, R = 0.1^2$
Measurement memory length (sec)	$T = 1$ and $T = 1.5$
Simulation duration (sec)	80
Model uncertainty	
First scenario	$\delta(t) = 0.6$ for $20\text{sec} \leq t \leq 25\text{sec}$
Second scenario	$\delta(t) = \begin{cases} 0.6 & \text{for } 20\text{sec} \leq t \leq 25\text{sec} \\ 0.3 & \text{for } 30\text{sec} \leq t \leq 35\text{sec} \end{cases}$

As shown in Figs. 2 and 3, the proposed state estimation filtering algorithm can be superior to an IMS filter, such as the Kalman filter, in terms of error magnitude and error convergence for both scenarios according to different types of temporary uncertainties. The estimation errors of the proposed state estimation filter are shown to be smaller than those of the IMS filter at the modeling uncertainty interval. After temporary modeling uncertainty disappears, the estimation error convergence is shown to be better than that of the IMS filter. In particular, the proposed state estimation filtering can be shown to outperform the IMS filter significantly when temporary uncertainties occur in succession. It is well known that the Kalman filter with the infinite memory structure utilizes all past measurements using a recursive formulation. Hence, estimation errors tend to accumulate over time in the IMS filter and thus in some severe cases can show the error divergence for temporary uncertainties. Thus, as shown in the second plots of Figs. 2 and 3, the estimation error due to the preceding model uncertainty can be propagated, and then the estimation error due to the following model uncertainty worsens although the



**Fig. 2.** Simulation results for a couple of scenarios for F404 engine system ( $T = 1$  second).



**Fig. 3.** Simulation results for a couple of scenarios for F404 engine system ( $T = 1.5$  seconds).

following model uncertainty,  $\delta(t) = 0.3$  for  $30\text{sec} \leq t \leq 35\text{sec}$ , is smaller than the preceding model uncertainty  $\delta(t) = 0.6$  for  $20\text{sec} \leq t \leq 25\text{sec}$ .

Therefore, when two estimation filters, IMS filter and proposed filter, are applied to temporarily uncertain systems, the proposed filtered estimate can be more robust than the IMS filtered estimate, although the robustness is not considered during the design process of the proposed estimation filter. If the effect of temporary modeling uncertainty completely disappears, the proposed state estimation filter is also shown to be comparable to the IMS filter. Moreover, it is also shown that the noise reduction of the FMS filter used in the proposed state estimation filtering can be greatly affected by the measurement memory length for past measurements. The estimation filter can have greater noise reduction with increasing the measurement memory length. On the other hand, the convergence speed of the estimation error worsens as the measurement memory length increases. This can be verified by the simulation results of Fig. 2 with  $T = 1$  second and Fig. 3 with  $T = 1.5$  seconds.

## 5. Concluding Remarks

This paper has developed an alternative state estimation filtering algorithm designed for continuous time systems with noises as well as control input. In the proposed algorithm, two types of estimation filters with different memory structures are operated selectively in order to take full advantage of both IMS and FMS filters. The IMS filter is operated for a certain continuous time system. On the other hand, the FMS filter is operated for temporarily uncertain continuous time system. Thus, one of FMS and IMS filtered estimates is operated selectively to obtain the valid estimate depending on the presence of uncertainty. A couple of test variables and a declaration rule have been developed to detect the presence of uncertainty, to perform the suitable choice from IMS and FMS filters, and to obtain ultimately the valid filtered estimate. Computer simulations for a continuous time aircraft engine system have shown that the proposed state estimation filtering algorithm can work well in a temporarily uncertain continuous time system as well as a certain continuous time system.

The measurement memory length  $T$  for the FMS filter can be considered as one of useful design parameters for the proposed state estimation filtering algorithm. Hence, it can be interesting issue to choose an appropriate measurement memory length that makes the filtering performance to be better. The noise reduction of the proposed state estimation filtering algorithm can be greatly affected by the measurement memory length, and it can lead to greater noise suppression as the measurement memory length increases, which enhances the filtering performance. In this paper, the measurement memory length has been selected with these observations, which is actually not a systematic method. Therefore, a more systematic method to determine the measurement memory length should be addressed as a future research topic.

## Acknowledgement

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. NRF-2017R1D1A1B03033024).

The work reported in this paper was conducted during the sabbatical year of Korea Polytechnic University in 2020.

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