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Original Article

New mathematical approach to calculate the geometrical efficiency using different radioactive sources with gamma-ray cylindrical shape detectors

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ABSTRACT

The geometrical efficiency of a source-to-detector configuration is considered to be necessary in the calculation of the full energy peak efficiency, especially for NaI(Tl) and HPGe gamma-ray spectroscopy detectors. The geometrical efficiency depends on the solid angle subtended by the radioactive sources and the detector surfaces. The present work is basically concerned to establish a new mathematical approach for calculating the solid angle and geometrical efficiency, based on conversion of the geometrical solid angle of a non-axial radioactive point source with respect to a circular surface of the detector to a new equivalent geometry. The equivalent geometry consists of an axial radioactive point source with respect to an arbitrary elliptical surface that lies between the radioactive point source and the circular surface of the detector. This expression was extended to include coaxial radioactive circular disk source. The results were compared with a number of published data to explain how significant this work is in the efficiency calibration procedure for the γ-ray detection systems, especially in case of using isotropic radiating γ-ray sources in the form of point and disk shapes.

1. Introduction

The majority of scientists are currently spotlighting their efforts on the improvement of the computational method to calculate the geometric efficiency. The measured calibration process for NaI(Tl) and HPGe gamma-ray detectors are complicated because of it is not easy to find exactly calibrated γ-sources of a variety of shape and medium. Also, the solid angle or geometrical factor is used in many different fields such as, nuclear interaction study, neutron scattering, angular distribution, optics, etc. This means that, it is very necessary to determine the solid angle with high accuracy, where data analysis and final results are based on its values in different applications. Determining the solid angle has become important during the last 60 years and many efforts were done by many techniques using different methods to obtain results with high accuracy. One of the most common techniques is the Monte Carlo method, which has been used by many authors [1–10].

The new mathematical approach to calculate the solid angle in the current work, was based on many published data. The analytical expressions in Refs. [11,12] with approximation series were explained a radioactive disk source with circular detector surface, also [13] was introduced an expression for rectangular slit shape. In Refs. [14,15] the solid angle was calculated approximately by an analytical expression for n-sided, regular polygon. A direct mathematical method to calculate the solid angle with high accuracy was deduced in Refs. [16–19]. The published work in Refs. [20,21] were generalized from the formula in Refs. [11,12] to include non-coaxial disk sources and use the vector potential to derive an integral for the solid angle subtended by point source to a detector of arbitrary shape and location, In Refs. [22–24] the actual alpha particles distribution in the solid angle were studied, and presented a series expansion of [11,12,20,21] solid-angle formulas [25–28], were gave an analytical expressions for the solid angle subtended by a right circular cylinder at a point source with cosine angular distribution in different cases [29], was derived an analytical formula for calculating the solid angle subtended by a cylinder at a point from the general solid angle equation. Based on the direct

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method [30,31] were presented a model for the solid angle subtended at a point by a circular disk and calculated the solid angle subtended at a point by closed elliptical contours [32,33], were introduced a new approach to reduce the mathematical formulas for coaxial shapes then non-axial shapes [34], was calculated the solid angle for different shapes of sources.

Identifying the geometrical solid angle is considered to be important in order to get the activity of the radioactive samples, in case the radioactive standard sources of the corresponding geometry are not on hand. This can be functional if the photons path lengths through the detector material were calculated. Every absorber between the source and the detector active medium should be taken into account in calculating the effective solid angle between the source and the detector, in addition to the self-attenuation coefficient of the source supplies. In this work, a new mathematical approach to calculate the solid angle and the detectors geometrical efficiency by the mathematical integration method was introduced, where isotropic radiating γ-ray sources (point and disk) located at different locations were considered.

The new approach was built on the transfer of the solid angle subtended by an arbitrary radioactive point source with respect to the circular detector surface to another equivalent geometry based on analytical expressions. This transfer the geometrical solid angle of a non-axial radioactive point source with respect to the circular detector surface to an equivalent geometric solid angle of axial radioactive point source with respect to an elliptical surface lies between the radioactive source and the detector plane. The expression was extended to include a coaxial radioactive disk source. This model was produced by elliptical integrals, with the advantage of the symmetry about the detector axis, which makes the model more accurate solution more accurate.

The arrangement of this paper is as follows: Section 2 presents in stepwise point the mathematical expressions for the calculation of the solid angle and the geometrical efficiency using isotropic radiating γ-ray sources (point and disk) located in the axial and non-axial locations. Section 3 includes the comparisons between the geometrical efficiency calculated by the current approach and the published results, followed by the Conclusions which were presented in Section 4.

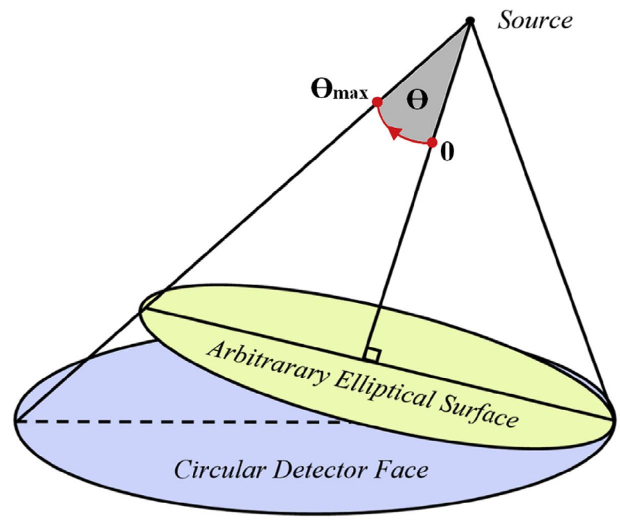

Fig. 1. Non-axial radioactive point source with respect to the circular detector face surface and its elliptical equivalent geometry.

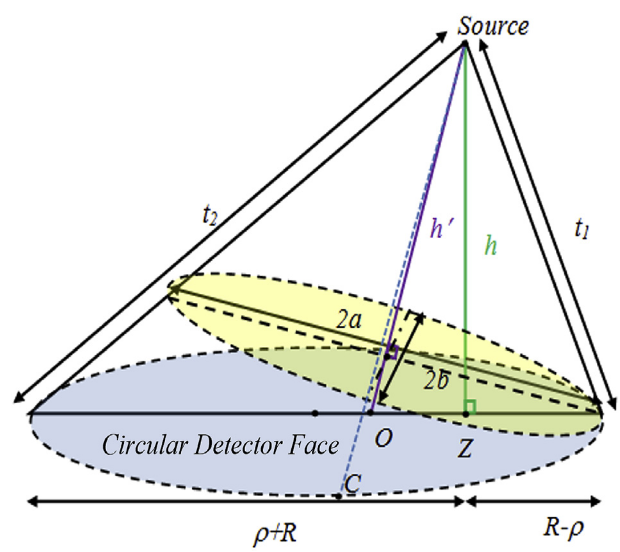

Fig. 2. Axial radioactive point source with respect to the elliptical equivalent geometry.

2. Mathematical model

Simply, the non-axial radioactive point source with respect to the circular detector surface and its elliptical equivalent geometry is shown in Fig. 1. Based on the mathematical technique, it is found that, the geometrical solid angle of a non-axial radioactive point source placed at a height, h, and at lateral distance, ρ, from the surface of the vertical cylindrical shaped detector, is equivalent to the geometrical solid angle of axial radioactive point source located at height h' from the surface of a virtual elliptical surface which lies between the source and the detector plan as shown in Fig. 2. The solid angle Ω_{geo} can be calculated by the following equation:

$$\Omega_{geo} = \int_0^{\phi}\int_0^{\theta_{max}} \sin\theta d\theta \, d\phi \tag{1}$$

Also, the geometrical efficiency *G* factor can be expressed as:

$$G = \frac{1}{4\pi}\int_0^{\phi}\int_0^{\theta_{max}} \sin\theta d\theta \, d\phi \tag{2}$$

Where ϕ is the azimuthal angle, whose value varies from 0 to 2π, while Θ is the polar angle for photons to enter detector's face and it takes values from 0 to θ_{max}. Where θ_{max} is the maximum polar angle for photon to enter the detector's face and it can be given by:

$$\theta_{max} = \tan^{-1}\left(\frac{R'}{h'}\right) \tag{3}$$

Where h′ is the source new height, based on the shape geometry in Fig. 3, and can be given by:

$$h' = \sqrt{t_1^2 - a^2} \tag{4}$$

where t_1 can be defined as:

$$t_1 = \sqrt{h^2 + (R - \rho)^2} \tag{5}$$

The parameter R′ is the varying ellipse radius, as shown in Fig. 4. It can be calculated depending on the semi-major axis *a* and the semi-minor axis *b* according to the following equation:

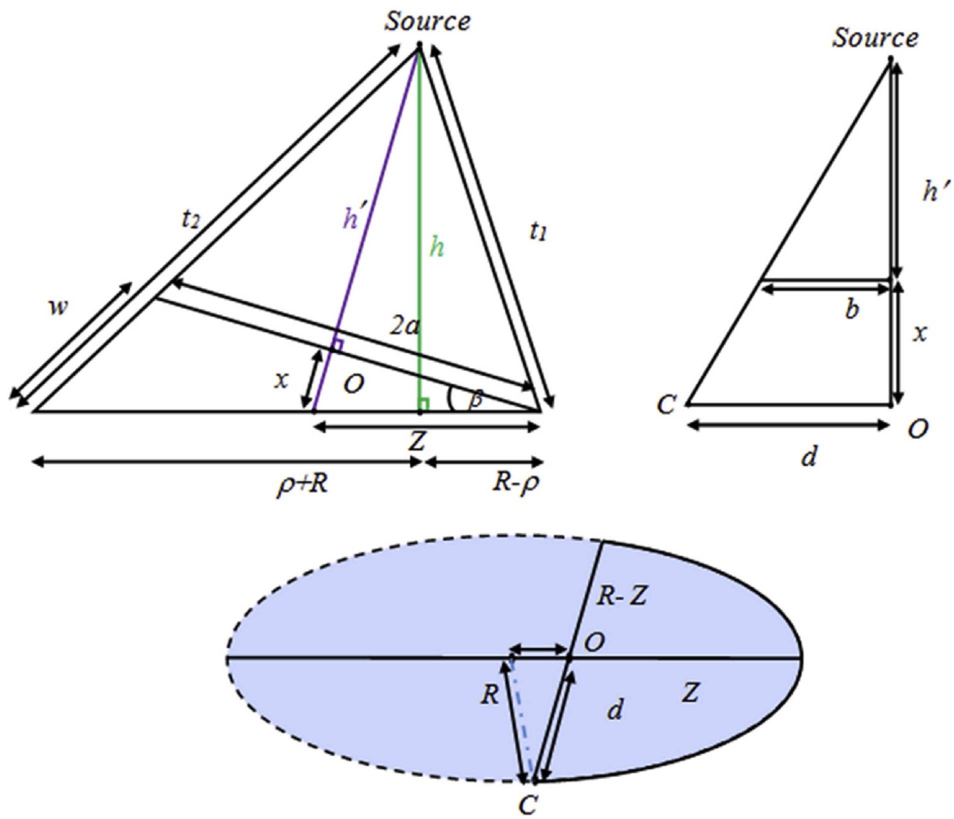

Fig. 3. Cross-sectional plans.

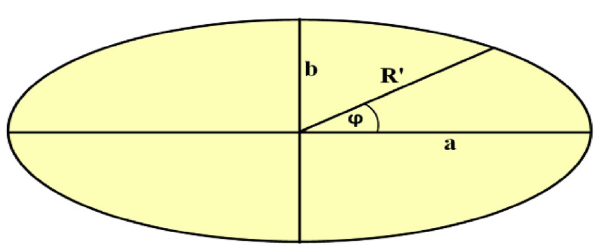

Fig. 4. The ellipse radius variation with the azimuthal angle ϕ.

$$R' = \frac{a.b}{\sqrt{a^2\sin^2\phi \ + \ b^2\cos^2\phi}} \tag{6}$$

The semi-major axis *a* can be calculated directly as:

$$a = \sqrt{\frac{4R^2 + w^2 - \frac{4Rw(R+\rho)}{t_2}}{4}} \tag{7}$$

where,

$$w \ = \ t_2 - \ t_1 \tag{8}$$

and t_2 can be expressed as:

$$t_2 = \sqrt{h^2 + (R+\rho)^2} \tag{9}$$

Also, the semi-minor axis *b* can be given as the following:

$$b = h'\left(\frac{d}{h' + x}\right) \tag{10}$$

where d and x can be expressed as:

$$d = \sqrt{Z\ (\ 2R \ - \ Z)} \qquad \& \qquad x \ = \ Z\sin\beta \tag{11}$$

Also, Z and β can be expressed as:

$$Z = \frac{8Ra^2}{4R^2 + \ 4a^2 - \ w^2} \text{ and } \beta = \cos^{-1}\left(\frac{a}{Z}\right) \tag{12}$$

By substituting the equations from (3) up to (12) in equations (1) and (2). The solid angle Ω_{geo} and the geometrical efficiency *G* can be calculated directly by using non-axial radioactive point source with respect to the circular disc surface of the detector. To get the solid angle Ω_{Disk} and the geometrical efficiency G_{Disk} for a coaxial radioactive disk source, the following equations can be used:

$$\Omega_{Disk} = \frac{1}{\pi S^2}\int_0^S \rho \ \Omega_{geo} d\rho \tag{13}$$

$$G_{Disk} = \frac{1}{\pi S^2}\int_0^S \rho \ G \ d\rho \tag{14}$$

All the above integrations are of an elliptical integrations type, so the calculations are numerically implemented by using the trapezoidal rule with Richardson's first order correction [36] in a FORTRAN program.

3. Validity of the new approach

In turn to show the strength of the present new approach, for the non-axial arrangements of sources-to-detectors, the comparison procedure was executed by using the values of the calculated geometrical efficiency and the published results. Geometrical efficiency obtained using the present work and the published one reported by Ref. [34] for an axial radioactive point source located at different heights from HPGe detector surface of outer radius 2.65 cm is presented in Fig. 5 with maximum deviation 7.66%. Also, the geometrical efficiency was calculated by the present new approach by using radioactive point source located at different heights and lateral distances from Canberra NaI(Tl) detector model number 802 with outer radius 3.81 cm the obtained results were compared with published data [34] as shown in Fig. 6 at heights 0.2 cm, 2.5 cm, and 5 cm from the detector surface and varying the lateral distance ρ from 0 cm to 3 cm, as shown in Fig. 6. The results are in good agreement with the published data and the maximum deviation between calculated and published results was found to be 1.15%.

Several calculations were achieved to show the behavior and the precision of the geometrical solid angle by the present new approach and compared with published data in Ref. [35]. The solid angle subtended by an ellipse with principal axes a and b at a co-axial distance h from the center of the ellipse was calculated and tabulated in Table 1. The maximum absolute deviation is 0.038%, where the deviation percentage $\Delta_1\%$ between the present new approach and published work were calculated by the following equation:

$$\Delta_1\% = \left| \frac{\Omega_{geo}(Published) - \Omega_{geo}(Present\ \ Work)}{\Omega_{geo}(Published)} \right| * 100 \qquad (15)$$

The calculations were also extended to the coaxial radioactive disk source with respect to circular detector surface and the results from the present approach were compared with published reported in Refs. [8,20]. The geometrical efficiency G_{Disk} was calculated using a coaxial radioactive disk source of radius S at height h. The range of each parameter was given as: $R = 1$ cm up to 8 cm, $h = 1$ cm up to 20 cm, and $S = 0.3$ cm up to 2 cm, the obtained results were tabulated in Table 2 with the published data. For more data analysis accuracy, the deviation percentage $\Delta_2\%$ between the

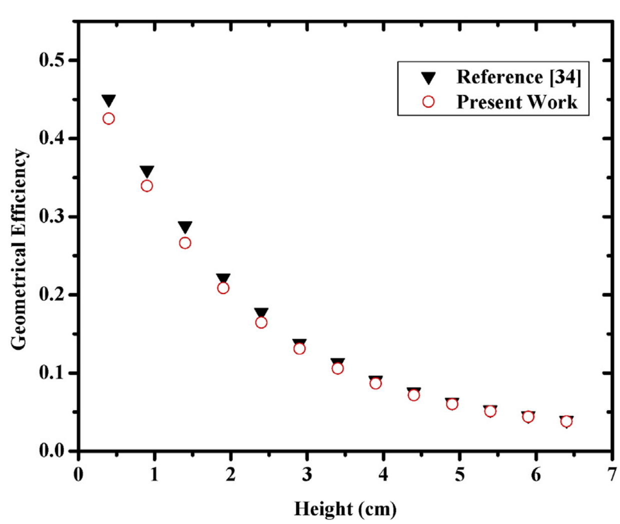

Fig. 5. Comparison between the geometrical efficiency calculated by using the present work and published results reported by Ref. [34] for an axial radioactive point source located at different axial heights from the HPGe detector.

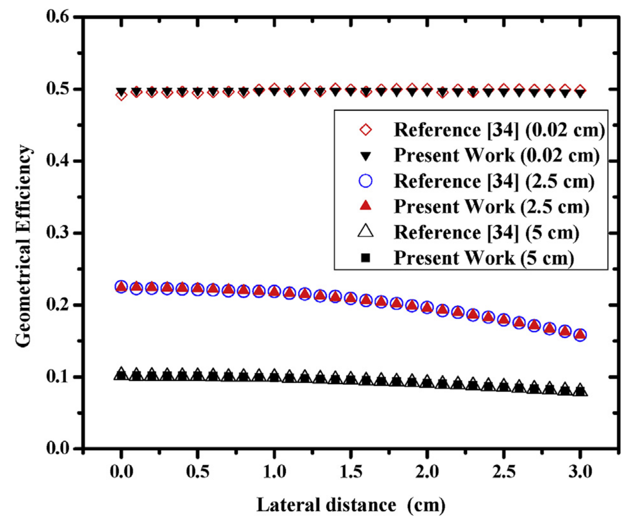

Fig. 6. Comparison between the geometrical efficiency variation of a radioactive point source located at different heights from the NaI(Tl) detector surface and at different lateral distances with the published results reported by Ref. [34].

Table 1
Comparison the geometrical solid angle calculated by the present new approach and published data in Ref. [35].

a (cm)	b (cm)	h (cm)	Geometrical Solid Angle (Ω_{geo})		$\Delta_1\%$
			Reference [35]	Present Work	
2	1	0	6.2831853	6.2827440	0.007
2	1	0.01	6.2347447	6.2346390	0.002
2	1	0.1	5.8004123	5.8001240	0.005
2	1	0.5	4.0425592	4.0423850	0.004
2	1	1	2.5164000	2.5163100	0.004
2	1	5	0.2341430	0.2341370	0.003
2	1	10	0.0616819	0.0616804	0.002
1.01	1	1	1.8513473	1.8512440	0.006
1.1	1	1	1.9453510	1.9452620	0.005
1.5	1	1	2.2659288	2.2658370	0.004
3	1	1	2.7764208	2.7763250	0.003
5	1	1	2.9695886	2.9694910	0.003
10	1	1	3.0847450	3.0846580	0.003
100	1	1	3.1405637	3.1404830	0.003
1000	1	1	3.1415778	3.1414940	0.003
1	1	1	1.8403024	1.8401740	0.007
1	0.99	1	1.8291325	1.8290270	0.006
1	0.9	1	1.7227569	1.7226750	0.005
1	0.5	1	1.1031842	1.1031500	0.003
1	0.2	1	0.4724316	0.4724257	0.001
1	0.1	1	0.2387594	0.2387612	0.001
1	0.01	1	0.0239619	0.0239632	0.005
1	0.001	1	0.0023963	0.0023972	0.038

present results and different published data was calculated by the following equation:

$$\Delta_2\% = \left| \frac{G_{Disk}^{Published} - G_{Disk}^{Present\ Work}}{G_{Disk}^{Published}} \right| * 100 \qquad (16)$$

The percentage deviation $\Delta_2\%$ between the present work and the published results reported by Ref. [8] is less than (1.8%), while with the numerical values is less than (0.8%). The percentage deviation $\Delta_2\%$ between the obtained results and published results reported by Ref. [20] is less than (0.35%) as shown in Table 2. These results confirm the validity of the present new approach for the computation of the solid angle and the geometrical efficiency.

Table 2
Comparison between the geometrical efficiency G_{Disk} calculated by the present new approach and published data in Refs. [8,20] in case of a coaxial radioactive disk source.

R (cm)	h (cm)	S (cm)	Geometrical Efficiency G_{Disk}						
			Present Work	Reference [8]		Reference [8]		Reference [20]	
				M. C.	Δ_2%	Numerical	Δ_2%	Calculated	Δ_2%
1	1	1	0.1165	0.1150	1.2641	0.1161	0.3445	0.1161	0.3001
2	1	1	0.2629	0.2606	0.8935	0.2621	0.3052	0.2621	0.3249
4	1	1	0.3774	0.3758	0.4209	0.3761	0.3457	0.3761	0.3323
1	5	1	0.0095	0.0094	0.8157	0.0094	0.8191	0.0094	0.3292
2	5	1	0.0351	0.0349	0.4776	0.0349	0.4871	0.0350	0.3299
4	5	1	0.1085	0.1080	0.4996	0.1082	0.2773	0.1082	0.3314
8	5	1	0.2350	0.2341	0.3738	0.2342	0.3416	0.2342	0.3328
2.54	5	0.3	0.0543	0.0541	0.3776	0.0541	0.3697	0.0541	0.3331
2.54	10	0.3	0.0154	0.0154	0.1963	0.0154	0.1948	0.0154	0.3332
2.54	15	0.3	0.0070	0.0070	0.5608	0.0070	0.5571	0.0070	0.3333
2.54	20	0.3	0.0040	0.0040	0.0814	0.0040	0.0750	0.0040	0.3333
2.54	5	2	0.0503	0.0499	0.8129	0.0501	0.4192	0.0501	0.3219
2.54	10	2	0.0150	0.0149	0.9065	0.0150	0.2667	0.0150	0.3294
2.54	15	2	0.0070	0.0069	0.7680	0.0069	0.7681	0.0069	0.3314
2.54	20	2	0.0040	0.0039	1.7547	0.0039	1.7436	0.0040	0.3322

4. Conclusion

The new approach for calculating the geometrical solid angle, and the geometrical efficiency by using a non-axial radioactive point and coaxial circular disc sources, with respect to the circular detector surface has been deduced. This technique is considered to be a simple and extremely quick process for the computation process, particularly in case of extended sources. Therefore, it is efficient and sufficiently powerful procedure after doing some adaptation to calculate the full-energy peak efficiency of the gamma-ray detectors with different shape of radioactive sources, in case of the paths of the photons through the detectors can be determined, taking the attenuation of all the surrounding materials through the setup of the measuring system into account. The calculated values in this work and the published results were established to be in a very good agreement based on the discrepancies value.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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