J. Appl. Math. & Informatics Vol. **38***(2020), No. 3 - 4, pp. 221 - 238*
https://doi.org/10.14317/jami.2020.221

GROUP S_3 CORDIAL REMAINDER LABELING OF SUBDIVISION OF GRAPHS

A. LOURDUSAMY*, S. JENIFER WENCY AND F. PATRICK

Abstract. Let $G = (V(G), E(G))$ be a graph and let $g : V(G) \to S_3$ be a function. For each edge xy assign the label r where r is the remainder when $o(g(x))$ is divided by $o(g(y))$ or $o(g(y))$ is divided by $o(g(x))$ according as $o(g(x)) \geq o(g(y))$ or $o(g(y)) \geq o(g(x))$. The function g is called a group S_3 cordial remainder labeling of G if $|v_g(i) - v_g(j)| \leq 1$ and $|e_g(1) - e_g(0)| \leq 1$, where $v_g(j)$ denotes the number of vertices labeled with j and $e_g(i)$ denotes the number of edges labeled with i $(i = 0, 1)$. A graph G which admits a group S_3 cordial remainder labeling is called a group S_3 cordial remainder graph. In this paper, we prove that subdivision of graphs admit a group S_3 cordial remainder labeling.

AMS Mathematics Subject Classification : 05C78.
Key words and phrases : Group S_3 cordial remainder labeling, star, fan graph.

1. Introduction

By a graph we mean fnite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ so that the order and size of G are $|V(G)|$ and $|E(G)|$ respectively. Terms not defined here are taken from Harary [3]. Graph labeling was first introduced in 1960's. Most of the graph labeling trace their origins in the paper presented by Alex Rosa in 1967 [8]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices then the labeling is called vertex labeling. If the domain is the set of edges then the labeling is called edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. The complete survey of graph labeling is in [2]. Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by I. Cahit in [1]. Let

Received December 2, 2019. Revised March 10, 2020. Accepted April 8, 2020. *Corresponding author.

f be a function from the vertices of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. f is called a cordial labeling of G if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Lourdusamy et al. [4] introduced the concept of group S_3 cordial remainder labeling and they proved path, cycle, star, bistar, complete bipartite, wheel, fan, comb and crown graphs are group S_3 cordial remainder graphs. In [5, 6, 7], Lourdusamy et al. discussed that the behaviour of group S_3 cordial remainder labeling of helm, flower, closed helm, gear, sunflower, triangular snake, quadrilateral snake, square of the path, duplication of a vertex by a new edge in path and cycle graphs, duplication of an edge by a new vertex in path and cycle graphs, total graph of cycle and path graphs, lotus inside a circle, double fan, ladder, slanting ladder and triangular ladder.

Definition 1.1. Let A be a group. The order of $a \in A$ is the least positive integer n such that $a^n = e$. We denote the order of a by $o(a)$.

Definition 1.2. Consider the symmetric group S_3. Let the elements of S_3 be e, a, b, c, d, f where

$$e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad a = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$
$$c = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \quad d = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad f = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

We have $o(e) = 1$, $o(a) = o(b) = o(c) = 2$, $o(d) = o(f) = 3$.

Definition 1.3. Let $G = (V(G), E(G))$ be a graph and let $g : V(G) \to S_3$ be a function. For each edge xy assign the label r where r is the remainder when $o(g(x))$ is divided by $o(g(y))$ or $o(g(y))$ is divided by $o(g(x))$ according as $o(g(x)) \geq o(g(y))$ or $o(g(y)) \geq o(g(x))$. The function g is called a group S_3 cordial remainder labeling of G if $|v_g(i) - v_g(j)| \leq 1$ and $|e_g(1) - e_g(0)| \leq 1$, where $v_g(j)$ denotes the number of vertices labeled with j and $e_g(i)$ denotes the number of edges labeled with i $(i = 0, 1)$. A graph G which admits a group S_3 cordial remainder labeling is called a group S_3 cordial remainder graph.

In this paper, we prove that subdivision of star, subdivision of bistar, subdivision of wheel, subdivision of comb, subdivision of crown, subdivision of fan and subdivision of ladder admit a group S_3 cordial remainder labeling.

We use the following definitions in the subsequent sections.

Definition 1.4. A bipartite graph is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. If every vertex of V_1 is adjacent with every vertex of V_2, then G is a complete bipartite graph. If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denoted by $K_{m,n}$.

Definition 1.5. $K_{1,n}$ is called a Star.

Definition 1.6. The Bistar $B_{m,n}$ is the graph obtained by joining the two central vertices of $K_{1,m}$ and $K_{1,n}$.

Definition 1.7. The Cartesian product $G_1 \times G_2$ of two graphs is defined to be the graph with vertex set $V_1 \times V_2$ and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent in $G_1 \times G_2$ if either $u_1 = v_1$ and u_2 is adjacent to v_2 or $u_2 = v_2$ and u_1 is adjacent to v_1.

Definition 1.8. The graph $L_n = P_n \times P_2$ is called a Ladder.

Definition 1.9. The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.

Definition 1.10. The graph $W_n = C_n + K_1$ is called a wheel. In a Wheel, a vertex of degree 3 on the cycle is called a rim vertex. A vertex which is adjacent to all the rim vertices is called the central vertex. The edges with one end incident with a rim vertex and the other incident with the central vertex are called spokes.

Definition 1.11. The graph $F_n = K_1 + P_n$ is called a fan.

Definition 1.12. The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 (with p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The graph $P_n \odot K_1$ is called a Comb. The graph $C_n \odot K_1$ is called a Crown.

Definition 1.13. The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.

2. Main results

Theorem 2.1. *$S(K_{1,n})$ is a group S_3 cordial remainder graph for every n.*

Proof. Let $G = S(K_{1,n})$. Let $V(G) = \{u, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{uv_i, v_iu_i : 1 \leq i \leq n\}$. Therefore G is of order $2n + 1$ and size $2n$. Define $g : V(G) \to S_3$ as follows:
Case 1. n is odd.

$$g(u) = a,$$

$$g(v_i) = \begin{cases} b & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq n \ , \end{cases}$$

$$g(u_i) = \begin{cases} d & \text{if } i \equiv 1\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 2\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 3\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 4\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 5\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 0\ (mod\ 6) \text{ and } 1 \leq i \leq n. \end{cases}$$

Case 2. n is even.

$g(u) = a,$

$$g(v_i) = \begin{cases} b & \text{if } i \equiv 1\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 2\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 3\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 4\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 5\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 0\ (mod\ 6) \text{ and } 1 \leq i \leq n\ , \end{cases}$$

$$g(u_i) = \begin{cases} d & \text{if } i \equiv 1\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 2\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 3\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 4\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 5\ (mod\ 6) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 0\ (mod\ 6) \text{ and } 1 \leq i \leq n. \end{cases}$$

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$	$e_g(0)$	$e_g(1)$
$6k\ (k \geq 1)$	$2k+1$	$2k$	$2k$	$2k$	$2k$	$2k$	$6k$	$6k$
$6k+1\ (k \geq 0)$	$2k+1$	$2k+1$	$2k$	$2k+1$	$2k$	$2k$	$6k+1$	$6k+1$
$6k+2\ (k \geq 0)$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k$	$2k+1$	$6k+2$	$6k+2$
$6k+3\ (k \geq 0)$	$2k+2$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$2k+1$	$6k+3$	$6k+3$
$6k+4\ (k \geq 0)$	$2k+2$	$2k+2$	$2k+1$	$2k+2$	$2k+1$	$2k+1$	$6k+4$	$6k+4$
$6k+5\ (k \geq 0)$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+1$	$6k+5$	$6k+5$

TABLE 1

From Table 1, it is easy to verify that $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$. Therefore g is a group S_3 cordial remainder labeling. □

Example 2.2. A group S_3 cordial remainder labeling of $S(K_{1,7})$ is given in Figure 1.

Theorem 2.3. *$S(W_n)$ is a group S_3 cordial remainder graph for $n \geq 3$.*

Proof. Let $V(S(W_n)) = \{u, u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(S(W_n)) = \{uw_i, w_iu_i, u_iv_i : 1 \leq i \leq n\} \bigcup \{v_iu_{i+1} : 1 \leq i \leq n-1\} \bigcup \{v_nu_1\}$. Therefore $S(W_n)$ is of order $3n+1$ and size $4n$. Define $g : V(S(W_n)) \to S_3$ as follows:

Case 1. $n \equiv 0\ (mod\ 6)$.

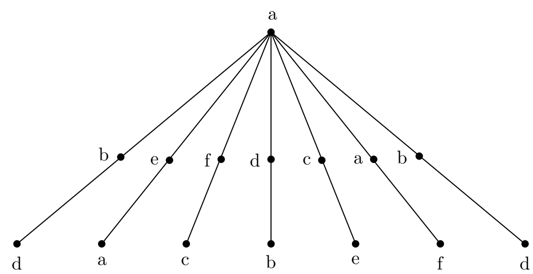

FIGURE 1

Let $n = 6k$ and $k \geq 1$.

$g(u_i) = d$,

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ , \end{cases}$$

$$g(v_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ , \end{cases}$$

$$g(w_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(e) = v_g(f) = 3k, v_g(d) = 3k + 1$ and $e_g(0) = e_g(1) = 12k$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 2. $n \equiv 5\ (mod\ 6)$.

Let $n = 6k + 5$ and $k \geq 0$. Assign the labels to the vertices u, u_i, v_i, w_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i)=\begin{cases} a & \text{if } i=6k+1\\ d & \text{if } i=6k+2\\ b & \text{if } i=6k+3\\ c & \text{if } i=6k+4\\ f & \text{if } i=6k+5\ , \end{cases} \qquad g(v_i)=\begin{cases} a & \text{if } i=6k+1\\ b & \text{if } i=6k+2\\ f & \text{if } i=6k+3\\ e & \text{if } i=6k+4\\ c & \text{if } i=6k+5\ , \end{cases}$$

$$g(w_i)=\begin{cases} a & \text{if } i=6k+1\\ b & \text{if } i=6k+2\\ c & \text{if } i=6k+3\\ d & \text{if } i=6k+4\\ e & \text{if } i=6k+5\ . \end{cases}$$

Here we have $v_g(a)=v_g(b)=v_g(c)=v_g(d)=3k+3, v_g(e)=v_g(f)=3k+2$ and $e_g(0)=e_g(1)=12k+10$. Therefore $|v_g(i)-v_g(j)|\leq 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\leq 1$.

Case 3. $n\equiv 4\ (mod\ 6)$.

Let $n=6k+4$ and $k\geq 0$. Assign the labels to the vertices u,u_i,v_i,w_i for $1\leq i\leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i)=\begin{cases} a & \text{if } i=6k+1\\ c & \text{if } i=6k+2\\ f & \text{if } i=6k+3\\ d & \text{if } i=6k+4\ , \end{cases} \qquad g(v_i)=\begin{cases} f & \text{if } i=6k+1\\ b & \text{if } i=6k+2\\ e & \text{if } i=6k+3\\ c & \text{if } i=6k+4\ , \end{cases}$$

$$g(w_i)=\begin{cases} d & \text{if } i=6k+1\\ b & \text{if } i=6k+2\\ a & \text{if } i=6k+3\\ e & \text{if } i=6k+4\ . \end{cases}$$

Here we have $v_g(a)=v_g(b)=v_g(c)=v_g(e)=v_g(f)=3k+2, v_g(d)=3k+3$ and $e_g(0)=e_g(1)=12k+8$. Therefore $|v_g(i)-v_g(j)|\leq 1$ for $i,j\in S_3$ and $|e_g(0)-e_g(1)|\leq 1$.

Case 4. $n\equiv 3\ (mod\ 6)$.

Let $n=6k+3$ and $k\geq 0$. Assign the labels to the vertices u,u_i,v_i,w_i for $1\leq i\leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i)=\begin{cases} a & \text{if } i=6k+1\\ d & \text{if } i=6k+2\\ e & \text{if } i=6k+3\ , \end{cases} \qquad g(v_i)=\begin{cases} b & \text{if } i=6k+1\\ c & \text{if } i=6k+2\\ f & \text{if } i=6k+3\ , \end{cases}$$

$$g(w_i)=\begin{cases} b & \text{if } i=6k+1\\ c & \text{if } i=6k+2\\ e & \text{if } i=6k+3\ . \end{cases}$$

Here we have $v_g(b)=v_g(c)=v_g(d)=v_g(e)=3k+2, v_g(a)=v_g(f)=3k+1$

and $e_g(0) = e_g(1) = 12k + 6$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 5. $n \equiv 2\ (mod\ 6)$.

Let $n = 6k + 2$ and $k \geq 1$. Assign the labels to the vertices u, u_i, v_i, w_i for $1 \leq i \leq 6k$ as in Case (i) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ f & \text{if } i = 6k+2\ , \end{cases} \qquad g(v_i) = \begin{cases} b & \text{if } i = 6k+1 \\ c & \text{if } i = 6k+2\ , \end{cases}$$

$$g(w_i) = \begin{cases} e & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2\ . \end{cases}$$

Here we have $v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 3k + 1, v_g(a) = 3k + 2$ and $e_g(0) = e_g(1) = 12k + 4$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 6. $n \equiv 1\ (mod\ 6)$.

Let $n = 6k + 1$ and $k \geq 1$. Assign the labels to the vertices u, u_i, v_i, w_i for $1 \leq i \leq 6k$ as in Case (1), except for the vertices $u_{6k+1}, v_{6k+1}, w_{6k+1}$ are labeled by b, f, e respectively. Here we have $v_g(a) = v_g(c) = 3k, v_g(b) = v_g(d) = v_g(e) = v_g(f) = 3k + 1$ and $e_g(0) = e_g(1) = 12k + 2$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Thus g is a group S_3 cordial remainder labeling. Hence $S(W_n)$ is a group S_3 cordial remainder graph for $n \geq 3$. $\square$

Example 2.4. A group S_3 cordial remainder labeling of $S(W_8)$ is given in Figure 2.

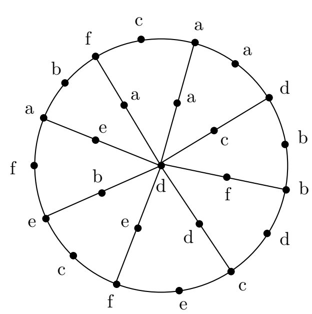

FIGURE 2

Theorem 2.5. *$S(B_{n,n})$ is a group S_3 cordial remainder graph for every n.*

Proof. Let $V(S(B_{n,n})) = \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\} \bigcup \{u, v, w\}$ and $E(S(B_{n,n})) = \{uw, wv\} \bigcup \{ux_i, x_iu_i, vy_i, y_iv_i : 1 \leq i \leq n\}$. Therefore G is of order $4n + 3$ and size $4n + 2$. Define $g : V(S(B_{n,n})) \to S_3$ as follows:

$$g(u) = a,\ g(w) = b,\ g(v) = d,$$

$$g(x_i) = \begin{cases} f & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \le i \le n \\ c & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \le i \le n \\ d & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \le i \le n\ , \end{cases}$$

$$g(u_i) = \begin{cases} b & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \le i \le n \\ e & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \le i \le n \\ f & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \le i \le n\ , \end{cases}$$

$$g(y_i) = \begin{cases} e & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \le i \le n \\ a & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \le i \le n \\ b & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \le i \le n\ , \end{cases}$$

$$g(v_i) = \begin{cases} c & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \le i \le n \\ d & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \le i \le n \\ a & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \le i \le n\ . \end{cases}$$

Nature of n	$v_g(a)$	$v_g(b)$	$v_g(c)$	$v_g(d)$	$v_g(e)$	$v_g(f)$	$e_g(0)$	$e_g(1)$
$3k+1$ $(k \ge 0)$	$2k-1$	$2k$	$2k-1$	$2k-1$	$2k-1$	$2k-1$	$6k+3$	$6k+3$
$3k+2$ $(k \ge 0)$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+2$	$2k+1$	$6k+5$	$6k+5$
$3k$ $(k \ge 1)$	$2k+1$	$2k+1$	$2k$	$2k+1$	$2k$	$2k$	$6k+1$	$6k+1$

TABLE 2

From Table 2, it is easy to verify that $|v_g(i) - v_g(j)| \le 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \le 1$. Therefore g is a group S_3 cordial remainder labeling.

$\square$

Example 2.6. A group S_3 cordial remainder labeling of $S(B_{3,3})$ is given in Figure 3.

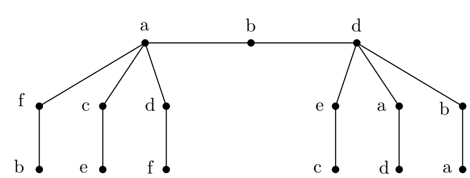

FIGURE 3

Theorem 2.7. *$S(C_n \odot K_1)$ is a group S_3 cordial remainder graph for $n \ge 3$.*

Proof. Let $G = S(C_n \odot K_1)$. Let $V(G) = \{u_i, v_i, x_i, y_i : 1 \le i \le n\}$ and $E(G) = \{x_n u_1\} \bigcup \{u_i x_i, u_i y_i, y_i v_i : 1 \le i \le n\} \bigcup \{x_i u_{i+1} : 1 \le i \le n-1\}$. Therefore G is of order $4n$ and size $4n$. Define $g : V(G) \to S_3$ as follows:
Case 1. $n \equiv 0$ $(mod\ 3)$.

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \leq i \leq n \ , \end{cases}$$

$$g(x_i) = \begin{cases} e & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ b & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ f & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \leq i \leq n \ , \end{cases}$$

$$g(v_i) = \begin{cases} c & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ d & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ e & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \leq i \leq n \ , \end{cases}$$

$$g(y_i) = \begin{cases} f & \text{if } i \equiv 1 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ a & \text{if } i \equiv 2 \ (mod\ 3) \text{ and } 1 \leq i \leq n \\ c & \text{if } i \equiv 0 \ (mod\ 3) \text{ and } 1 \leq i \leq n \ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 2k$ and $e_g(0) = e_g(1) = 6k$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 2. $n \equiv 1 \ (mod\ 3)$.

Let $n = 3k + 1$ and $k \geq 1$. Assign the labels to the vertices u_i, x_i, v_i, y_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels: $g(u_{3k+1}) = a$; $g(x_{3k+1}) = e$; $g(v_{3k+1}) = c$; $g(y_{3k+1}) = f$. Here we have $v_g(a) = v_g(c) = v_g(e) = v_g(f) = 2k + 1, v_g(b) = v_g(d) = 2k$ and $e_g(0) = e_g(1) = 6k + 2$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 3. $n \equiv 2 \ (mod\ 3)$.

Let $n = 3k + 2$ and $k \geq 1$. Assign the labels to the vertices u_i, x_i, v_i, y_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels: $g(u_{3k+1}) = a$; $g(u_{3k+2}) = d$; $g(x_{3k+1}) = c$; $g(x_{3k+2}) = b$; $g(v_{3k+1}) = f$; $g(v_{3k+2}) = e$; $g(y_{3k+1}) = b$; $g(y_{3k+2}) = a$. Here we have $v_g(c) = v_g(d) = v_g(e) = v_g(f) = 2k + 1, v_g(a) = v_g(b) = 2k + 2$ and $e_g(0) = e_g(1) = 6k + 4$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Therefore g is a group S_3 cordial remainder labeling. Hence subdivision of crown $S(C_n \odot K_1)$ is a group S_3 cordial remainder graph for $n \geq 3$.

□

Example 2.8. A group S_3 cordial remainder labeling of $S(C_5 \odot K_1)$ is given in Figure 4.

Corollary 2.9. *$S(P_n \odot K_1)$ is a group S_3 cordial remainder graph for every n.*

Proof. Let $V(S(P_n \odot K_1)) = \{u_i, v_i, y_i : 1 \leq i \leq n\} \bigcup \{x_i : 1 \leq i \leq n-1\}$ and $E(S(P_n \odot K_1)) = \{u_i y_i, y_i v_i : 1 \leq i \leq n\} \bigcup \{u_i x_i, x_i u_{i+1} : 1 \leq i \leq n-1\}$. We define $g : V(S(P_n \odot K_1)) \to S_3$ as follows.

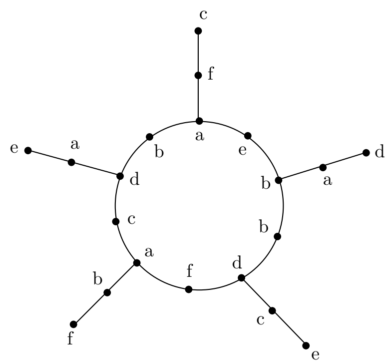

FIGURE 4

For $n = 1$, we assign the labels a, c, f to the vertices u_1, y_1, v_1 respectively. Clearly $S(P_1 \odot K_1)$ is a group S_3 cordial remainder graph. For $n = 2$, we assign the labels a, d, c, b, a, f, e to the vertices $u_1, u_2, x_1, y_1, y_2, v_1, v_2$ respectively. Clearly $S(P_2 \odot K_1)$ is a group S_3 cordial remainder graph.

For $n \geq 3$, the subdivision of comb graph $S(P_n \odot K_1)$ is obtained by removing the edges $u_n x_n$ and $x_n u_1$ in Theorem 2.7. Then we use the same labeling techniques as in Theorem 2.7. Clearly g is a group S_3 cordial remainder labeling for $n \geq 3$.

Hence the subdivision of comb graph $S(P_n \odot K_1)$ is a group S_3 cordial remainder graph for every n.

□

Theorem 2.10. *$S(F_n)$ is a group S_3 cordial remainder graph for $n \geq 2$.*

Proof. Let $V(S(F_n)) = \{u, u_i, w_i : 1 \leq i \leq n\} \bigcup \{v_i : 1 \leq i \leq n-1\}$ and $E(S(F_n)) = \{uw_i, w_i u_i, u_i v_i : 1 \leq i \leq n\} \bigcup \{v_i u_{i+1} : 1 \leq i \leq n-1\}$. Therefore $S(F_n)$ is of order $3n+1$ and size $4n-2$. Define $g : V(S(F_n)) \to S_3$ as follows:

Case 1. $n = 2$.

$g(u) = d,\ g(v_1) = f,$

$$g(u_i) = \begin{cases} a & \text{if } i = 1 \\ b & \text{if } i = 2\ , \end{cases} \qquad g(w_i) = \begin{cases} c & \text{if } i = 1 \\ e & \text{if } i = 2\ . \end{cases}$$

It is easy to verify that g is a group S_3 cordial remainder graph.

Case 2. $n = 3$.

$g(u) = d,$

$$g(u_i) = \begin{cases} a & \text{if } i = 1 \\ d & \text{if } i = 2 \\ e & \text{if } i = 3\ , \end{cases} \qquad g(w_i) = \begin{cases} f & \text{if } i = 1 \\ c & \text{if } i = 2 \\ e & \text{if } i = 3\ , \end{cases}$$

$$g(v_i) = \begin{cases} b & \text{if } i = 1 \\ c & \text{if } i = 2\ . \end{cases}$$

It is easy to verify that g is a group S_3 cordial remainder graph.

Case 3. $n = 4$.

$g(u) = d$,

$$g(u_i) = \begin{cases} a & \text{if } i = 1 \\ c & \text{if } i = 2 \\ f & \text{if } i = 3 \\ d & \text{if } i = 4 \end{cases}, \qquad g(w_i) = \begin{cases} c & \text{if } i = 1 \\ b & \text{if } i = 2 \\ a & \text{if } i = 3 \\ e & \text{if } i = 4 \end{cases},$$

$$g(v_i) = \begin{cases} f & \text{if } i = 1 \\ b & \text{if } i = 2 \\ e & \text{if } i = 3 \end{cases}.$$

It is easy to verify that g is a group S_3 cordial remainder graph.

Case 4. $n = 5$.

$g(u) = d$;

$$g(u_i) = \begin{cases} a & \text{if } i = 1 \\ d & \text{if } i = 2 \\ b & \text{if } i = 3 \\ c & \text{if } i = 4 \\ f & \text{if } i = 5 \end{cases}; \qquad g(w_i) = \begin{cases} a & \text{if } i = 1 \\ b & \text{if } i = 2 \\ c & \text{if } i = 3 \\ d & \text{if } i = 4 \\ e & \text{if } i = 5 \end{cases};$$

$$g(v_i) = \begin{cases} a & \text{if } i = 1 \\ b & \text{if } i = 2 \\ f & \text{if } i = 3 \\ e & \text{if } i = 4 \end{cases}.$$

It is easy to verify that g is a group S_3 cordial remainder graph.

Case 5. $n \geq 6$.

Subcase 5.1. $n \equiv 0 \ (mod\ 6)$.

Let $n = 6k$ and $k \geq 1$.

$g(u) = f$,

$$g(u_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \end{cases},$$

$$g(w_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \ , \end{cases}$$

$$g(v_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(e) = v_g(d) = v_g(f) = 3k$ and $e_g(0) = e_g(1) = 12k-1$. Therefore $|v_g(i)-v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0)-e_g(1)| \leq 1$.

Subcase 5.2. $n \equiv 5 \ (mod\ 6)$.

Let $n = 6k + 5$ and $k \geq 1$. Assign the labels to the vertices u, u_i, w_i, v_i for $1 \leq i \leq 6k$ as in Subcase (5.1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ d & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3 \\ e & \text{if } i = 6k+4 \\ b & \text{if } i = 6k+5 \ , \end{cases} \qquad g(w_i) = \begin{cases} c & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \\ a & \text{if } i = 6k+5 \ , \end{cases}$$

$$g(v_i) = \begin{cases} e & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ f & \text{if } i = 6k+4 \ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(f) = 3k + 3, v_g(c) = v_g(d) = v_g(e) = 3k + 2$ and $e_g(0) = e_g(1) = 12k + 9$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Subcase 5.3. $n \equiv 4 \ (mod\ 6)$.

Let $n = 6k + 4$ and $k \geq 1$. Assign the labels to the vertices u, u_i, w_i, v_i for $1 \leq i \leq 6k$ as in Subcase (5.1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ d & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3 \\ e & \text{if } i = 6k+4 \ , \end{cases} \qquad g(w_i) = \begin{cases} c & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \ , \end{cases}$$

$$g(v_i) = \begin{cases} e & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 3k+2$ and $e_g(0) = e_g(1) = 12k+7$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Subcase 5.4. $n \equiv 3\ (mod\ 6)$.

Let $n = 6k+3$ and $k \geq 1$. Assign the labels to the vertices u, u_i, w_i, v_i for $1 \leq i \leq 6k$ as in Subcase (5.1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ d & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3\ , \end{cases} \qquad g(w_i) = \begin{cases} c & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3\ , \end{cases}$$

$$g(v_i) = \begin{cases} e & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(f) = 3k+2, v_g(c) = v_g(d) = v_g(e) = 3k+1$ and $e_g(0) = e_g(1) = 12k+5$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Subcase 5.5. $n \equiv 2\ (mod\ 6)$.

Let $n = 6k+2$ and $k \geq 1$. Assign the labels to the vertices u, u_i, w_i, v_i for $1 \leq i \leq 6k$ as in Subcase (5.1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} a & \text{if } i = 6k+1 \\ d & \text{if } i = 6k+2\ , \end{cases} \qquad g(w_i) = \begin{cases} c & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2\ , \end{cases}$$

$$g(v_i) = \begin{cases} e & \text{if } i = 6k+1\ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 3k+1$ and $e_g(0) = e_g(1) = 12k+3$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Subcase 5.6. $n \equiv 1\ (mod\ 6)$.

Let $n = 6k+1$ and $k \geq 1$. Assign the labels to the vertices u, u_i, v_i for $1 \leq i \leq 6k$ as in Subcase (5.1), except for the vertices u_{6k+1}, w_{6k+1} are labeled by b, a respectively. Here we have $v_g(a) = v_g(b) = v_g(f) = 3k+1, v_g(c) = v_g(d) = v_g(e) = 3k$ and $e_g(0) = e_g(1) = 12k+1$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Thus g is a group S_3 cordial remainder labeling. Hence, $S(F_n)$ is a group S_3 cordial remainder graph for $n \geq 2$. □

Example 2.11. A group S_3 cordial remainder labeling of $S(F_4)$ is given in Figure 5.

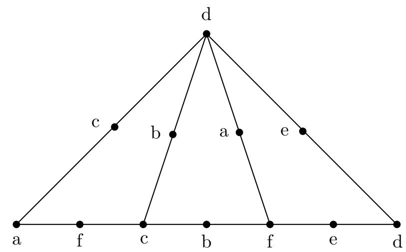

FIGURE 5

Theorem 2.12. *$S(L_n)$ is a group S_3 cordial remainder graph for every n.*

Proof. Let $u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n$ be the vertices of the ladder L_n. Let $V(S(L_n)) = \{u_i, v_i, z_i : 1 \leq i \leq n\} \bigcup \{x_i, y_i : 1 \leq i \leq n-1\}$ and $E(S(L_n)) = \{u_i x_i, v_i y_i, u_i z_i, z_i v_i, : 1 \leq i \leq n\} \bigcup \{x_i u_{i+1}, y_i v_{i+1} : 1 \leq i \leq n-1\}$. Therefore $S(L_n)$ is of order $5n-2$ and size $6n-2$. Define $g : V(S(L_n)) \to S_3$ as follows:
Case 1. $n \equiv 0 \ (mod\ 6)$.
Let $n = 6k$ and $k \geq 1$.

$$g(u_i) = \begin{cases} d & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ a & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ , \end{cases}$$

$$g(v_i) = \begin{cases} b & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ a & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ , \end{cases}$$

$$g(x_i) = \begin{cases} c & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ a & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k\ , \end{cases}$$

$$g(y_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \ , \end{cases}$$

$$g(z_i) = \begin{cases} a & \text{if } i \equiv 1 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ b & \text{if } i \equiv 2 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ d & \text{if } i \equiv 3 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ c & \text{if } i \equiv 4 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ f & \text{if } i \equiv 5 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \\ e & \text{if } i \equiv 0 \ (mod\ 6) \text{ and } 1 \leq i \leq 6k \ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(e) = 5k, v_g(d) = v_g(f) = 5k - 1$ and $e_g(0) = e_g(1) = 18k - 2$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 2. $n \equiv 1 \ (mod\ 6)$.

Let $n = 6k + 1$ and $k \geq 0$. Assign the labels to the vertices u_i, v_i, x_i, y_i, z_i for $1 \leq i \leq 6k$ as in Case (1), except for the vertices $u_{6k+1}, z_{6k+1}, v_{6k+1}$, are labeled by d, a, b respectively. Here we have $v_g(a) = v_g(b) = v_g(d) = 5k + 1, v_g(c) = v_g(e) = v_g(f) = 5k$ and $e_g(0) = e_g(1) = 18k + 1$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 3. $n \equiv 2 \ (mod\ 6)$.

Let $n = 6k + 2$ and $k \geq 0$. Assign the labels to the vertices u_i, v_i, x_i, y_i, z_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} d & \text{if } i = 6k + 1 \\ e & \text{if } i = 6k + 2 \ , \end{cases} \qquad g(v_i) = \begin{cases} b & \text{if } i = 6k + 1 \\ f & \text{if } i = 6k + 2 \ , \end{cases}$$

$$g(z_i) = \begin{cases} a & \text{if } i = 6k + 1 \\ b & \text{if } i = 6k + 2 \ , \end{cases} \qquad g(x_{6k+1}) = c; \ g(y_{6k+1}) = a.$$

Here we have $v_g(a) = v_g(b) = 5k + 2, v_g(c) = v_g(d) = v_g(e) = v_g(f) = 5k + 1$ and $e_g(0) = e_g(1) = 18k + 4$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 4. $n \equiv 3 \ (mod\ 6)$.

Let $n = 6k + 3$ and $k \geq 0$. Assign the labels to the vertices u_i, v_i, x_i, y_i, z_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} d & \text{if } i = 6k + 1 \\ e & \text{if } i = 6k + 2 \\ f & \text{if } i = 6k + 3 \ , \end{cases} \qquad g(v_i) = \begin{cases} b & \text{if } i = 6k + 1 \\ f & \text{if } i = 6k + 2 \\ d & \text{if } i = 6k + 3 \ , \end{cases}$$

$$g(z_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ c & \text{if } i = 6k+3 \ , \end{cases} \qquad g(x_i) = \begin{cases} c & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2 \ , \end{cases}$$

$$g(y_i) = \begin{cases} a & \text{if } i = 6k+1 \\ e & \text{if } i = 6k+2 \ . \end{cases}$$

Here we have $v_g(a) = 5k+3, v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 5k+2$ and $e_g(0) = e_g(1) = 18k+7$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 5. $n \equiv 4$ $(mod\ 6)$.

Let $n = 6k+4$ and $k \geq 0$. Assign the labels to the vertices u_i, v_i, x_i, y_i, z_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} d & \text{if } i = 6k+1 \\ e & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \ , \end{cases} \qquad g(v_i) = \begin{cases} b & \text{if } i = 6k+1 \\ f & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ e & \text{if } i = 6k+4 \ , \end{cases}$$

$$g(z_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ c & \text{if } i = 6k+3 \\ f & \text{if } i = 6k+4 \ , \end{cases} \qquad g(x_i) = \begin{cases} c & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \ , \end{cases}$$

$$g(y_i) = \begin{cases} a & \text{if } i = 6k+1 \\ e & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3 \ . \end{cases}$$

Here we have $v_g(a) = v_g(b) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 5k+3$ and $e_g(0) = e_g(1) = 18k+10$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Case 6. $n \equiv 5$ $(mod\ 6)$.

Let $n = 6k+5$ and $k \geq 0$. Assign the labels to the vertices u_i, v_i, x_i, y_i, z_i for $1 \leq i \leq 6k$ as in Case (1) and for the remaining vertices assign the following labels:

$$g(u_i) = \begin{cases} d & \text{if } i = 6k+1 \\ e & \text{if } i = 6k+2 \\ f & \text{if } i = 6k+3 \\ c & \text{if } i = 6k+4 \\ a & \text{if } i = 6k+5 \ , \end{cases} \qquad g(v_i) = \begin{cases} b & \text{if } i = 6k+1 \\ f & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ e & \text{if } i = 6k+4 \\ c & \text{if } i = 6k+5 \ , \end{cases}$$

$$g(z_i) = \begin{cases} a & \text{if } i = 6k+1 \\ b & \text{if } i = 6k+2 \\ c & \text{if } i = 6k+3 \\ f & \text{if } i = 6k+4 \\ e & \text{if } i = 6k+5\ , \end{cases} \qquad g(x_i) = \begin{cases} c & \text{if } i = 6k+1 \\ a & \text{if } i = 6k+2 \\ d & \text{if } i = 6k+3 \\ f & \text{if } i = 6k+4\ , \end{cases}$$

$$g(y_i) = \begin{cases} a & \text{if } i = 6k+1 \\ e & \text{if } i = 6k+2 \\ b & \text{if } i = 6k+3 \\ d & \text{if } i = 6k+4\ . \end{cases}$$

Here we have $v_g(a) = v_g(c) = v_g(d) = v_g(e) = v_g(f) = 5k+4, v_g(b) = 5k+3$ and $e_g(0) = e_g(1) = 18k+13$. Therefore $|v_g(i) - v_g(j)| \leq 1$ for $i, j \in S_3$ and $|e_g(0) - e_g(1)| \leq 1$.

Thus g is a group S_3 cordial remainder labeling. Hence, $S(L_n)$ is a group S_3 cordial remainder graph every for n. $\square$

Example 2.13. A group S_3 cordial remainder labeling of $S(L_5)$ is given in Figure 6.

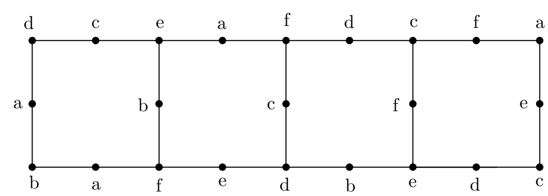

FIGURE 6

REFERENCES

1. I. Cahit, *Cordial graphs: A weaker version of graceful and harmonious graphs*, Ars Combin. **23** (1987), 201-207.
2. J.A. Gallian, *A Dyamic Survey of Graph Labeling*, The Electronic J. Combin. **22** (2019), # DS6.
3. F. Harary, *Graph Theory*, Addison-wesley, Reading, Mass, 1972.
4. A. Lourdusamy, S. Jenifer Wency and F. Patrick, *Group S_3 cordial remainder labeling*, International Journal of Recent Technology and Engineering **8** (2019), 8276-8281.
5. A. Lourdusamy, S. Jenifer Wency and F. Patrick, *Group S_3 cordial remainder labeling for wheel and snake related graphs*, (Submitted for Publication).
6. A. Lourdusamy, S. Jenifer Wency and F. Patrick, *Group S_3 cordial remainder labeling for path and cycle related graphs*, (Submitted for Publication).
7. A. Lourdusamy, S. Jenifer Wency and F. Patrick, *Some Result on Group S_3 cordial remainder graphs*, (Submitted for Publication).

8. A. Rosa, *On certain valuations of the vertices of a graph*, Theory of Graphs, (Internat Sympos., Rome, 1966) New York, Gordon and Breach 1967, 349-355.

A. Lourdusamy

Department of Mathematics, St. Xavier's College (Autonomous), Palayamkottai-627002, Tamil Nadu, India.
e-mail: lourdusamy15@gmail.com

S. Jenifer Wency

Research Scholar, Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli, Tamil Nadu, India.
e-mail: jeniferwency@gmail.com

F. Patrick

Department of Mathematics, St. Xavier's College (Autonomous), Palayamkottai-627002, Tamil Nadu, India.
e-mail: patrick881990@gmail.com