



Original Article

Pulse pileup correction method for gamma-ray spectroscopy in high radiation fields

Minju Lee ^a, Daehee Lee ^b, Eunbie Ko ^a, Kyeongjin Park ^a, Junhyuk Kim ^a, Kilyoung Ko ^a, Manish Sharma ^c, Gyuseong Cho ^{a,*}^a Department of Nuclear and Quantum Engineering, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon, 34141, Republic of Korea^b Fuze Laboratory, Agency for Defense Development, Yuseong, P.O. Box 35-5, Daejeon, 305-600, Republic of Korea^c Department of Nuclear Engineering, Khalifa University, Abu Dhabi, 127788, United Arab Emirates

ARTICLE INFO

Article history:

Received 7 August 2019

Received in revised form

25 November 2019

Accepted 3 December 2019

Available online 6 December 2019

Keywords:

Gamma spectroscopy

Scintillation detector

Pulse pileup correction

Scintillation pulse model

ABSTRACT

The detector suffers from pulse pileup by overlapping of the signals when it was used in high radiation fields. The pulse pileup deteriorates the energy spectrum and causes count losses due to random coincidences, which might not resolve within the resolving time of the detection system. In this study, it is aimed to propose a new pulse pileup correction method. The proposed method is to correct the start point of the pileup pulse. The parameters are obtained from the fitted exponential curve using the peak point of the previous pulse and the start point of the pileup pulse. The amplitude at the corrected start point of the pileup pulse can be estimated by the peak time of the pileup pulse. The system is composed of a NaI (TI) scintillation crystal, a photomultiplier tube, and an oscilloscope. A 61 μCi ^{137}Cs check-source was placed at a distance of 3 cm, 5 cm, and 10 cm, respectively. The gamma energy spectra for the radioisotope of ^{137}Cs were obtained to verify the proposed method. As a result, the correction of the pulse pileup through the proposed method shows a remarkable improvement of FWHM at 662 keV by 29, 39, and 7%, respectively.

© 2019 Korean Nuclear Society, Published by Elsevier Korea LLC. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

Gamma spectroscopy is the quantitative study of the energy spectra of gamma rays emitted from radioactive sources [1]. It can be used to identify radioactive nuclides because it analyzes the characteristics of the gamma-ray emitting material. One of the most commonly used detectors in this field is the scintillation detector. The scintillation detector based on sodium iodide is widely used for the gamma-ray measurement with two main advantages [1]: It has good yielding efficiency by producing large crystals, and it produces exceptional blasts of light contrasted with other scintillation crystals. The scintillation detector produces an electrical signal by the interaction of gamma rays and handles signals in the signal processing circuit. Finite signal processing time is required to handle a single gamma ray. In low dose environments, there is no problem in processing gamma rays. The detector suffers, however, from pulse pileup by overlapping of the signals in high dose environments. Each comparator of the signal processing circuit is

recognized as a single gamma ray even though it has two inputs, resulting in count losses [2,3]. The pulse pileup events are formed by random coincidence due to photoelectric absorption, Compton scattering, backscatter interactions, and so on. The pulse pileup ratio (PPR) depends on the count rate and the dead time of the detector. A large dead time of the detector increases the probability of the pileup. The pileup effect also causes distortion of the amplitude of the pulse, resulting in deterioration of the energy spectrum [2–5]. Various methods are being studied to solve the pulse pileup problem.

The hardware method is to reduce the pixel size of the detector for a large number of pixels in the same area [6]. Then, the problem of the pulse pileup can be solved by parallel signal processing. However, integrated circuit and signal processing to achieve good performance requires a hard work. It also takes a long time to read out the signals because of increased pixels.

The shaping method reduces the probability of pulse pileup by shortening the long tail of the pulse [7,8]. The current pulse from the detector is amplified and converted to the voltage pulse by a preamplifier. The width of the voltage pulse determined by the decay constant of the scintillation crystal can be reduced by a shaping amplifier. Even though this method is widely used in

* Corresponding author.

E-mail address: gscho@kaist.ac.kr (G. Cho).

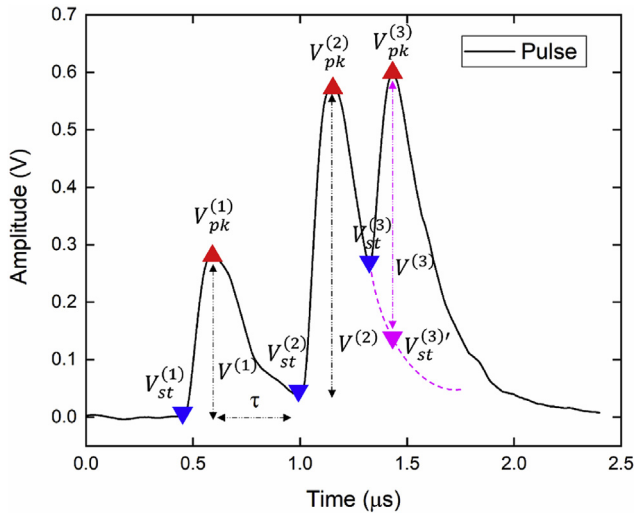


Fig. 1. A detailed description of the pulse pileup correction.

gamma spectroscopy [9–11], the energy resolution may degrade if the amplitude of the original pulse is not preserved [12]. Additionally, the use of the shaping amplifier for pulse shaping may lead to a poor signal-to-noise ratio.

The rejection method removes all pileup events. It can be implemented in two ways: One, pileup events can be eliminated by adding a pulse pileup rejector (PUR) in the signal processing circuit. The other can exclude the pileup events using a post-processing algorithm. This method is the best solution for solving the energy distortion problem. However, the data acquisition time needs to be

increased to overcome count losses, which is the disadvantage of this method [12,13].

The pulse pileup correction approaches recover each single pulse by subtracting the extrapolated remains from the pileup pulse [12]. This method has the advantage of being able to overcome the distortion of the pulse shape and count rate losses, which is not accomplished by the shaping method and the rejection method [12,14,15]. The pulse model should be determined for this method [12,16]. For example, the scintillation crystal produces a light proportional to the energy of the incident radiation. The light is collected and amplified by a photomultiplier tube (PMT) and converted into an exponential pulse. This is why a scintillation pulse model is widely adopted to correct the pulse pileup in the scintillation detector. The pulse model can be expressed by the parameters such as the normalization constant that is proportional to the energy of the incident radiation, the decay constant of the scintillation crystal, and the time constant of the PMT anode [16]. The parameters can be predetermined experimentally by investigating the pulses of the detector [12]. The variation in pulse amplitude is determined by the uniformity of light collection in the scintillator [17]. Often for larger scintillator, optical self-absorption within the scintillator and light losses at the scintillator surfaces lead to the non-uniformity of light collection [17]. The predetermined parameters of the pulse model may change with large variation in light collection [12]. This problem can be solved by calculating the parameters of each event pulse using the recorded samples [14]. In previous studies, four parameters of the pulse model are needed to correct the pulse pileup [14]. However, it takes a long time to compute the four parameters.

The purpose of this study is to propose a new approach for pulse pileup correction. The core of the proposed method is to correct the

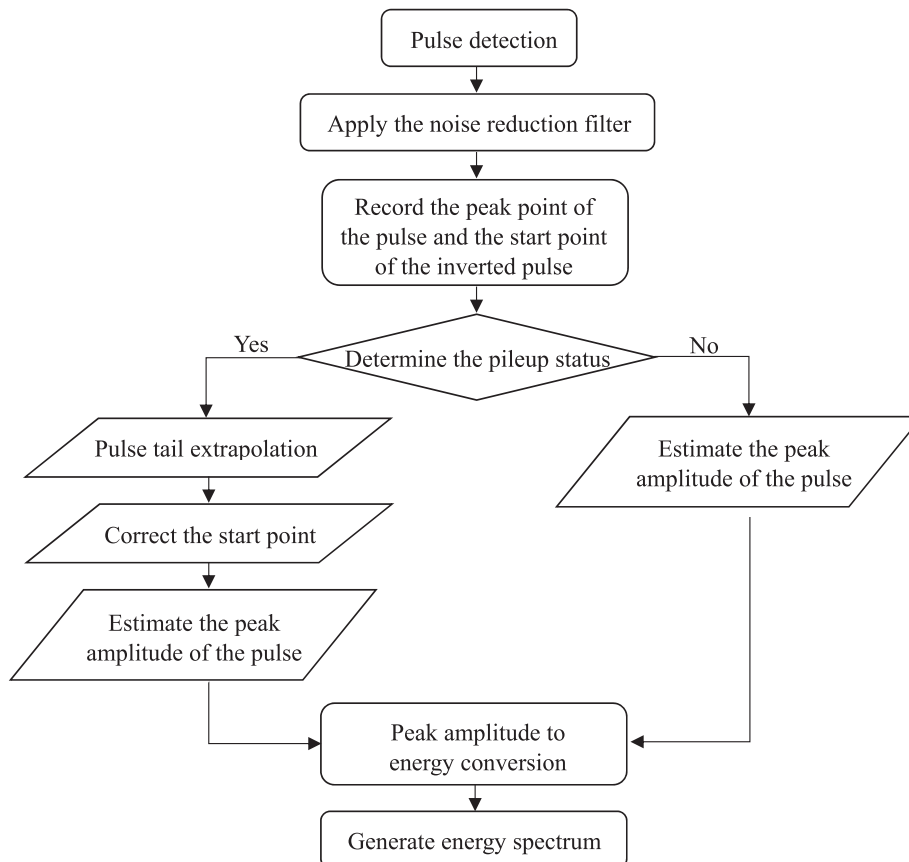


Fig. 2. A flowchart of the pulse pileup correction in the proposed method.

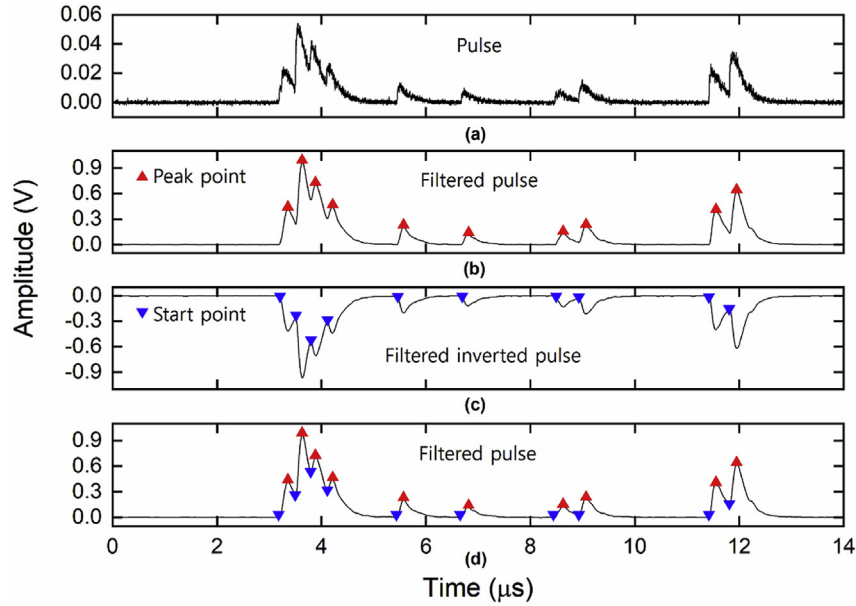


Fig. 3. Process of finding the peak point of the filtered pulse and the start point of the filtered inverted pulse.

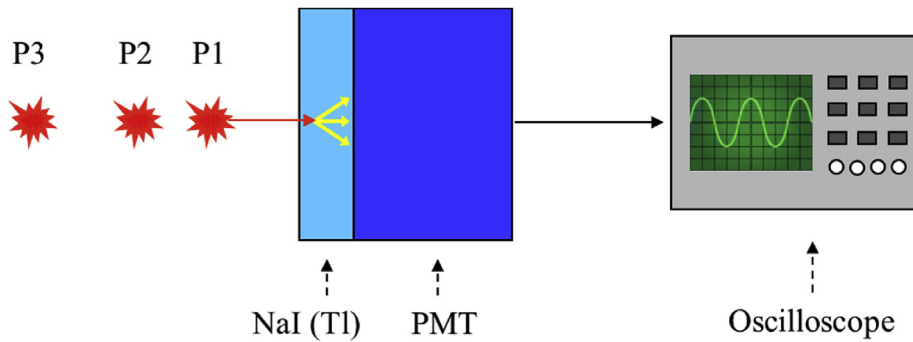


Fig. 4. A schematic of the system to verify the proposed method.

start point of the pileup pulse. Only two parameters are required for this. The parameters are obtained from the fitted exponential curve using the information at the peak point of the previous pulse and the information at the start point of the pileup pulse. The amplitude at the corrected start point of the pileup pulse can be estimated by the peak time of the pileup pulse.

2. Materials and method

2.1. Proposed pulse pileup correction method

The proposed method for pulse pileup correction is based on a scintillation pulse model. The scintillation crystal interacts with radiation and generates a light proportional to the energy of the incident radiation. The light is collected and amplified by the PMT and converted into an electrical signal. The electron current arriving at the PMT anode is expressed as [17].

$$i(t) = i_0 e^{-\lambda t} \tag{1}$$

where λ is the decay time constant of the scintillator and i_0 is the initial current. The initial current i_0 expressed in terms of total charge Q is described as

$$Q = \int_0^{\infty} i(t) dt = i_0 \int_0^{\infty} e^{-\lambda t} dt = \frac{i_0}{\lambda} \tag{2}$$

Therefore

$$i_0 = \lambda Q \tag{3}$$

and the initial current i_0 is described by combining Eq. (1) and Eq. (3):

Table 1
Information of the experiments for different source-to-detector positions.

| Position | Pulses sampling recorded | Radioisotope | Activity (μCi) | Pileup fraction (%) |
|------------|--------------------------|-------------------|-----------------------------|---------------------|
| P1 (3 cm) | 8000 | ^{137}Cs | 61 | 52.6 |
| P2 (5 cm) | 8000 | ^{137}Cs | 61 | 49.1 |
| P3 (10 cm) | 8000 | ^{137}Cs | 61 | 32.2 |

$$i(t) = \lambda Q e^{-\lambda t} \quad (4)$$

The voltage pulse $V(t)$ is represented by a scintillation pulse model [14].

$$V(t) = \begin{cases} 0, & t < t_s \\ V_p \frac{(t - t_s)}{(t_p - t_s)}, & t_s \leq t < t_p \\ V_p e^{-(t-t_p)/\lambda}, & t \geq t_p \end{cases} \quad (5)$$

where V_p is the peak value of the pulse, t_s is the start time of the event, t_p is the peak time of the event, and λ is the decay time constant of the scintillator.

If the leading edge of the pulse is fast, the voltage pulse $V(t)$ can be simplified as follows [14]:

$$V(t) = \begin{cases} 0, & t < t_s \\ K_1 \times t + K_2, & t_s \leq t < t_p \\ K_3 e^{-K_4 \times t}, & t \geq t_p \end{cases} \quad (6)$$

where K_1 is the slope of the straight line, K_2 is its intercept on the V -axis, K_3 is a scale coefficient of the exponential curve, and K_4 is the exponent of the curve.

In previous studies, the four parameters $K_1 - K_4$ are computed by fitting procedure to correct the pulse pileup [14]. The parameters K_1 and K_2 are obtained from the fitted straight line using the samples on the leading edge of the pulse and the parameters K_3 and K_4 are obtained from the fitted exponential curve using the samples on the trailing edge of the pulse [14]. However, it takes a long time to compute the four parameters.

Fig. 1 shows a detailed description of the pulse pileup correction in the proposed method. The core of the proposed method is to correct the start point of the pileup pulse. The two parameters K_3 and K_4 are required for this. The parameters are obtained from the fitted exponential curve using the information at the peak point $(V_{pk}^{(i-1)}, t_{pk}^{(i-1)})$ of the previous pulse and the information at the start point $(V_{st}^{(i)}, t_{st}^{(i)})$ of the pileup pulse. The amplitude at the corrected start point $(V_{st}^{(i)'})$ of the pileup pulse can be estimated by the peak time $(t_{pk}^{(i)'})$ of the pileup pulse. The peak amplitude $(V^{(i)})$ can be computed by subtracting the amplitude at the corrected start point $(V_{st}^{(i)'})$ and the amplitude at the peak point $(V_{pk}^{(i)})$.

The strategy of the pulse pileup correction requires several steps, as shown in Fig. 2. The first step is to detect pulses from the scintillators. The second step is to apply the noise reduction filter, as shown in Fig. 3. The noise should be minimized because the PMT suffers from dark noise, flux-generated noise, etc [18,19]. A low pass filter was used in this study. The third step is to record the start point of the inverted pulse and the peak point of the pulse, as shown in Fig. 3. The fourth step is to determine the pileup status of the pulse using the fall time (τ) of the pulse. If the time difference at the peak points of a continuous pulse is larger than the fall time, it is considered as a pileup free pulse; otherwise a pileup pulse. In the case of the pileup pulse, the amplitude at the start point is corrected by tail extrapolation. The fifth step is to calculate the peak amplitude by subtracting the amplitude of the start point from the amplitude of the peak point. Finally, the energy of the incident radiation is estimated from the energy-peak amplitude relationship and the energy spectrum is generated.

Table 2
Information on the radioisotope point source.

| Radioisotope | ^{22}Na | ^{137}Cs | ^{60}Co |
|-----------------------------|------------------|-------------------|------------------|
| Activity (μCi) | 1.010 | 1.002 | 0.9876 |
| Energy 1 (keV) | 511 | 661.7 | 1173.2 |
| Energy 2 (keV) | 1274.5 | | 1332.5 |

2.2. Experimental setup

A schematic of the system that verifies the proposed method is shown in Fig. 4. The system is composed of a NaI(Tl) scintillation crystal (Epic Crystal, $\varnothing 2 \times 2$ inch), a PMT (Hamamatsu, H7195), and an oscilloscope (Tektronix, MSO 3014). The contact surface of the NaI (Tl) and the PMT were coupled using optical grease (Saint Gobain, BC-630) to minimize the loss of photons. The operational voltage of the PMT was set to -1300 V. The output pulse of the PMT was digitized by an oscilloscope with a sampling rate of 2.5 GHz and a bandwidth of 100 MHz. The digitized pulse was read out using a

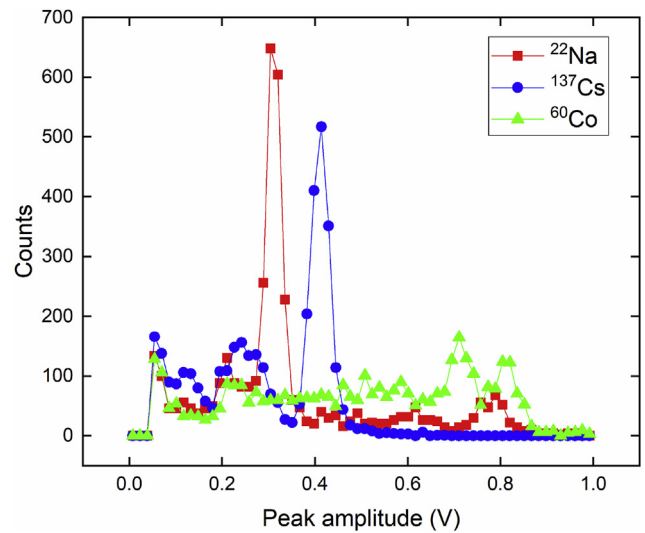


Fig. 5. The peak amplitude spectra on multiple radioisotopes.

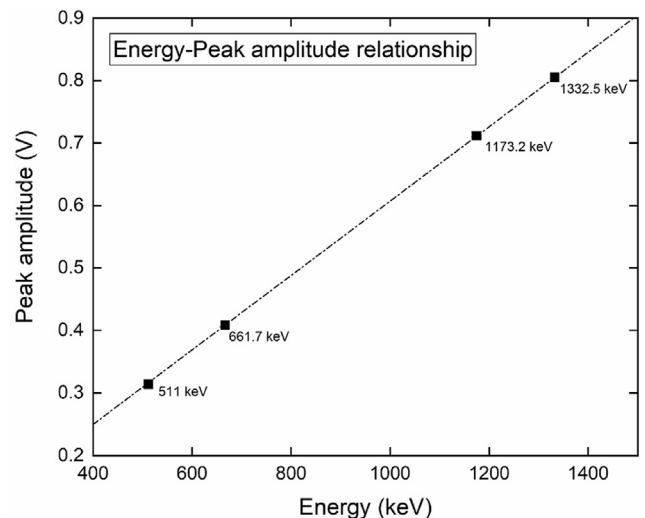


Fig. 6. The relationship between the energy of the gamma rays and the peak amplitude of the pulse.

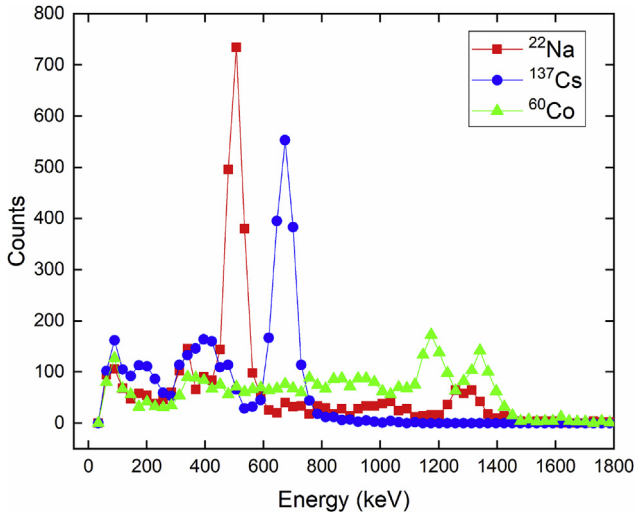


Fig. 7. The calibrated energy spectra on multiple radioisotopes.

computer. The experiment was tested in an aluminium dark box.

The experiment was performed by changing the position of the radioisotope check-source to evaluate the pulse pileup correction effect of the proposed method. A 61 μCi ^{137}Cs check-source was placed at P1, P2, and P3. The positions were located at a distance of 3 cm, 5 cm, and 10 cm from the scintillation crystal, respectively. The pileup fraction was defined as the ratio of the number of pileup events and the number of total events.

$$\text{Pileup fraction (\%)} = \frac{N_{\text{pileup}}}{N_{\text{total}}} \times 100 \quad (7)$$

where N_{pileup} is the number of the pileup events and N_{total} is the

number of total events. The pileup fractions of the positions were 52.6, 49.1, and 32.3%, respectively. A total of 8000 samples were obtained using an oscilloscope. Details of the experiment used in this work are described in Table 1.

The peak amplitude spectra were acquired using a multiple radioisotopes to evaluate the correlation between the peak amplitude of the pulse and the energy of the incident radiation. The ^{137}Cs , ^{22}Na , and ^{60}Co point source were located 10 cm from the scintillation crystal and the pileup free pulses were obtained by an oscilloscope. We found the peak amplitude of each pulse and measured the amplitude spectrum. The mean of the photo absorption peak of the amplitude spectrum was estimated by fitting a Gaussian distribution. The relationship between the peak amplitude of the pulse and the energy of the incident radiation was derived by fitting a linear regression with the energy of the gamma rays emitted from the radioisotope and the mean. Details of the point source used in this experiment are described in Table 2.

3. Results and discussion

The peak amplitude spectra for the radioisotopes of ^{22}Na , ^{137}Cs , and ^{60}Co were obtained to correlate the peak amplitude of the pulse and the energy of the gamma rays. Fig. 5 shows the peak amplitude spectra on multiple radioisotopes.

The centroid of the photo absorption peak in the peak amplitude spectrum for the radioisotopes was estimated by fitting a Gaussian distribution. The peak amplitude of the pulse and the energy of the gamma rays showed a linear relationship, as shown in Fig. 6. This was used to calibrate the energy from the peak amplitude of the pulse. Fig. 7 shows the calibrated energy spectra on multiple radioisotopes. As a result, the errors of conversion to energy from the peak amplitude for the radioisotopes of ^{22}Na , ^{137}Cs , and ^{60}Co were 1.57, 1.54, 0.36, and 0.05%, respectively.

In the proposed method, the start point of the pileup pulse

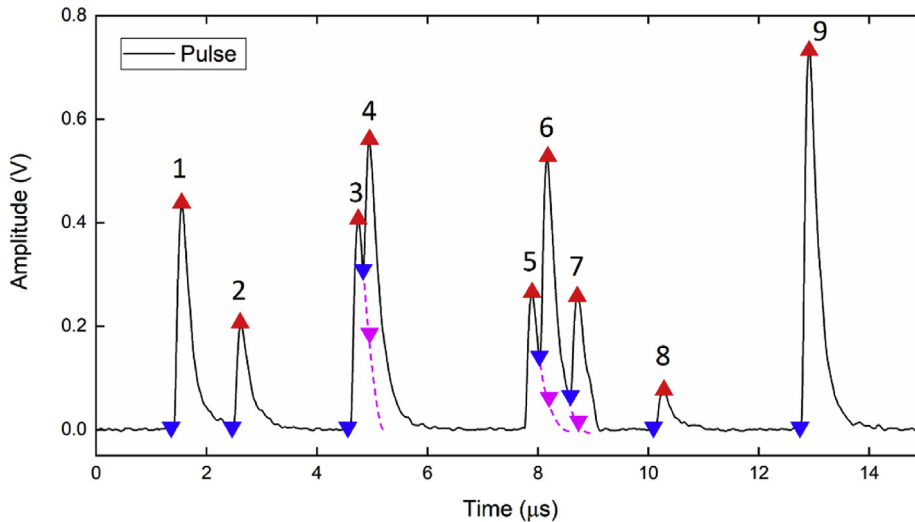


Fig. 8. Tail extrapolation to correct the start point of the pileup pulse.

Table 3
The correction results of the pulse pileup by the proposed method.

| Status of pileup | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------------------------------|-----|-----|-----|-----|-----|-----|-----|----|------|
| | X | X | X | 0 | X | 0 | 0 | X | X |
| Energy (keV) without correction | 708 | 322 | 660 | 922 | 422 | 863 | 414 | 98 | 1205 |
| Energy (keV) with correction | 708 | 322 | 660 | 632 | 422 | 799 | 402 | 98 | 1205 |

needed to be corrected in order to achieve the goal. This was performed by extrapolating the peak point of the previous pulse and the start point of the pileup pulse. In Fig. 8, the correction of the start point of the fourth pulse was carried out using the peak time of the fourth pulse after the tail extrapolation. The peak amplitude was computed by subtracting the amplitude of the corrected start point from the amplitude of the peak point. The energy of the gamma rays was estimated by the energy-peak amplitude relationship. Consequently, the energy corresponding to the fourth pulse was corrected from 922 keV to 632 keV. The remaining pileup pulses in Fig. 8 were corrected in the same way. Table 3 shows the correction results of the pulse pileup by the proposed method.

The gamma energy spectra for the radioisotope of ^{137}Cs were obtained to verify the proposed method. The uncorrected spectrum, corrected spectrum, and rejected spectrum were compared to evaluate the correction effect of the pulse pileup. The uncorrected spectrum was acquired using the peak amplitude of all events. The corrected spectrum was obtained by calibrating the pulse pileup using the peak point of the pulse and the start point of the inverted pulse. The rejected spectrum was obtained by removing the pileup pulse from all events. Fig. 9 shows the energy spectra for the radioisotope of ^{137}Cs with the pileup fractions of 52.6, 49.1, and 32.3%, respectively.

The main interactions of the gamma rays in the scintillator are photoelectric absorption, Compton scattering, and pair production. The probability of pair production is extremely slim for the radioisotope of ^{137}Cs because the energy of the gamma rays does not exceed twice the rest-mass energy of an electron [17]. Additionally, backscatter interactions are caused by Compton scattering with the gamma rays and the materials surrounding the detector [17]. Therefore, the energy spectra for the radioisotope of ^{137}Cs often show a Compton continuum and peaks by photoelectric absorption and backscatter interaction [17]. The detector suffers, however, from the pulse pileup by overlapping of the events in high dose environments. The pulse pileup can be caused by overlapping of the pulses of Compton scattering and photoelectric absorption, overlapping of the pulses by photoelectric absorption, and overlapping of the pulses of photoelectric absorption and backscatter interaction. The contribution of the pulse pileup in the energy spectra is shown at an energy of the gamma rays higher than the photopeak, as shown in Fig. 9.

Fig. 9a indicates the energy spectra for the radioisotope of ^{137}Cs with a pileup fraction of 52.6%. The energy resolutions of uncorrected, corrected, and rejected were compared to evaluate the correction effect of the pulse pileup. The energy resolution was defined as the full width at half maximum (FWHM) divided by the peak centroid [17]. The energy resolution of uncorrected, corrected, and rejected were 16.9, 13.1, and 15.6%, respectively. The improvement of FWHM at 662 keV at each position were 29, 39, and 7%, respectively after the proposed method was applied.

The results confirmed that the contribution of the photopeak has increased after the correction of the pileup pulse. In this way, the correction of the pileup pulse through the proposed method shows improved energy resolution. The energy resolution of rejected spectrum is improved because all pileup events are corrected. However, the rejection method may increase the statistical noise due to reduction of the throughput rate of the detector system [20], and degrades the energy resolution [12]. Fig. 9b and c shows the energy spectra for the radioisotope of ^{137}Cs with pileup fractions of 49.1 and 32.3%. Similarly, the energy resolution of corrected spectrum was improved compared to the energy resolution of uncorrected spectrum. Table 4 indicates the energy resolution of uncorrected, corrected, and rejected according to the pileup fraction. From the results, we can clearly see that the correction of the pulse pileup by the proposed method prevents

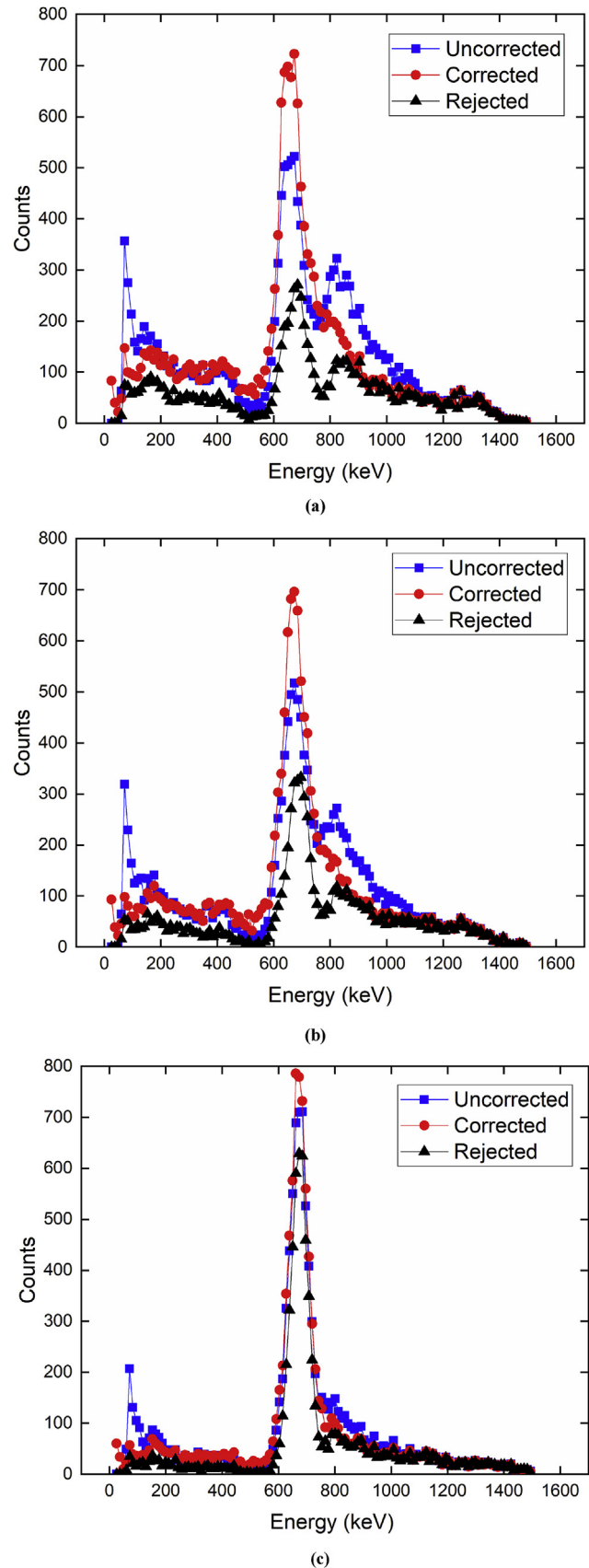


Fig. 9. The gamma energy spectra for the radioisotope of ^{137}Cs with pileup fractions of (a) 52.6, (b) 49.1, and (c) 32.3%. The uncorrected spectrum (blue square), corrected spectrum (red circle), and rejected spectrum (black triangle) were compared to evaluate the correction effect of the pulse pileup. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Table 4

The energy resolution of uncorrected, corrected, and rejected according to the pileup fraction.

| Position | Energy resolution (%) | | | Pileup fraction (%) |
|----------|-----------------------|-----------|----------|---------------------|
| | Uncorrected | Corrected | Rejected | |
| P1 | 16.9 | 13.1 | 15.6 | 52.6 |
| P2 | 16.6 | 12.0 | 12.7 | 49.1 |
| P3 | 11.8 | 11.0 | 10.6 | 32.3 |

distortion of the energy spectrum and improves the energy resolution.

4. Conclusions

In this manuscript, we proposed a new method to correct the pulse pileup in gamma spectroscopy. The core of the proposed method is to correct the start point of the pileup pulse. Only two parameters are required for this. The parameters are obtained from the fitted exponential curve using the information at the peak point of the previous pulse and the information at the start point of the pileup pulse. The amplitude at the corrected start point of the pileup pulse can be estimated by the peak time of the pileup pulse. The peak amplitude of the pulse is computed by subtracting the amplitude at the corrected start point from the amplitude at the peak point. The energy of the gamma rays is estimated by the peak amplitude–energy relationship. As a result, the correction of the pulse pileup by the proposed method prevents distortion of the energy spectrum and improves the energy resolution by 29% at 662 keV at 3 cm source –to detector measurements. The count losses after the proposed method is implemented is around 9% at 3 cm source –to detector measurements.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This research was supported by the KUSTAR-KAIST Institute, Khalifa University of Science & Technology Faculty Start-Up Fund (Grant No. 8474000158), and the Nuclear Research & Development Program of the National Research Foundation of Korea (NRF) grant funded by the Ministry of Science, ICT and Future Planning (MISP) (NRF-2018M2A8A5023361).

Appendix A. Supplementary data

Supplementary data to this article can be found online at

<https://doi.org/10.1016/j.net.2019.12.003>.

References

- [1] Shultis, K. John, Faw, E. Richard, *Fundamentals of Nuclear Science and Engineering*, CRC Press, 2007.
- [2] Daehee Lee, Kyungtaek Lim, Kyungjin Park, Changyeop Lee, Gyuseong Cho, A new cross-detection method for improved energy-resolving photon counting under pulse pile-up, *Nucl. Instrum. Methods Phys. Res. A* 867 (21) (2017) 154–162.
- [3] D. Lee, K. Lim, K. Park, C. Lee, An innovative method to reduce count loss from pulse pile-up in a photon-counting pixel for high flux X-ray applications, *J. Instrum.* 12 (2017) 1–15.
- [4] M. Vencelj, K. Bucar, R. Novak, H.J. Wortche, Event by event pile-up compensation in digital timestamped calorimetry, *Nucl. Instrum. Methods Phys. Res. A* 607 (3) (2009) 581–586.
- [5] Daehee Lee, Kyungjin Park, Kyungtaek Lim, Gyuseong Cho, Energy-correction photon counting pixel for photon energy extraction under pulse pile-up, *Nucl. Instrum. Methods Phys. Res. A* 856 (1) (2017) 36–46.
- [6] Yuichiro Ueno, Isao Takahashi, Takafumi Ishitsu, Takahiro Tadokoro, Koichi Okada, Yasushi Nagumo, Yasutake Fujishima, Yutaka Kometani, Yasuhiko Suzuki, Kikuo Umegaki, Spectroscopic gamma camera for use in high dose environments, *Nucl. Instrum. Methods Phys. Res. A* 822 (2016) 48–56.
- [7] Juan Lanchares, Oscar Garnica, L. Jose, Risco-Martin, J. Ignacio Hidalgo, Regadio Alberto, Real-time evolvable pulse shaper for radiation measurements, *Nucl. Instrum. Methods Phys. Res. A* 727 (1) (2013) 73–83.
- [8] Valentin T. Jordanov, Glenn F. Knoll, Digital synthesis of pulse shapes in real time for high resolution radiation spectroscopy, *Nucl. Instrum. Methods Phys. Res. A* 345 (2) (1994) 337–345.
- [9] Valentin T. Jordanov, Glenn F. Knoll, Alan C. Huber, John A. Pantazis, Digital techniques for real-time pulse shaping in radiation measurements, *Nucl. Instrum. Methods Phys. Res. A* 353 (1994) 261–264.
- [10] Vahid Esmaili-sani, Ali Moussavi-zarandi, Nafiseh Akbar-ashrafi, Behzad Boghrati, Hossein Afarideh, Neutron-gamma discrimination based on bipolar trapezoidal pulse shaping using FPGAs in NE213, *Nucl. Instrum. Methods Phys. Res. A* 694 (2012) 113–118.
- [11] Vahid Esmaili-sani, Ali Moussavi-zarandi, Nafiseh Akbar-ashrafi, Behzad Boghrati, Triangle bipolar pulse shaping and pileup correction based on DSP, *Nucl. Instrum. Methods Phys. Res. A* 665 (2011) 11–14.
- [12] Mohammad-Reza Mohammadian-Behbahani, Shahyar Saramad, Pile-up correction algorithm based on successive integration for high count rate medical imaging and radiation spectroscopy, *Nucl. Instrum. Methods Phys. Res. A* 897 (2018) 1–7.
- [13] S.N. Ahmed, *Physics and Engineering of Radiation Detection*, Academic Press, 2007.
- [14] Xi Wang, Qingguo Xie, I.E.E.E. Member, Yuanbao Chen, Ming Niu, Peng Xiao, Member, IEEE, “advantages of digitally sampling scintillation pulses in pileup processing in PET, *IEEE Trans. Nucl. Sci.* 59 (3) (2012) 498–506.
- [15] M. Nakhostin, Zs Podolyak, P.H. Regan, P.M. Walker, A digital method for separation and reconstruction of pile-up events in germanium detectors, *Rev. Sci. Instrum.* 81 (2010) 1–5.
- [16] Weijun Guo, Robin P. Gardner, Charles W. Mayo, A study of the real-time deconvolution of digitized waveforms with pulse pile up for digital radiation spectroscopy, *Nucl. Instrum. Methods Phys. Res. A* 544 (2005) 668–678.
- [17] Knoll, F. Glenn, *Radiation Detection and Measurement*, John Wiley & Sons, 2010.
- [18] E.H. Everhardt, Noise in photomultiplier tubes, *IEEE Trans. Nucl. Sci.* 14 (2) (1967) 7–14.
- [19] W. Shockley, J.R. Pierce, A theory of noise for electron multiplier, *Proc. IRE* 26 (1938) 321–332.
- [20] Roman Novak, Matjaz Vencelj, Gauss-seidal iterative method as a real-time pile-up solver of scintillation pulses, *IEEE Trans. Nucl. Sci.* 56 (6) (2009) 3680–3687.