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Original Article

A new approach for calculation of the neutron noise of power reactor based on Telegrapher's theory: Theoretical and comparison study between Telegrapher's and diffusion noise

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ABSTRACT

The telegrapher's theory was used to develop a new formulation for the neutron noise equation. Telegrapher's equation is supposed to demonstrate a more realistic approximation for neutron transport phenomena, especially in comparison to the diffusion theory. The physics behind such equation implies that the signal propagation speed is finite, instead of the infinite as in the case of ordinary diffusion. This paper presents the theory and results of the development of a new method for calculation of the neutron noise using the telegrapher's equation as its basis. In order to investigate the differences and strengths of the new method against the diffusion based neutron noise, a comparison was done between the behaviors of two methods. The neutron noise based on SN transport considered as a precision measuring point. The Green's function technique was used to calculate the neutron noise based on telegrapher's and diffusion methods as well as the transport. The amplitude and phase of Green's function associated with the properties of the medium and frequency of the noise source were obtained and their behavior was compared to the results of the transport. It was observed, the differences in some cases might be considerable. The effective speed of propagation for the noise perturbations were evaluated accordingly, resulting in considerable deviations in some cases.

1. Introduction

The neutron noise is the deviation between the time dependent neutron flux and its expected value, assuming that all process is stationary and ergodic in time. The power reactor noise is used for diagnostic purposes as well as surveillance and monitoring in nuclear reactor core. The neutron noise is calculated using the Greens' function technique. The transfer function, Green's function, obtained by this technique depends on the parameters of the unperturbed core, so the derived Green's function is independent of the perturbation type [1]. The neutron noise equations developed by diffusion, Telegrapher's and SN transport theory have different accuracies. More accurate calculation of Green's function and consequently the neutron noise is important in interoperation of detector signals as well as localizing noise source.

The diffusion theory is a widespread approximation because of its simplicity, being adequately realistic and fast enough to allow programing. Thereby, numerous studies focused on the diffusion noise [2–4]. Theory of diffusion have the essential feature that all disturbances, propagate with “infinite” velocity; that is, any disturbance at any point in the system is felt everywhere instantly.

For further study on neutron noise and increasing its validity in heterogeneous media, the neutron noise equation based on SN transport was developed and advantages and disadvantages of its equation were reported [5]. Despite less simplifications and more compatibility of this approximation, there has not been many studies on the neutron transport noise. It is expected, a survey in different approximations to clarify some new aspects in the neutron noise field.

The telegrapher's equation is a time and space dependent, linear partial differential equation. This equation comes from Oliver Heaviside who in the 1880s developed the transmission line theory (also known as the “telegrapher's equations”). The unique feature of this equation is that it describes physical phenomena which exhibit both wavelike characteristics and residual disturbance effects [6]. In this equation, the first order term in *t*, represents the residual disturbance effect, and the second order term in *t*, gives the wave like behavior of the phenomena. The telegrapher's theory has

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been used widely in time dependent electromagnetic, heat transfer and particle transport problems. Recently [7], has reviewed some physics-based derivations of Telegrapher's equation, as well as its mathematical derivation of it and also came up with solutions to one and multi-dimensional Telegrapher's equation based on persistent random walk. The most recent work on Telegrapher's equation is related to the investigation of the effects of resetting mechanisms on random processes that follow the Telegrapher's equation instead of the usual diffusion equation [8].

The use of the Telegrapher's equation in the field of reactor theory was seriously considered after the publication of the [9] article about wavelike behavior of neutrons. Most of the works done based on telegrapher's theory, focused on the development of a neutron population model from the kinetic theory which led to a general approach to the Telegrapher's equation by Ref. [10] for monokinetic neutron wave for the first time [11]. used the two-group one-dimensional Telegrapher's equations to obtain the space-dependent reactor transfer function for a zero-power reactor.

Although the use of the Telegrapher's theory of neutron kinetics models has been relatively widely considered due to its strengths derived from its hyperbolic nature, there have been no significant effort made to use the telegraph theory to calculate neutron noise especially in power reactors. In addition, no research has been found that surveyed the propagation speed of neutron noise. Accordingly, this study was conducted with two main objectives. Initially, the neutron noise based on telegrapher's equation was developed and a comparison was made against the diffusion theory. The SN transport model was used to verify the neutron noise based on telegrapher's equation. Then an assessment was conducted on the effect of nuclear parameters of medium and frequency of noise source on Green's function behavior.

This paper has been divided into five sections. In section 2, the derivation of the neutron noise based on diffusion, Telegrapher's and SN transport in 2 groups is represented. The results of the behavior of Green's function of these methods associated with properties of media and source frequency is given in section 3. In this section, the validity range of Telegrapher's approximation is presented. Section 4 and 5 discuss the differences resulting from the calculation of neutron noise based on the three methods mentioned above.

2. Mathematical formulation

The noise theory is based on the assumption that the stationary fluctuation occurring in the system is small enough, so that the system in average will remain unchanged. This means that the linearized equation can describe such processes, namely the Langevin equation. To obtain the induced neutron noise, all time dependent parameters $X(r,t)$ would be decomposed into a time-independent mean value, $X_0(r)$, which corresponds to the stationary value of the parameter, plus a small time-dependent fluctuation component, reading as follows:

$$X(r,t) = X_0(r) + \delta X(r,t). \tag{1}$$

The standard procedure to obtain the neutron noise equation, based on a transport model (e.g., telegrapher's equation) described by $L(r,\omega)$ is as follows:

- Substituting this generic expression given by last equation in static equations,
- Subtracting the static equation, and neglecting second order terms based on the assumption that these terms play no role in the linear system,
- Eliminating the delayed neutron precursors,
- Finally performing a temporal Fourier transform.

Such a process is demanded to result the following equation

$$\mathbf{L}(r,\omega)\,\delta\varphi(r,\omega) = S(r,\omega), \tag{2}$$

where $\delta\varphi(r,\omega)$ denotes the space- and frequency-dependent neutron noise, $S(r,\omega)$ is noise source and $\mathbf{L}(r,\omega)$ denotes the transport operator which corresponds to the unperturbed state of the system.

Green's function method is a useful mathematical technique for analysis of nonhomogeneous linear differential equations. The Green function obtained by this method is an alternative kernel representing the inverse operator $\mathbf{L}^{-1}$. Utilizing the Green function method for solving Eq. (2), one arrives at the induced neutron noise:

$$\delta\varphi(r,\omega) = \int G(r,r',\omega)S(r',\omega)dr'. \tag{3}$$

This means that, as long as the superposition principle remains valid, the induced neutron noise can be calculated by the Green's function (the so-called dynamic transfer function).

2.1. Green's function based on diffusion equation

The diffusion theory is an approximation for the transport equation based on Fick's law. Since, the diffusion equation is relatively simple, while it preserves enough accuracy of the transport phenomena, it is still widely in use for most nuclear reactor problems (Bell and Glasstone 1970). Eventually, it appears that most of the neutron noise studies are solely based on the diffusion approximation. The noise equation based on the two-group diffusion model could be written as follows [2]:

$$\begin{bmatrix} D_1\Delta - \Sigma_1(\omega) & \nu\Sigma_{f2}\left(1-\dfrac{i\omega\beta}{i\omega+\lambda}\right) \\ \Sigma_{s1\to2} & D_2\Delta - \Sigma_{a2} + \dfrac{i\omega}{\nu_2} \end{bmatrix} \begin{bmatrix} \delta\varphi_1(x,\omega) \\ \delta\varphi_2(x,\omega) \end{bmatrix} = \begin{bmatrix} \delta S_1(x,\omega) \\ \delta S_2(x,\omega) \end{bmatrix} \tag{4}$$

Where

$$\Sigma_1(\omega) \equiv \left(\Sigma_{a1} + \frac{i\omega}{\nu_1}\right) - \nu\Sigma_{f1}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right), \tag{5}$$

and the standard notation were used [2]. Applying the Green's function technique to Eq. (4) we have

$$\begin{bmatrix} D_1\Delta - \Sigma_1(\omega) & \nu\Sigma_{f2}(\omega) \\ \Sigma_{s1\to2} & D_2\Delta - \Sigma_{a2}(\omega) \end{bmatrix} \times \begin{bmatrix} G_{g\to1}(x,x',\omega) \\ G_{g\to2}(x,x',\omega) \end{bmatrix} = \begin{bmatrix} \delta(x-x') \\ 0 \end{bmatrix}_{g=1} \text{or} \begin{bmatrix} 0 \\ \delta(x-x') \end{bmatrix}_{g=2}. \tag{6}$$

where

$$\nu\Sigma_{f2}(\omega)=\nu\Sigma_{f2}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right) \tag{7}$$

$$\Sigma_{a2}(\omega)=\Sigma_{a2}+\frac{i\omega}{\nu_2} \tag{8}$$

With the Green's function of Eq. (6), the neutron noise can be expressed as:

$$\begin{bmatrix}\delta\varphi_1(x,\omega)\\ \delta\varphi_2(x,\omega)\end{bmatrix}=\begin{bmatrix}\int[G_{1\to1}(x,x',\omega)S_1(x',\omega)+G_{2\to1}(x,x',\omega)S_2(x',\omega)]dx'\\ \int[G_{1\to2}(x,x',\omega)S_1(x',\omega)+G_{2\to2}(x,x',\omega)S_2(x',\omega)]dx'\end{bmatrix} \tag{9}$$

2.2. Green's function based on telegrapher's equation

The diffusion equation implies infinite speed of propagation due to its parabolic nature; demanding that particles could propagating from a pulse of neutron source could appear at any distant point instantaneously. It is unphysical as we know that neutrons travel with a finite velocity in nuclear reactor. To overcome this deficiency, parabolic form should be upgraded to a hyperbolic form. The telegrapher's equation is of hyperbolic nature and keeps the speed of particles to a finite value, while it still has certain deviations. The Telegrapher's equation could be written as follows [7].

$$\frac{\partial^2 p}{\partial t^2}+\frac{1}{T}\frac{\partial p}{\partial t}=c^2\nabla^2 p, \tag{10}$$

where $p(r,t)$ is the probability density for the location of diffusing particle at time t, T is a characteristic time and c is a characteristic speed. This equation could be regarded as an interpolation between the wave equation and the diffusion equation. When $T\to\infty$, with the fixed speed c, the telegrapher's equation converts to a wave equation and when both $T\to 0$ and $c\to\infty$ with the constraint that $c^2T\to D$ to be a constant, it reduces to the diffusion equation. The telegrapher's equation could also be derived from the transport equation [12]. When representing the total flux as a function of space and time, it reads as follows:

$$\frac{3D}{\nu^2}\frac{\partial^2}{\partial t^2}\varphi(r,t)+\left(\frac{1+3D\Sigma_a}{\nu}\right)\frac{\partial}{\partial t}\varphi(r,t)=D\nabla^2\varphi(r,t)-\Sigma_a\varphi(r,t)+\left(\frac{3D}{\nu}\frac{\partial}{\partial t}+1\right)S(r,t), \tag{11}$$

where $\varphi(r,t)$ is the neutron flux at point r and time t, $S(r,t)$ is the neutron source strength, D shows the diffusion coefficient, Σ_a is the macroscopic absorption cross section, and ν is the neutron speed. To obtain the induced noise equation, the four steps mentioned in Section 2 were applied. It was assumed that $\frac{1}{\nu}\nabla\left(\frac{1}{\Sigma_t}\frac{\partial J}{\partial t}\right)$ is neglected, which is reasonable approximation. Assuming the two-group Telegrapher's equation as a basis, the neutron noise equation could be derived from telegrapher's equation as follows:

$$\begin{bmatrix} D_1\Delta-\Sigma_1(\omega)\left(1+\frac{i\omega}{\nu_1\Sigma_{t1}}\right) & \nu\Sigma_{f2}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\left(1+\frac{i\omega}{\nu_1\Sigma_{t1}}\right)\\ \Sigma_{s1\to2}\left(1+\frac{i\omega}{\nu_2\Sigma_{t2}}\right) & D_2\Delta-\left(\Sigma_{a2}+\frac{i\omega}{\nu_2}\right)\left(1+\frac{i\omega}{\nu_2\Sigma_{t2}}\right)\end{bmatrix}\times\begin{bmatrix}\delta\varphi_1(x,\omega)\\ \delta\varphi_2(x,\omega)\end{bmatrix}\begin{bmatrix}\delta S_1(x,\omega)\\ \delta S_2(x,\omega)\end{bmatrix}. \tag{12}$$

As it can be seen from Eq. (12), a new parameter as follows

$$1+\frac{i\omega}{\nu_g\Sigma_{tg}}$$

appears in elements of coefficient matrix. It is expected that this parameter to considered as a correcting factor. Applying the Green's function technique to Eq. (12) we have

$$\begin{bmatrix} D_1\Delta-\Sigma_1(\omega)\left(1+\frac{i\omega}{\nu_1\Sigma_{t1}}\right) & \nu\Sigma_{f2}(\omega)\left(1+\frac{i\omega}{\nu_1\Sigma_{t1}}\right)\\ \Sigma_{s1\to2}\left(1+\frac{i\omega}{\nu_2\Sigma_{t2}}\right) & D_2\Delta-\Sigma_{a2}(\omega)\left(1+\frac{i\omega}{\nu_2\Sigma_{t2}}\right)\end{bmatrix}\times\begin{bmatrix}G_{g\to1}\left(x,x',\omega\right)\\ G_{g\to2}\left(x,x',\omega\right)\end{bmatrix}=\begin{bmatrix}\delta\left(x-x'\right)\\ 0\end{bmatrix}_{g=1}\text{or}\begin{bmatrix}0\\ \delta\left(x-x'\right)\end{bmatrix}_{g=2}. \tag{13}$$

With the Green's function of Eq. (13), the neutron noise can be expressed as Eq. (9).

2.3. Green's function based on transport equation

The most fundamental equation in reactor physics is the neutron transport equation. The discrete ordinate method, SN, is one of the oldest and still the most widely used methods in nuclear reactor studies due to its computational time efficiency, easy implementation, and relative accurate results. In this method, the direction of neutron motion divided into a number of ordinates, to each of which is associated a weight. The compatible quadrature set is applied for ordinate discretization, where μ_n implies abscissas for N ordinates in the interval $-1<\mu_n<1$ and w_n is its corresponding weight. The resulting equations for the neutron noise in SN transport method is given in Eq. (14) [5].

$$\Omega.\nabla\begin{bmatrix}\delta\psi_{n1}\\ \delta\psi_{n2}\end{bmatrix}-\begin{bmatrix}\frac{i\omega}{\nu_1}+\Sigma_{t1} & 0\\ 0 & \frac{i\omega}{\nu_2}+\Sigma_{t2}\end{bmatrix}\begin{bmatrix}\delta\psi_{n1}\\ \delta\psi_{n2}\end{bmatrix}-\sum_{l=0}^{L}(2l+1)P_l(\mu_n)\begin{bmatrix}\Sigma_{sl,1\to1} & 0\\ \Sigma_{sl,1\to2} & \Sigma_{sl,2\to2}\end{bmatrix}\begin{bmatrix}\delta\varphi_1^l\\ \delta\varphi_2^l\end{bmatrix}$$
$$-\begin{bmatrix}\nu\Sigma_{f1}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right) & \nu\Sigma_{f2}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta\varphi_1\\ \delta\varphi_2\end{bmatrix}=\begin{bmatrix}\delta S_{n1}(x,\omega)\\ \delta S_{n2}(x,\omega)\end{bmatrix}, \tag{14}$$

where ψ is angular flux; $\varphi_g=\sum_{n=1}^{N}w_n\psi_{ng}$ and $\varphi_g^l=\sum_{n=1}^{N}w_nP_l(\mu_n)\psi_{ng}$ are scalar flux and moments of scalar flux respectively. The symbols g and g' represent the neutron energy group indexes; and index n designates ordinate. Using the Green's function technique to solve Eq. (14) one finds that:

$$\Omega.\nabla\begin{bmatrix}G_{ns\to n,g\to1}(\mathrm{x},\mathrm{x}',\omega)\\ G_{ns\to n,g\to1}(\mathrm{x},\mathrm{x}',\omega)\end{bmatrix}-\begin{bmatrix}\frac{i\omega}{V_1}+\Sigma_{t1} & 0\\ 0 & \frac{i\omega}{V_2}+\Sigma_{t2}\end{bmatrix}\begin{bmatrix}G_{ns\to n,g\to1}(\mathrm{x},\mathrm{x}',\omega)\\ G_{ns\to n,g\to1}(\mathrm{x},\mathrm{x}',\omega)\end{bmatrix}$$
$$-\sum_{l=0}^{L}(2l+1)P_l(\mu_n)\begin{bmatrix}\Sigma_{sl,1\to1} & 0\\ \Sigma_{sl,1\to2} & \Sigma_{sl,2\to2}\end{bmatrix}\begin{bmatrix}\sum_n w_nP_l(\mu_n)G_{ns\to n,g\to1}(\mathrm{x},\mathrm{x}',\omega)\\ \sum_n w_nP_l(\mu_n)G_{ns\to n,g\to2}(\mathrm{x},\mathrm{x}',\omega)\end{bmatrix} \tag{15}$$
$$-\begin{bmatrix}\nu\Sigma_{f1}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right) & \nu\Sigma_{f2}\left(1-\frac{i\omega\beta}{i\omega+\lambda}\right)\\ 0 & 0\end{bmatrix}\begin{bmatrix}\sum_n w_nG_{ns\to n,g\to1}(\mathrm{x},\mathrm{x}',\omega)\\ \sum_n w_nG_{ns\to n,g\to2}(\mathrm{x},\mathrm{x}',\omega)\end{bmatrix}=\begin{bmatrix}\delta(\mathrm{x}-\mathrm{x}')\\ 0\end{bmatrix}_{g=1,ns=n}\ or\ \begin{bmatrix}0\\ \delta(\mathrm{x}-\mathrm{x}')\end{bmatrix}_{g=2,ns=n},$$

where ns index represents ordination and $G_{gs\to g}(\mathrm{x},\ \mathrm{x}',\ \omega)=\sum_{ns}\sum_{n}w_nG_{ns\to n,gs\to g}(\mathrm{x},\mathrm{x}',\omega)$.

The neutron noise is calculated from the obtained Green's function, from Eq. (15), and knowing the noise source. The resulting equation is presented in Eq. (16).

$$\begin{bmatrix}\delta\psi_{n,1}(\mathrm{x},\omega)\\ \delta\psi_{n,2}(\mathrm{x},\omega)\end{bmatrix}=\begin{bmatrix}\int\left[\sum_{ns}\left(G_{ns\to n,1\to1}(\mathrm{x},\mathrm{x}_0,\omega)\delta S_{n1}(\mathrm{x}_0,\omega)+G_{ns\to n,2\to1}(\mathrm{x},\mathrm{x}_0,\omega)\delta S_{n2}(\mathrm{x}_0,\omega)\right)\right]dx_0\\ \int\left[\sum_{ns}\left(G_{ns\to n,1\to2}(\mathrm{x},\mathrm{x}_0,\omega)\delta S_{n1}(\mathrm{x}_0,\omega)+G_{ns\to n,2\to2}(\mathrm{x},\mathrm{x}_0,\omega)\delta S_{n2}(\mathrm{x}_0,\omega)\right)\right]dx_0\end{bmatrix} \tag{16}$$

It is obvious that $\delta\varphi_g(\mathrm{x},\omega)=\sum_n\delta\psi_{n,1}(\mathrm{x},\omega)\ \ w_n$.

3. The impact of constant coefficients on Green's function amplitude

Three equations were obtained based on diffusion (section 2.1), Telegrapher's (section 2.2) and Sn transport equation (section 2.3) to calculate Green's function. Despite the similarity in the general form of the matrix, the elements of the coefficient matrix obtained from these methods are not identical in mathematical formulation. The value of the Green's function is a complex number. So, In order to evaluate the behavior of the Green's function based on the three methods mentioned in Section 2, the amplitude and the phase of it must be considered simultaneously.

To make a comparison between the three methods, an evaluation of the impact of the constant coefficients on Green's function amplitude in the wide range of physical parameters (neutron

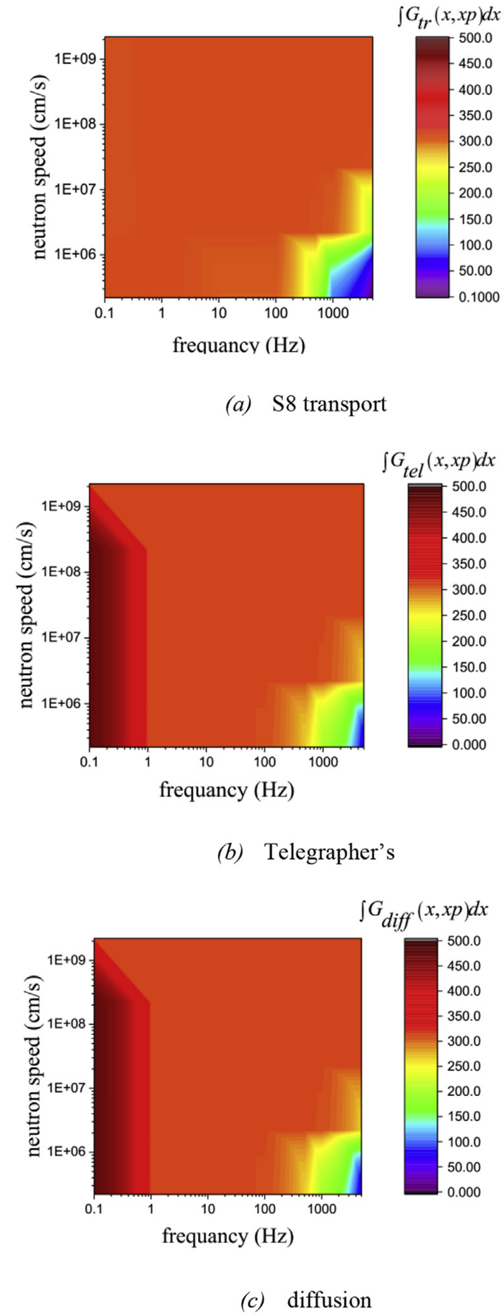


Fig. 1. The integral of Green's function amplitude based on SN transport, Telegrapher's and diffusion associated with the neutron speed and the frequency of source

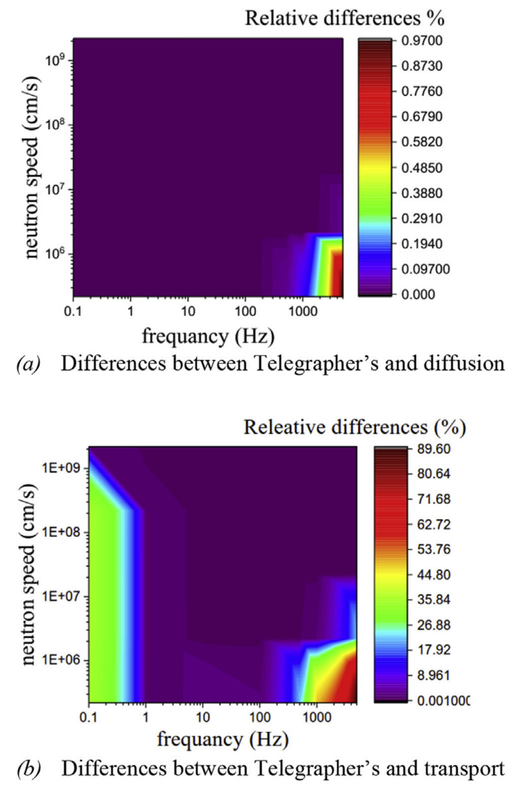


Fig. 2. Relative Differences between methods.

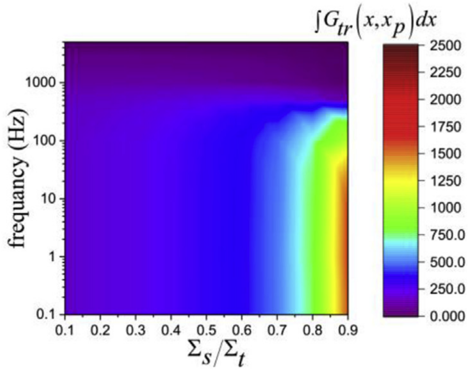
energy and frequency of the source of the neutron noise) for diffusion, Telegrapher's and S8 noise equation was done. In this relation, the Green's function amplitude in the energy range from 0.025 eV (associated with 2.2 e5 cm/s thermal neutron speed), to 2 MeV (associated with 2e9 cm/s fast neutron speed), and different frequency from 0.1 to 5000 was investigated. In order to study the effect of the changes in velocity (energy) and frequency, on the Green's function behavior; a large medium, with $\Sigma_s = \upsilon\Sigma_f$ was considered to minimize the boundary conditions effect. The results obtained of these methods are shown in Fig. 1:

As can be seen from Fig. 1, the area under the Green's function curve $\int G(x, x_p)dx$, in the wide range of the frequency and neutron speed has slight variations. But as the frequency of noise source increases to over 1000 and the neutron speed decreases to the thermal neutron speed, the area under the Green's function varies extremely. These variations in $\int G(x, x_p)dx$ indicate high sensitivity to speed and frequency in low neutron speed, (thermal neutron range) and high frequency (more than 1000 Hz). Also as can be seen in Fig. 1, there are notable changes in low frequency, 0.1 Hz in both Telegrapher's and diffusion noise equation, indicating a high sensitivity of the diffusion and telegrapher's to noise frequencies less than 0.1 Hz, while such sensitivity isn't observed in transport noise equation.

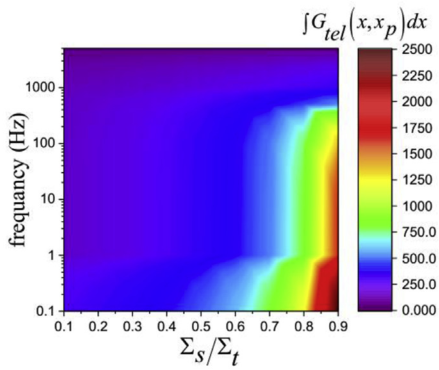
For better clarification, the differences between the area under the Green's function for Telegrapher's and diffusion and for Telegrapher's and transport is demonstrated in Fig. 2. According to Fig. 2, the differences between diffusion and Telegrapher's is under 0.1% in wide range of frequency and speed of neutron, but the relative difference approaches 1% in thermal neutron speed and frequency beyond 1000 Hz. Also the differences between transport and Telegrapher's approaches 35% for frequencies less than 1 Hz and 90% for frequencies beyond 1000 Hz and in thermal speed of neutron respectively.

Behavior of Green's function relative to scattering ratio Σ_s/Σ_t of media and frequency of noise source investigated in two cases, thermal and fast speed of neutron. Like the example previously stated, a large medium with $\Sigma_t = 1 \ cm^{-1}$ was considered. The value of $\int G(x, x_p)dx$ is calculated in different frequency and different scattering ratio in two case of fast and thermal neutron. The results obtained of three methods are shown in Fig. 3.

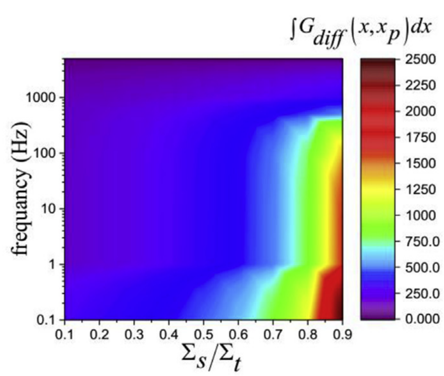
As can be seen in Figs. 3 and 4, the Telegrapher's and diffusion noise is of same behavior in amplitude for space independent problems. In these two methods, the changes of area under Green's function is noticeably different in low frequency, 0.1 Hz. These results reveal that behavior of integral value of Green's function is different in thermal and fast speed of neutron. In thermal speed of neutron, as the frequency increases, the integral of Green's function amplitude decreases. In contrast, the integral of Green's function



(a) The Amplitude of integral value of Green's function based on SN transport

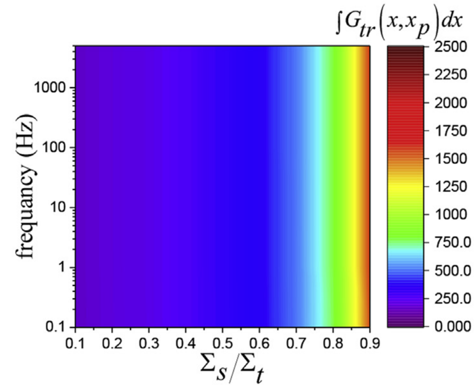


(b) The Amplitude of integral value of Green's function based on telegrapher's

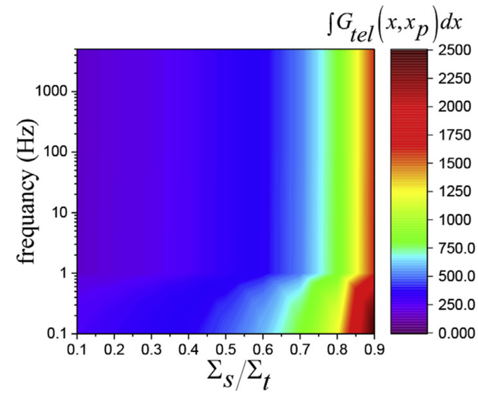


(c) The Amplitude of integral value of Green's function based on diffusion

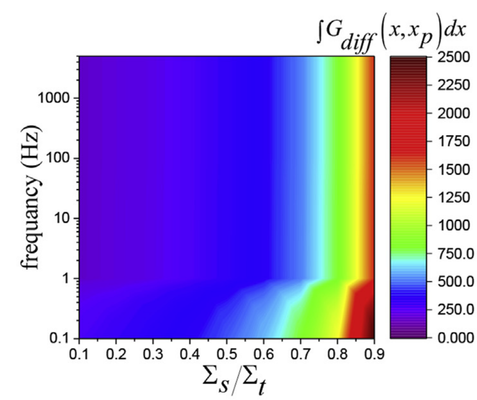
Fig. 3. The Amplitude of integral value of Green's function associated with $\Sigma_s/\ \Sigma_t$ and the frequency of source in the thermal speed of neutron



(a) The Amplitude of integral value of Green's function based on SN transport



(b) The Amplitude of integral value of Green's function based on telegrapher's

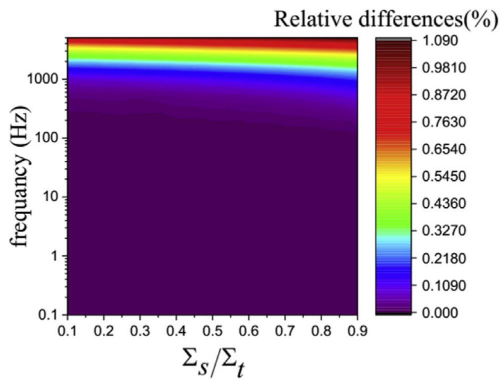


(c) The Amplitude of integral value of Green's function based on diffusion

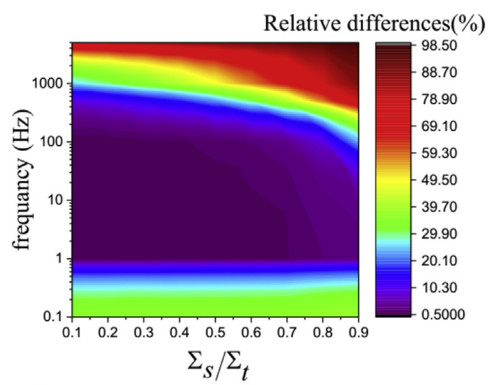
Fig. 4. The Amplitude of integral value of Green's function associated with Σ_s/Σ_t and the frequency of source in the fast speed of neutron

amplitude changes slightly associated with frequency in fast speed of neutron.

The differences between diffusion and Telegrapher's and also between transport and Telegrapher's are presented in Figs. 5 and 6 for thermal and fast speed of neutron. At thermal speed of neutron, the relative difference between diffusion and Telegrapher's reaches 1% on frequency more than 1000 Hz; the relative difference between transport and Telegrapher's illustrate a significant differences in frequencies less than 1 Hz and more than 1000 Hz. By increasing of scattering ratio (Σ_s/Σ_t), the relative difference intensified. At fast speed of neutron, the relative difference between diffusion and Telegrapher's is noticeable on frequencies lower than 1 Hz and reaches around 38% on 0.1 Hz. The differences between diffusion and Telegrapher's are negligible, lower than 0.001% for fast speed of neutron, so the figure is not provided. At fast and thermal speed of neutron, the relative difference rises by increasing Σ_s/Σ_t.



(a) Relative differences between diffusion and Telegrapher's



(b) Relative differences between transport and Telegrapher's

Fig. 5. Relative differences between represented methods in thermal speed of neutron.

4. The Green's function based on diffusion, telegrapher's and transport equation in extremely vacuum media

Different methods used to develop noise equation can lead to different accuracy in results of neutron noise. Differences between neutron noise equations based on diffusion and transport equation asserted in Ref. [5]. In this section, a comparison between neutron noise based on diffusion, Telegrapher's and transport was performed. A heterogeneous media was considered that includes 3 regions; it is demonstrated in Fig. 7. The middle region is of near

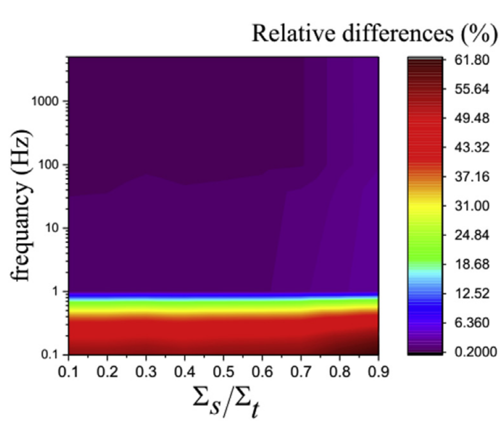


Fig. 6. Relative differences between Telegrapher's and transport in fast speed of neutron.

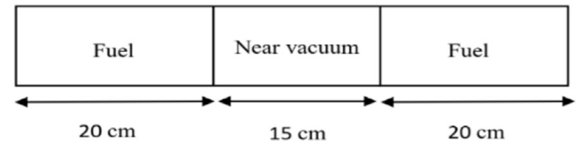


Fig. 7. Arrangement of 3-region problem.

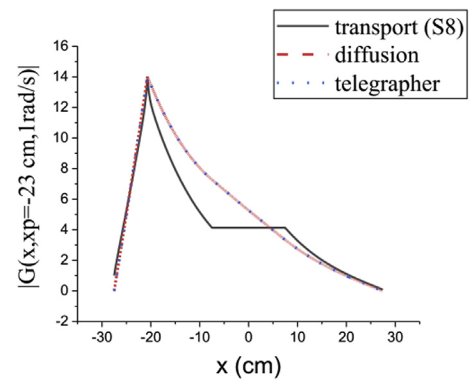


Fig. 8. Space dependent of the Green's function amplitude obtained by S8 transport, diffusion and Telegrapher's

vacuum properties, the total cross section equal 0.0001 cm^{-1}. Two other fissile regions have same properties, $\Sigma_t = 1 \quad cm^{-1}$ and $\nu\Sigma_f = \Sigma_s$. The amplitude and phase of green's function obtained by diffusion, Telegrapher's and transport (S8) represented in Figs. 8 and 9 respectively.

As it was expected, there is not any factor to change the amplitude and phase in near vacuum region. Transport noise predict this behavior correctly, in contrast, diffusion and Telegrapher's noise both failed.

5. The speed of neutron noise propagation based on diffusion, Telegrapher's and transport equation

The diffusion equation is a parabolic equation; the disadvantage of the parabolic equation is that it predicts the particle velocity as infinite. The Telegrapher's equation has a hyperbolic nature that predict the finite velocity. In order to obtain the propagation speed, the phase differences was calculated based on diffusion, Telegrapher's and transport equation and accordingly propagation

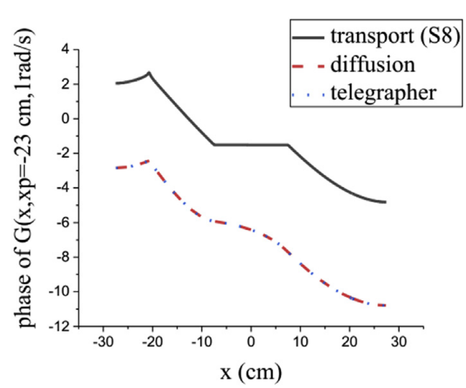


Fig. 9. Space dependent of the Green's function phase obtained by S8 transport, diffusion and Telegrapher's

Table 1
The speed of propagation calculated by diffusion, Telegrapher's and SN transport in the thermal speed of neutron, 1000 Hz and $\Sigma_t = 0.001 \quad cm^{-1}$.

Method	Diffusion	Telegrapher's	S2	S4	S8	S12	S16
Speed of propagation $[cm/s]$	1.82e5	7.45e4	5.73e4	3.90e4	2.81e4	2.38e4	2.24e4

Table 2
The speed of propagation calculated by diffusion, Telegrapher's and SN transport in the fast speed of neutron, 1000 Hz and $\Sigma_t = 0.001 \quad cm^{-1}$.

Method	Diffusion	Telegrapher's	S2	S4	S8	S12	S16
Speed of propagation $[cm/s]$	2.08e8	2.08e8	1.06e8	8.91e7	1.12e8	3.78e7	3.61e7

speed is achieved.

Heizler (2010) asserts that the angular dependence of the Boltzmann equation like PN and SN reaches the exact solution when N goes to large integer, so it is expected that the real propagation speed is obtained. The Speed of propagation is a function of neutron speed, frequency of noise source, cross sections, decay constant of precursors and delayed neutron fraction. However, neutron speed and frequency of noise source have the greatest impact among these factors. Two examples are given to compare the propagation speed in relatively large media. In both example, the frequency of noise source is 1000 Hz and the properties of media is $\Sigma_t = 0.001 \quad cm^{-1}$ and $\nu\Sigma_f = \Sigma_s$. In the first example, the speed of neutron is 2.2e5 cm/s and in the second one is 2e9 $cm/\ s$. The speed of propagation obtained from diffusion, Telegrapher's and transport is represented in Tables 1 and 2.

The distinction between diffusion and Telegrapher's is obvious when the value of $\omega/\nu\Sigma_t$ is noticeable. In the first example, thermal media, the value of $\omega/\nu\Sigma_t$ is considerable and the difference between propagation speed of Telegrapher's and diffusion is clear. However, there is a good agreement between results of Telegrapher's and S2 approximation. In the second example, fast media, there is not any distinction between the results.

As the results are obvious, the speed of propagation obtained from transport are less than diffusion approximation.

6. Discussion and conclusion

In this paper, the neutron noise is derived based on the Telegrapher's equation for the first time and a comparison is made between this method and the diffusion method. The neutron noise based on transport equation is used as a performance check criterion for Telegrapher's neutron noise. Although the transport theory is an accurate method for calculating neutron noise, but the complexity and time consuming of its calculations require other methods such as diffusion and telegrapher's approximation. On the other hand, even though the diffusion equation is simple and fast method for calculating neutron noise but its precision and performance decreases significantly near the noise source and in problems with small dimensions (less than 100 mean free path), high noise frequencies, and high heterogeneity. So the Telegrapher's neutron noise could be used as an alternative to diffusion equation when a fast and more reliable method is required to identify induced neutron noise. In order to investigate the capabilities of the Telegrapher's approximation in calculating neutron noise, Telegrapher's noise behavior was considered in three case:

1. Large media, wide range of noise frequency, neutron velocity and scattering ratio
2. Extremely vacuum media
3. Noise propagation speed

First the behavior of the green's function associated with frequency, speed of neutron and scattering ratio was surveyed. Although there is no significant differences between behaviors of the Green's function based on diffusion and Telegrapher's, the incompatibility was observed between diffusion and Telegrapher's with transport method in some cases.

In high frequency, more than 1000 Hz, and low speed of neutron, speed of thermal neutron, the amplitude and width of the Green's function curve collapse dramatically. In low frequency, 0.1 Hz or less, the Green's function based on Telegrapher's and diffusion method increases noticeably whilst this behavior does not appear in transport method. It was also shown that the Green's function rises with the scattering property of medium. With the increasing of the proportion of scattering cross section to total cross section, the rate of variation for Green's function was intensified.

Second, One distinction observed between these methods was the different ability to obtain the actual neutron noise in a medium with extremely small cross sections. In such a medium, changes on amplitude and phase of Green's function should be negligible. However, Telegrapher's and diffusion method do not provide actual response and considerable changes in amplitude and phase observed.

Third, the propagation speed of noise improved by telegrapher's and transport methods. When the term $\omega/\nu\Sigma_t$ is significant, the Telegrapher's noise result approaches S2 transport noise results, otherwise it is approximately identical to diffusion noise result. The findings of this study suggest that the telegrapher's method tends to S2 in calculating propagation speed provided that the term $\omega/\nu\Sigma_t$ is significant.

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