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Original Article

The first application of modified neutron source multiplication method in subcriticality monitoring based on Monte Carlo

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ABSTRACT

The control rod drive mechanism needs to be debugged after reactor fresh fuel loading. It is of great importance to monitor the subcriticality of this process accurately. A modified method was applied to the subcriticality monitoring process, in which only a single control rod cluster was fully withdrawn from the core. In order to correct the error in the results obtained by Neutron Source Multiplication Method, which is based on one point reactor model, Monte Carlo neutron transport code was employed to calculate the fission neutron distribution, the iterated fission probability and the neutron flux in the neutron detector. This article analyzed the effect of a coarse mesh and a fine mesh to tally fission neutron distributions, the iterated fission probability distributions and to calculate correction factors. The subcriticality before and after modification is compared with the subcriticality calculated by MCNP code. The modified results turn out to be closer to calculation. It's feasible to implement the modified NSM method in large local reactivity addition process using Monte Carlo code based on 3D model.

1. Introduction

The control rod drive mechanism needs to be debugged after fresh fuel loading in pressurized water reactor (PWR). In this process, a single control rod cluster is fully withdrawn from the fully inserted state. Large reactivity is locally added. So, it is necessary to monitor the subcriticality of the nuclear reactor to ensure the safety of the reactor core. Since the startup physical experiment hasn't been carried out, accurate subcriticality monitoring of the process above is of great importance.

The method used to monitor the subcriticality is neutron source multiplication method (NSM method) which is based on one point reactor model. This model assumes that the neutron flux distribution in the reactor core is the fundamental mode of the neutron diffusion equation, and it keeps the same shape in different subcritical states. However, in real situation, the higher modes, spatial effect and the disturbance of neutron importance field could affect the result in the measurement of the subcriticality. When large reactivity is added locally, the neutron importance distribution and the fundamental mode distribution would change heavily. At the same time, the higher modes of neutron source would affect the detection of neutron signal.

In order to reduce these effects, Tsuji [1] proposed a modified NSM method based on fundamental mode extraction, and proved its validity by calculation. Then, based on Tsuji's work, Win Naing [2] applied this method to PWR, when large reactivity is inserted homogenously by control rod bank in the reactor core. They investigated the sensitivity of neutron source distribution on the modified results. Truchet [3] made research on the modification of NSM method applied to small local reactivity insertion process in prototype fast breeder reactor Monju, by measuring the subcriticality when the central control rod is inserted at different height. The results obtained by modified NSM method agree well with those obtained by other kinetic method. In the research above, the modified NSM method is performed by deterministic code. These research verified the feasibility of the modified NSM method in subcritical system with large homogenous reactivity addition and small local reactivity addition. But the feasibility to implement the modified NSM method in large local reactivity addition process based on Monte Carlo has not been verified.

In the control rod drive mechanism debugging process, since the startup physical experiment has not been carried out, the critical control rod position is unknown, it is impossible to use the critical state or any other subcritical states close to critical as the reference state. Besides, the withdrawn of the control rod is strictly limited in

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this process, only control rod all in and a single control rod cluster out state are allowed. Thus, the control rod all in state is the only choice as the reference state to monitor the subcriticality in this process.

So, in order to modify the subcriticality of deep subcritical state as accurately as possible, we need to calculate the physical parameters such as the fission neutron distribution, the iterated fission probability and the neutron flux in the neutron detector as accurately as possible.

Since most of the modification work has been done through deterministic method calculation code, and good results are obtained. It is a trial to perform the modification through Monte Carlo method. Besides, subcriticality monitoring is always performed in some deep subcritical states, in which core configuration is changing, such as, fuel loading and fuel unloading, etc. Since the Monte Carlo code is more precise in calculation modeling, and it's results are always considered as benchmark, it is more likely to get satisfactory physical parameter calculation results of these subcritical states using high-fidelity Monte Carlo calculation model. Although more calculation time is needed, it is worth trying.

However, the calculation accuracy and mesh accuracy are limited by the calculation efficiency of Monte Carlo code, which is much higher in the calculation of deterministic code. It is necessary to investigate the sensitivity of the mesh accuracy of Monte Carlo code in the modification and the accuracy of the modification performed in different mesh accuracy.

In summary, the feasibility to implement the modified NSM method in large local reactivity addition process using Monte Carlo code has not been verified in literature and this article will carry out an in-depth study of it.

In this article, the modified NSM method was applied to monitor the subcriticality of control rod drive mechanism debugging process, in which large reactivity is locally added. By modifying the experimental results with calculation, we evaluated the accuracy of the modified NSM method applying to large local reactivity addition process using Monte Carlo code based on 3D model. Sensitivity analysis was carried out using Monte Carlo code on different mesh accuracy. The feasibility to implement the modified NSM method in large local reactivity addition process using Monte Carlo code based on 3D model was verified.

2. Modified method of subcriticality monitoring

From the steady state neutron diffusion equation with an external neutron source and the neutron diffusion equation defined as an eigenvalue problem:

$$(F-M)\phi = S, \tag{1}$$

$$M\phi_i^{\lambda} = \frac{1}{k_i} F\phi_i^{\lambda} \qquad i = 1,2,\cdots,\infty, \tag{2}$$

where M is the neutron destruction operator, F is the production operator, ϕ refers to the neutron flux, S refers to the external neutron source. k_i refers to the i-th eigenvalue and ϕ_i^{λ} refers to the corresponding eigenfunction. The maximum eigenvalue of equation (2) k_1 is considered to be the neutron multiplication factor k_{eff}, and ϕ_1^{λ} is the fundamental mode.

For equation (2), the adjoint equations are defined as:

$$M^*\phi_i^{\lambda^*} = \frac{1}{k_i} F^*\phi_i^{\lambda^*} \qquad i = 1,2,\cdots,\infty, \tag{3}$$

where M^* and F^* are the adjoint operators for M and F; $\phi_i^{\lambda^*}$ is the adjoint eigenfunction of the i-th eigenvalue k_i.

Here we use the assumption that the eigenfunctions form a complete, orthogonal basis.The orthogonal relationship between ϕ_i^{λ} and $\phi_i^{\lambda^*}$ is as follows [1,2,5].

$$\left(\phi_i^{\lambda^*}, F\phi_j^{\lambda}\right) = 0 \qquad if \;\; i \neq j, \tag{4}$$

$$\left(\phi_i^{\lambda^*}, F\phi_j^{\lambda}\right) \neq 0 \qquad if \;\; i = j. \tag{5}$$

ϕ of equation (1) can be expanded with the eigenfunctions of equation (2) as follows:

$$\phi = \sum_{i=1}^{\infty} a_i \phi_i^{\lambda}. \tag{6}$$

Multiplication of equation (6) with $\phi_j^{\lambda^*}F$ and integrating with respect to space and energy (written in scalar product notation) gives:

$$\left(\phi_j^{\lambda^*}, F\phi\right) = \sum_{i=1}^{\infty} a_i \left(\phi_j^{\lambda^*}, F\phi_i^{\lambda}\right). \tag{7}$$

From equations (4) and (5), equation (7) could be written as:

$$\left(\phi_j^{\lambda^*}, F\phi\right) = a_j \left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right), \tag{8}$$

$$a_j = \frac{\left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right)}{\left(\phi_j^{\lambda^*}, F\phi\right)}. \tag{9}$$

Substitute equation (6) into equation (1). Then, multiplication of equation (1) with $\phi_j^{\lambda^*}$ and integrating with respect to space and energy (written in scalar product notation) gives:

$$\sum_{i=1}^{\infty} a_i \left(\phi_j^{\lambda^*}, M\phi_i^{\lambda}\right) = \sum_{i=1}^{\infty} a_i \left(\phi_j^{\lambda^*}, F\phi_i^{\lambda}\right) + \left(\phi_j^{\lambda^*}, S\right). \tag{10}$$

From equations (4) and (5), equation (10) could be written as:

$$a_j \left(\phi_j^{\lambda^*}, M\phi_j^{\lambda}\right) = a_j \left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right) + \left(\phi_j^{\lambda^*}, S\right), \tag{11}$$

$$a_j \frac{\left(\phi_j^{\lambda^*}, M\phi_j^{\lambda}\right)}{\left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right)} = a_j \frac{\left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right)}{\left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right)} + \frac{\left(\phi_j^{\lambda^*}, S\right)}{\left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right)}, \tag{12}$$

$$a_j = \frac{k_j}{1-k_j} \frac{\left(\phi_j^{\lambda^*}, S\right)}{\left(\phi_j^{\lambda^*}, F\phi_j^{\lambda}\right)}. \tag{13}$$

By substituting equation (13) into equation (6), the fundamental mode of ϕ could be described as:

$$\phi_1 = \frac{-1}{\rho} \frac{\left(\phi_1^{\lambda^*}, S\right)}{\left(\phi_1^{\lambda^*}, F\phi_1^{\lambda}\right)} \phi_1^{\lambda}. \tag{14}$$

By substituting equation (9) into equation (6), the fundamental mode of ϕ also could be described as:

$$\phi_1 = \frac{\left(\phi_1^{\lambda^*}, F\phi\right)}{\left(\phi_1^{\lambda^*}, F\phi_1^{\lambda}\right)} \phi_1^{\lambda}. \tag{15}$$

From (14) and (15):

$$\frac{(\phi_1^{\lambda^*}, F\phi)}{(\phi_1^{\lambda^*}, F\phi_1^{\lambda})}\phi_1^{\lambda} = \frac{-1}{\rho}\frac{(\phi_1^{\lambda^*}, S)}{(\phi_1^{\lambda^*}, F\phi_1^{\lambda})}\phi_1^{\lambda}, \tag{16}$$

$$\rho = -\frac{(\phi_1^{\lambda^*}, S)}{(\phi_1^{\lambda^*}, F\phi)}. \tag{17}$$

The neutron detector signal could be described as:

$$N = \Sigma\phi(r_d). \tag{18}$$

Equation (17) could be written as:

$$\rho = -\frac{(\phi_1^{\lambda^*}, S)}{(\phi_1^{\lambda^*}, F\phi)}\frac{\Sigma\phi(r_d)}{N}. \tag{19}$$

For the subcritical states "0" and "1" considered as the initial and final states in a subcritical reactor, the subcriticality are described as follows:

$$\rho_0 = \frac{-\left(\phi_{0,1}^{\lambda^*}, S_0\right)}{\left(\phi_{0,1}^{\lambda^*}, F_0\phi_0\right)}\cdot\frac{\Sigma_0\phi_0(r_d)}{N_0}, \tag{20}$$

$$\rho_1 = \frac{-\left(\phi_{1,1}^{\lambda^*}, S_1\right)}{\left(\phi_{1,1}^{\lambda^*}, F_1\phi_1\right)}\cdot\frac{\Sigma_1\phi_1(r_d)}{N_1}. \tag{21}$$

If ρ_0 is known, ρ_1 could be described as:

$$\rho_1 = \frac{\left(\phi_{1,1}^{\lambda^*}, S_1\right)}{\left(\phi_{0,1}^{\lambda^*}, S_0\right)}\frac{\left(\phi_{0,1}^{\lambda^*}, F_0\phi_0\right)}{\left(\phi_{1,1}^{\lambda^*}, F_1\phi_1\right)}\frac{\Sigma_1\phi_1(r_d)}{\Sigma_0\phi_0(r_d)}\frac{N_0}{N_1}\rho_0. \tag{22}$$

The subcriticality of a certain state can also be expressed as:

$$\rho_1 = C\frac{N_0}{N_1}\rho_0, \tag{23}$$

Where:

$$C = \frac{\left(\phi_{1,1}^{\lambda^*}, S_1\right)}{\left(\phi_{0,1}^{\lambda^*}, S_0\right)}\frac{\left(\phi_{0,1}^{\lambda^*}, F_0\phi_0\right)}{\left(\phi_{1,1}^{\lambda^*}, F_1\phi_1\right)}\frac{\Sigma_1\phi_1(r_d)}{\Sigma_0\phi_0(r_d)}, \tag{24}$$

where $F\phi$ is the fission neutron distribution, ϕ^{λ^*} is the adjoint flux distribution, $\phi(r_d)$ is the neutron flux at the neutron detector, S is the neutron source distribution. Σ is the efficiency of the neutron detector, which is considered to be the same in most of the case-sand (a, b) refers to scalar product.

In Eq. (23), the correction factor C is determined by numerical calculations in advance, ρ_0 could be determined by numerical calculations or evaluated with experimental method(it's determined by numerical calculations in this research because of the characteristic of the monitoring process), and N is given with measured neutron count rates.

Eq. (24) is the correction factor. As can be seen from this equation, when the neutron source distribution is known, only fission neutron distribution, adjoint flux distribution and neutron flux at the detector position need to be calculated.

3. Application to PWR based on 3D model

In this study, application to PWR control rod drive mechanism debugging process was carried out. Reactivity is locally added in the core by withdrawing a single control rod cluster. Five control cod clusters are chosen in this research, as shown in Fig. 1. Each time, only a single control rod cluster was fully withdrawn from the fully inserted state. In-core Cf-252 neutron sources "P" in the core is located around the control rod. BF_3 detector is used to measure the neutron count rate, which is located outside of the reactor core, drawn by a circle with character "D" in Fig. 1.

Since the startup physical experiment has not been carried out, the critical control rod position is unknown. So, the rod all in state is chosen to be the reference state. The flow diagram of the modified subcriticality monitoring method is shown in Fig. 2.

First, calculate the subcriticality of the reference state and the correction factors of the subcritical states to be measured. Second, measure the neutron count rate of the reference state with neutron detector. Third, change the control rod position, and measure the neutron count rate of a certain subcritical state to be measured. Fourth, calculate the subcriticality with Eq. (23). At last, repeat step 3–4 until all of the subcritical states are measured.

3.1. Calculation model

In this study, all of the physical parameters in Eq. (24) were calculated by MCNP code, including: fission neutron distribution, adjoint flux distribution and neutron flux at the detector position. Considering the calculation cost, the calculation were performed in 2 ways of mesh accuracy: (1) coarse mesh: assembly-wise, (2) relatively fine mesh: assembly-wise in radial and 10 cm mesh grid in axial.

The actual calculation geometry is shown in Fig. 3. In this study, The 3D calculation model was verified by physical start up test data. Since the detector is far from the core, the efficiency of the neutron detector Σ is considered to be unchanged in all of the states, the neutron flux at detector position $\phi(r_d)$ appears as ratio in Eq. (24), the neutron detector in the model is simplified as a cylinder with its shell around it.

In real application, since the startup physical experiment has not been carried out, the critical control rod position and other physical parameters are unknown, verification of calculation model need to be done by other data, such as, experiment data of the same type reactor or calculation data obtained by other calculation code

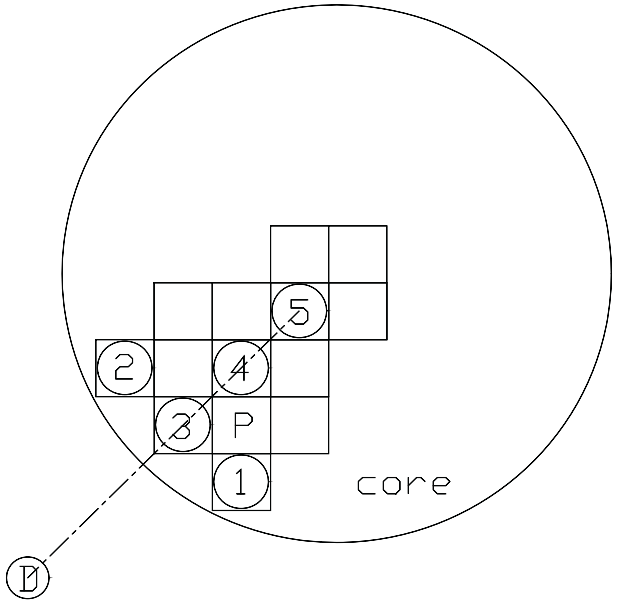

Fig. 1. Diagrammatic drawing of the core configuration.

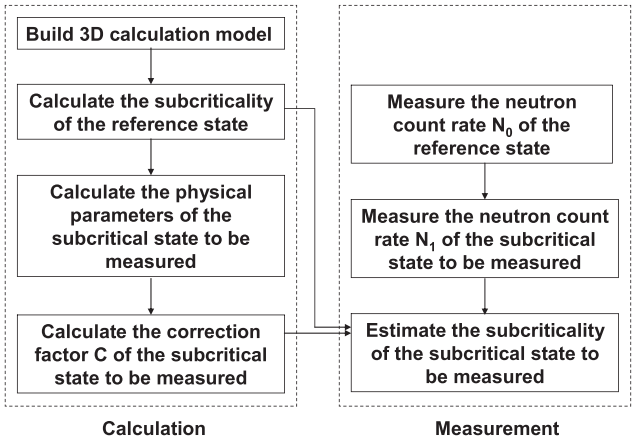

Fig. 2. Flow diagram of the modified method.

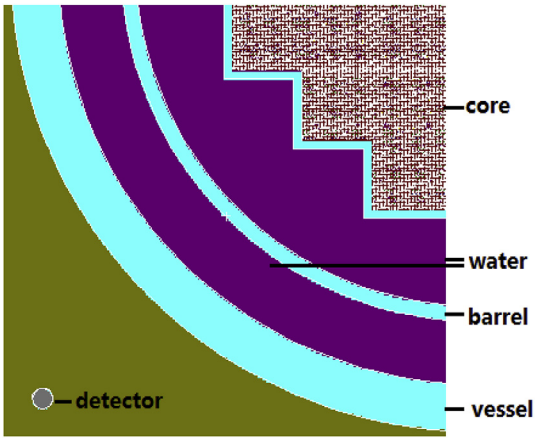

Fig. 3. The actual calculation geometry.

calculated in high precise calculation model.

3.2. Calculation of physical parameters

3.2.1. Fission neutron distribution

The calculation of fission neutron distribution $F\phi$ was realized by neutron flux tally card and tally multiplier card. Since fission reaction only occurs in the fuel pellet, only fuel pellet region was considered in the calculation.

Neutron flux tally card is employed to determine the neutron flux in the fuel pellet region of a fuel assembly, by multiplying the neutron flux to the average fission neutron number υ and total fission cross section Σ_f with FMn card, the normalized fission neutron number in a fuel assembly was obtained. The fission neutron distribution was evaluated in coarse mesh and fine mesh.

3.2.2. Adjoint flux

The calculation for the $\phi^{\lambda *}$ adjoint flux, which is the eigenfunction of the static adjoint equation, is difficult to perform using MCNP code. Since it has been considered that the iterated fission probability (I_{FP}) is proportional to adjoint flux [4,6], and the adjoint flux in equation (24) appears in the form of ratio, so the iterated fission probability is calculated instead of the adjoint flux. The criticality source card (kcode card) of MCNP code was employed to do the eigenvalue calculation.

3.2.2.1. The iterated fission probability. Suppose there is a critical core of zero power without external sources. Let a neutron be injected at phase space position $\theta = (r, E, \Omega)$. The neutron stochastically induces a fission reaction and yields neutrons in the next generation. We call the yielded neutrons progenies. The progenies induce chain reactions and produce new progenies in subsequent generations. As the generation number, λ, increases, distributions of the neutron flux $\phi^{(\lambda)}$ and the fission neutron emission (source) produced by the progenies converge to those of the fundamental mode. Assume that convergence is achieved in generation L. The reactor power in generation L, where $\lambda \geq L$, approaches quite a definite level. The asymptotic power originating in the neutron at θ is called the iterated fission probability ($I_{FP}(\theta)$) [4].

3.2.2.2. Calculation of I_{FP}. The proportionality of I_{FP} to adjoint flux, verification of calculation of I_{FP} are proven [6]. In this research, I_{FP} is determined from the number of fission neutrons, it could be calculated with the formula below [6].

$$I_{FP}^{(\lambda)}(\theta_0) = \frac{s_{\theta_0}^{(1)}}{s_{\theta_0}^{(0)}} \frac{s_{\theta_0}^{(2)}}{s_{\theta_0}^{(1)}} \cdot \cdot \cdot \frac{s_{\theta_0}^{(\lambda-1)}}{s_{\theta_0}^{(\lambda-2)}} \frac{s_{\theta_0}^{(\lambda)}}{s_{\theta_0}^{(\lambda-1)}}, \tag{25}$$

$$I_{FP}^{(\lambda)}(\theta_0) = k_{\theta_0}^{(1)} k_{\theta_0}^{(2)} \cdot \cdot \cdot k_{\theta_0}^{(\lambda-1)} k_{\theta_0}^{(\lambda)}. \tag{26}$$

The ratio $s_{\theta_0}^{(i)}/s_{\theta_0}^{(i-1)}$ could be estimated by MCNP conventional eigenvalue calculation [7]. I_{FP} depends on initial point source θ_0, so we write the corresponding ratio $s_{\theta_0}^{(i)}/s_{\theta_0}^{(i-1)}$ as $k_{\theta_0}^{(i)}$. Thus, the I_{FP} of $\theta_0 = (r_0, E_0, \Omega_0)$ could be estimated by calculation given initial source points at θ_0.

In the I_{FP} calculation using MCNP, a sigle eigenvalue calculation determines the I_{FP} of a single nodal. In every eigenvalue calculation, hundreds of source points were located in the corresponding mesh grid homogeneously, and a sufficient number of neutrons are sampled. The k-collision is regarded as $k_{\theta_0}^{(i)}$ to determine the $I_{FP}(r_0)$. The k-collision could be read directly from the output file, and the $I_{FP}(r_0)$ is obtained with equation (26). Here the E and Ω are considered to be integrated.

3.2.2.3. The uncertainty of I_{FP}. Strictly speaking, we should calculate I_{FP} at the same source points by different random numbers for lots of times to determine the uncertainty (sample standard deviation) of I_{FP}. But in real application, it is hard to realize, for the huge computational cost. So an approximation method is proposed to evaluate the relative uncertainty with the k in one critical calculation.

We use the assumption that the $k_{\theta_0}^{(i)}$ is almost relatively independent in MCNP calculation, the correlation between $k_{\theta_0}^{(i)}$ and $k_{\theta_0}^{(i+1)}$ (or $k_{\theta_0}^{(i-1)}$) is negligible.

In Refs. [8,9], the uncertainty is estimated the standard deviation, and a first order Taylor's formula is used to linearize and calculate an approximation of the uncertainty. So, the relative uncertainty of I_{FP} could be inferred as follows according to Refs. [8,9]:

$$ln\left(I_{FP}^{(\lambda)}\right) = ln\left(k_{\theta_0}^{(1)} k_{\theta_0}^{(2)} \cdot \cdot \cdot k_{\theta_0}^{(\lambda-1)} k_{\theta_0}^{(\lambda)}\right), \tag{27}$$

$$u\left[ln\left(I_{FP}^{(\lambda)}\right)\right] = u\left[ln\left(k_{\theta_0}^{(1)}\right) + ln\left(k_{\theta_0}^{(2)}\right) + \cdot \cdot \cdot + ln\left(k_{\theta_0}^{(\lambda)}\right)\right]. \tag{28}$$

The left hand side of the equation is:

$$u\left[In\left(I_{FP}^{(\lambda)}\right)\right]=\frac{\partial\left[In\left(I_{FP}^{(\lambda)}\right)\right]}{\partial I_{FP}^{(\lambda)}}u\left(I_{FP}^{(\lambda)}\right)=\frac{u\left(I_{FP}^{(\lambda)}\right)}{I_{FP}^{(\lambda)}}. \tag{29}$$

The right hand side of the equation is:

$$u\left[ln\left(k_{\theta_0}^{(1)}\right)+ln\left(k_{\theta_0}^{(2)}\right)+\cdots+ln\left(k_{\theta_0}^{(\lambda)}\right)\right] =\sqrt{\sum_{i=1}^{\lambda}\left[\frac{\partial\left[In\left(I_{FP}^{(\lambda)}\right)\right]}{\partial k_{\theta_0}^{(i)}}u\left(k_{\theta_0}^{(i)}\right)\right]^2}. \tag{30}$$

The equation (28) could be written as:

$$\frac{u\left(I_{FP}^{(\lambda)}\right)}{I_{FP}^{(\lambda)}}=\sqrt{\sum_{i=1}^{\lambda}\left[\frac{u\left(k_{\theta_0}^{(i)}\right)}{k_{\theta_0}^{(i)}}\right]^2}. \tag{31}$$

Technically, we should run the critical card at the same source points by different random numbers for lots of times to determine the uncertainty (sample standard deviation) of each $k_{\theta_0}^{(i)}$. But in each criticality calculation, we use the assumption that the relative uncertainties of $k_{\theta_0}^{(i)}$ from different cycles are the same, for the calculation condition is unchanged, even though the source points change in different cycles. Here $\frac{u(k_{\theta_0}^{(i)})}{k_{\theta_0}^{(i)}}$ is assumed to be the same in one critical calculation. Then we get the relative uncertainty of I_{FP}:

$$\frac{u\left(I_{FP}^{(\lambda)}\right)}{I_{FP}^{(\lambda)}}=\sqrt{\lambda}\,\frac{u\left(k_{\theta_0}^{(i)}\right)}{k_{\theta_0}^{(i)}}. \tag{32}$$

Since the source points changes a lot in the first few dozen cycles in one critical calculation, the k fluctuates heavily, the relative uncertainty would be overvalued with these cycles. So the cycles with nearly the same source condition are used to evaluate the relative uncertainty. As the neutron generation propagating, the fission neutron emission (source) produced by the progenies converge to the fundamental mode, the k calculated in these fundamental mode source conditon are considered to be in the same simulation condition, and they could be used to determine the relative uncertainty of k. Here we skip 50 cycles and use the k of the last 50 cycles to evaluate the relative uncertainty of k.

3.2.3. Neutron Flux at Detector Position

As the ex-core detector is small and located at the position far from the core region, the neutron flux at detector position $\phi(r_d)$ is evaluated by forward transport calculation based on high precise reactor calculation model. The F4:n card is employed to calculate the neutron flux at the detector position. In order to improve the efficiency of the calculation, geometry split technique is carried out by increasing the neutron importance of each area from the core region to the neutron detector outside the reactor vessel to make the calculation.

3.3. Uncertainty analysis

In MCNP output, the relative error is evaluated as relative standard deviation, which is the relative uncertainty in Ref. [8]. So, the relative uncertainty of fission neutron distribution $F\phi$ and neutron flux at the neutron detector $\phi(r_d)$ is evaluated with the relative error in MCNP output file. The relative uncertainty of the adjoint flux distribution $\phi^{\lambda*}(I_{FP})$ could be obtained in the way mentioned above.

The relative uncertainties of the physical parameters are propagated to C. With eq. (24), the relative uncertainty of C could be inferred as follows:

$$u(C)=u\left[\frac{\left(\phi_{1,1}^{\lambda*},S_1\right)}{\left(\phi_{0,1}^{\lambda*},S_0\right)}\frac{\left(\phi_{0,1}^{\lambda*},F_0\phi_0\right)}{\left(\phi_{1,1}^{\lambda*},F_1\phi_1\right)}\frac{\Sigma_1\phi_1(r_d)}{\Sigma_0\phi_0(r_d)}\right]=u\left(\frac{X_1}{X_0}\frac{Y_0}{Y_1}\frac{Z_1}{Z_0}\right). \tag{33}$$

Where X, Y, Z refer to corresponding part, and each of them are relatively independent from each other. So $u(C)$ could be inferred as:

$$In(C)=In(X_1)-In(X_0)+In(Y_0)-In(Y_1)+In(Z_1)-In(Z_0), \tag{34}$$

$$u[In(C)]=u[In(X_1)-In(X_0)+In(Y_0)-In(Y_1)+In(Z_1)-In(Z_0)]. \tag{35}$$

The left hand side of the equation is:

$$u[In(C)]=\frac{\partial[In(C)]}{\partial C}u(C)=\frac{u(C)}{C}. \tag{36}$$

The right hand side of the equation is:

$$u[In(C)]=\sqrt{\sum_{n=0}^{1}\left[\frac{\partial[In(C)]}{\partial X_n}u(X_n)\right]^2+\sum_{n=0}^{1}\left[\frac{\partial[In(C)]}{\partial Y_n}u(Y_n)\right]^2+\sum_{n=0}^{1}\left[\frac{\partial[In(C)]}{\partial Z_n}u(Z_n)\right]^2}, \tag{37}$$

$$u[In(C)]=\sqrt{\sum_{n=0}^{1}\left[\frac{u(X_n)}{X_n}\right]^2+\sum_{n=0}^{1}\left[\frac{u(Y_n)}{Y_n}\right]^2+\sum_{n=0}^{1}\left[\frac{u(Z_n)}{Z_n}\right]^2}, \tag{38}$$

where n refers to reference state (0) and measurement state (1).

So the relative uncertainty of C is:

$$\frac{u(C)}{C}=\sqrt{\sum_{n=0}^{1}\left[\frac{u(X_n)}{X_n}\right]^2+\sum_{n=0}^{1}\left[\frac{u(Y_n)}{Y_n}\right]^2+\sum_{n=0}^{1}\left[\frac{u(Z_n)}{Z_n}\right]^2}. \tag{39}$$

Since the neutron source S_n is constant, the relative uncertainty of X could be written as:

$$\frac{u(X_n)}{X_n}=\sqrt{\sum_{m}\left[S_{n,m}\frac{\partial X_n}{\partial\phi_{n,1,m}^{\lambda*}}u\left(\phi_{n,1,m}^{\lambda*}\right)\right]^2}, \tag{40}$$

where m refers to the m-th calculation grid.

Because of the higher-order terms in Y, the relative uncertainty of Y could be written as:

$$\frac{u(Y_n)}{Y_n}=\frac{\sqrt{\sum_m\left\{\left[\phi^{\lambda^*}_{n,1,m}u(F_{n,m}\phi_{n,m})\right]^2+\left[\phi^{\lambda^*}_{n,1,m}u(F_{n,m}\phi_{n,m})\right]^2+\left[u\left(\phi^{\lambda^*}_{n,1,m}\right)u(F_{n,m}\phi_{n,m})\right]^2\right\}}}{\sum_m\left(\phi^{\lambda^*}_{n,1,m}F_{n,m}\phi_{n,m}\right)}. \tag{41}$$

Since Σ_n is considered to be constant, the relative uncertainty of Z could be written as:

$$\frac{u(Z_n)}{Z_n}=\frac{u[\phi_n(r_d)]}{\phi_n(r_d)}. \tag{42}$$

4. Results and discussion

4.1. Subcriticality monitoring results obtained by NSM method

Conventional NSM method based on one point model is employed to monitor the subcriticality of single control rod cluster out state ($1^{\#}$~$5^{\#}$). The neutron count rate of each state is measured in 100 s by the BF_3 neutron detector in Fig. 1, the results are shown in Table 1.The measurement parameters, such as high voltage value and amplification factor, are kept unchanged in the experiment. The subcriticality of each subcritical state is calculated by MCNP code as theoretical results.

Control rod all in state is considered to be the reference state (neutron count rate 5.1 cps), its subcriticality ρ_0 is calculated by MCNP code based on 3D reactor calculation model, which is −7320pcm. The subcriticality ρ_1 of conventional NSM method is obtained with measured neutron count rate N and calculated ρ_0 by the formula below:

$$\rho_1=\frac{N_0}{N_1}\rho_0. \tag{43}$$

As shown in Table 1, the relative error between results obtained by NSM method and MCNP code is large: the error of $5^{\#}$ rod is about 104%, which is the largest one. The error of the rods located around the Cf-252 neutron source ($1^{\#}$, $3^{\#}$, $4^{\#}$) is large, among which the largest one is 52.30%. However, the error of the rod located far from the Cf-252 neutron source ($2^{\#}$) is quite small.

Obviously, the conventional NSM method based on one point model is limited in the subcriticality monitoring process that large reactivity is locally added by control rod cluster. Modification needs to be done to make the results more accurate.

4.2. Modified results based on 3D reactor model

From Eq. (24) and the physical parameters obtained by MCNP code, the correction factors are determined, as shown in Fig. 4. And the modified subcriticalty of single control rod cluster fully withdrawn state was obtained. The calculation uncertainties are evaluated with the method mentioned in chapter 3.

The physical parameters are calculated in 2 ways of mesh accuracy: (1) coarse mesh: assembly-wise, (2) relatively fine mesh: assembly-wise in radial and 10 cm mesh grid in axial. The modification is performed in 4 ways: (1) modification on coarse mesh (relatively high uncertainty data), (2) modification on coarse mesh with fine mesh data (relatively low uncertainty data), (3) modification on fine mesh with relatively low uncertainty data, (4) modification on fine mesh with the adjoint flux equal to 1. Corresponding results are shown in Table 2 (Modified results 1–4), where the relative error is the relative error between MCNP and modified results, the relative uncertainty is the relative uncertainty of calculation in the modification, the computation time is for a single subcritical state with a single CPU. It is read from MCNP output files.

As can be seen from the modified results 1–3 in Table 2, the subcriticality of $1^{\#}$~$5^{\#}$ rod out state becomes closer to theoretical analysis(the results of MCNP), the modified results is more helpful and valid for nuclear reactor subcriticality monitoring: the

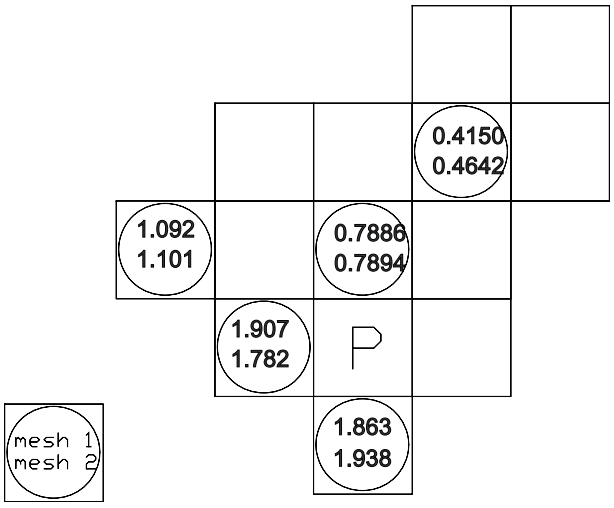

Fig. 4. Correction factors of different mesh.
In this figure, mesh 1 refers to coarse mesh, and mesh 2 refers to fine mesh.

Table 1
Subcriticality of conventional NSM method.

Subcritical state	Neutron count rate(cps)	MCNP Code (pcm)	Conventional NSM method	
			Subcriticality (pcm)	Relative error
$1^{\#}$ out	13.5	−5798	−2765	−52.30%
$2^{\#}$ out	6.6	−5721	−5657	−1.12%
$3^{\#}$ out	11.3	−5470	−3304	−39.60%
$4^{\#}$ out	22.1	−1319	−1689	28.06%
$5^{\#}$ out	14.7	−1243	−2540	104.28%

Table 2
Comparison of modified results.

Subcritical state		1# out	2# out	3# out	4# out	5# out	Total Computation Time (hours)
MCNP Code	Subcriticality (pcm)	−5798	−5721	−5470	−1319	−1243	/
Modified results 1 (coarse mesh modification)	C	1.863	1.092	1.907	0.7886	0.4150	568.8
	Subcriticality (pcm)	−5153	−6175	−6299	−1332	−1054	
	Relative error ± Relative uncertainty	−11.1% ±6.7%	7.9% ±7.3%	15.2% ±6.7%	1.0% ±7.7%	−15.2% ±8.2%	
Modified results 2 (coarse mesh modification with fine mesh data)	C	2.073	1.048	1.917	0.9111	0.4582	6531.7
	Subcriticality (pcm)	−5732	−5928	−6332	−1539	−1164	
	Relative error ± Relative uncertainty	−1.1% ±4.4%	3.6% ±4.4%	15.8% ±4.4%	16.7% ±4.9%	−6.4% ±4.8%	
Modified results 3 (fine mesh modification)	C	1.938	1.101	1.782	0.7894	0.4642	11556.2
	Subcriticality (pcm)	−5360	−6228	−5888	−1334	−1179	
	Relative error ± Relative uncertainty	−7.5% ±4.8%	8.9% ±4.7%	7.6% ±4.7%	1.1% ±5.0%	−5.2% ±4.7%	
Modified results 4 (fine mesh Modification with the adjoint flux equal to 1)	C	1.836	1.187	1.723	1.057	0.6947	6249.9
	Subcriticality (pcm)	−5077	−6716	−5694	−1785	−1764	
	Relative error ± Relative uncertainty	−12.4% ±1.6%	17.4% ±1.8%	4.1% ±1.7%	35.3% ±1.8%	41.9% ±2.0%	

subcriticality of $5^{\#}$ rod is smaller than that of $4^{\#}$, the subcriticality relative error of the rods located symmetrically ($1^{\#}$, $2^{\#}$) reduced from about 2900pcm to about just 1000pcm, only to make the results of these two states closer.

For further analysis, the accuracy and validity of the modified method are investigated based on the relative error of different calculating mesh grid. As shown in Table 2, after modification, the relative error between measured results and MCNP calculation is largely reduced. With modification by calculating the parameters with MCNP code on coarse mesh (in assembly-wise) based on 3D reactor model, the relative error between measurement and MCNP calculation islargely reduced, which is among 0.99%–15.23% (absolute value in Modified results 1). With modification on coarse mesh with fine mesh data (lower uncertainty), some of the relative error is smaller, while the others isn't, the relative error is among 1.13%–16.67%(absolute value in Modified results 2). With modification on fine mesh (assembly-wise in radial and 10 cm mesh grid in axial), the relative error is smaller, which is among 1.09%–8.87%(absolute value in Modified results 3).

As can be seen from the analysis above, the results obtained by the modified method is closer to the calculation results, the error of most of the modified results is smaller. The results indicate that the modified NSM method is capable of reducing the effects of higher modes, spatial effect and the disturbance of neutron importance field in the subcriticality monitoring of nuclear reactor, the results obtained by the modified method turn out to be more accurate and valid.

4.3. The effect of calculation uncertainty

At first, in order to reduce the computational cost, the coarse mesh is used to modify the measurement, and the relative uncertainties of calculation data are kept relatively high. Then, in order to modify the results more precisely, fine mesh calculation was performed with the uncertainty kept in lower level. It is quite time consuming to get such low uncertainty fine mesh data, whose computation time is about 20 times of the coarse mesh. Since the uncertainties of the fine mesh data are lower than the coarse mesh, the low uncertainty fine mesh data ($F\phi(r_d)$ and $\phi(r_d)$) are used to create a low uncertainty coarse mesh data to make the modification.

Compare modified results 1 to 2 in Table 2, the relative error stay in the same level, we can see the improvement in uncertainty for coarse mesh would not reduce the relative error. For high uncertainty coarse mesh modification (Modified results 1), the relative uncertainties are about 1.5–2 times of the low uncertainty coarse mesh modification. The computation time of high uncertainty is about 5500 min for a single subcritical state, which is about only 1/10 of low uncertainty coarse mesh. So, for coarse mesh, the modification is limited by mesh accuracy when uncertainty and error are in the same level, relatively high uncertainty coarse mesh data can improve most of the NSM results, lower uncertainty would bring higher cost without better results.

Compare modified results 2 to 3, the relative error is largely reduced, we can see the improvement in mesh accuracy under the same calculation uncertainty would reduce the relative error. Although it will cost more time, finer mesh modification with almost the same uncertainty would improve most of the results, this will make the results more reliable. The computation time of low uncertainty fine mesh (modified results 3) is about twice as Modified results 2, for its finer mesh in adjoint flux computing. Although the computation time of fine mesh is huge, it could be largely shorten in parallel computing.

From the uncertainty and error in modified results 3, we can infer that such uncertainty for fine mesh is enough, since the error is low and the uncertainty is about only 1/2 of error in most of the states.

4.4. The effect of adjoint flux

In order to study the effect of adjoint flux, C was calculated with the adjoint flux equal to 1. The modified results are shown in Table 2 (modified results 4). Here we can see the C with the adjoint flux equal to 1 is different from the C with real adjoint flux, and most of the modified results are worse. So the change induced by the adjoint flux in the core could affect the results heavily.

4.5. Discussion

As can be seen from the modified results 1–4 in Table 2, most of the results are improved with the application of modified NSM method. Since the modified NSM method is based on 3D reactor model, the results are modified with correction factor C, thus the effects of higher modes, spatial effect and the disturbance of neutron importance field are largely reduced, so most of the results are improved compared to conventional NSM method.

However, the results of rod 2# become worse after modification in the modified results 1–3, and very few of the results are getting worse with the mesh accuracy and uncertainty improves in the modified results 1–3. We think the factors that affect the results are as follows:

(1) The mesh accuracy, which is limited by computation time.
(2) That the calculation uncertainties exist in C.
(3) The calculation model bias, which could not be eliminated.
(4) The simplification in the calculation, such as: energy mesh simplification, detector simplification, etc.

But we can see from the data, the effect of the items above is small. In order to avoid the worse results after modification, the modification might be omitted for the state whose C is close to 1.

Compared to the unmodified results, most of the modified ones are obviously improved. For subcritical monitoring, the coarse mesh (modified results 1) is recommended if the requirement for accuracy is not very high, it is computationally faster, and the error is largely reduced. The coarse mesh with fine mesh data (modified results 2) is not recommended, for it is more time consuming but not more accurate than coarse mesh. The fine mesh (modified results 3) is recommended for its advantage in accuracy and uncertainty if the computation efficiency is high enough.

5. Conclusions

MCNP code was employed to calculate the physical parameters based on high precision 3D calculation model. Modified NSM method was employed to modify the subcriticality monitoring process, in which large reactivity is locally added. The conclusions are as follows:

(1) In the subcriticality monitoring process that large reactivity is locally added by control rod cluster, the conventional NSM method is limited and the modified NSM method is capable of reducing the effects of higher modes, spatial effect and the disturbance of neutron importance field in the subcriticality monitoring of nuclear reactor. The results obtained by the modified method turn out to be closer to calculation results.
(2) By calculating the parameters with MCNP code on coarse mesh (in assembly-wise) based on 3D reactor model, the error between measurement and calculation is largely reduced. By calculating the parameters on fine mesh (assembly-wise in radial and 10 cm mesh grid in axial) to modify the results, the measured results would be closer to calculation.
(3) It is feasible to implement the modified NSM method in large local reactivity addition process using Monte Carlo code based on 3D model.
(4) If the calculation uncertainty is relatively low, the improvement in uncertainty for coarse mesh would not reduce the relative error and the improvement in mesh accuracy under the same calculation uncertainty would make the relative error smaller.
(5) The change induced by the adjoint flux in the core could affect the results heavily.

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