Print ISSN: 1738-3110 / Online ISSN 2093-7717
http://dx.doi.org/10.15722/jds.18.1.202001.85

Investigation on How VMI affects Ongoing Performance of Supply Chain System

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Received: December 11, 2019 Revised: December 03, 2020 Accepted: December 05, 2020

Abstract

Purpose: This study investigates the influence of VMI on the supply chain performance over time. By examining each supply chain member's ongoing performance, this study figures out how VMI allows the vendor to overcome the initial loss and eventually provides the benefit to every supply chain member. **Research design, data, and methodology**: The proposed mathematical model describes the supply chain system where a manufacturer and a retailer make the operational decisions to maximize their own profits. By using the numerical examples with arbitrary data, VMI and non-VMI are compared in terms of their profit changes over time. **Results**: The numerical analysis shows that VMI results in greater overall profits for both manufacturer and retailer than non-VMI, while the manufacturer make a loss in the early stage of VMI implementation. This study also examines the impacts of certain conditional factors on the performance of VMI. **Conclusions**: This study supports the idea that VMI leads to manufacturer's initial loss but it brings greater profits to both manufacturer and retailer than non-VMI after all. In addition, the result of this study provides the managerial implications about the particular condition that allows VMI to achieve a significant financial performance improvement over non-VMI.

Keywords: Supply Chain Collaboration, Vendor-Managed Inventory, Optimization Model, Optimal Control.

JEL Classification Code: M11, M19, M21, M29.

1. Introduction

Vendor-Managed Inventory (VMI) is one of the famous programs and is well known to lead to an effective supply chain system by realizing the strong collaboration between its members (Lee, Cho, & Paik, 2016). While the substantial advantages of VMI have been supported by many academic research studies as well as industry applications, the most of them measure merely the concluding outcome of VMI and fail to provide continuing observation on its performance changes over time (De Toni & Zamolo, 2005; Fry, Kapuscinski, & Olsen, 2001; Lee & Cho, 2018).

This study is motivated by the research question of whether VMI always has a positive impact on every supply chain member. While VMI has been recognized to be beneficial for every supply chain member (Hong, Chunyuan, Xu, & Diabat, 2016; Smaros & Holmstrom, 2000; Williams, 2000), the vendor's initial performance is questionable due to his burden of managing buyer's inventories under VMI (Kannan, Grigore, Devika, & Senthilkumar, 2013; Yao, Evers, & Dresner, 2007) . This study pays attention to this particular issue, because even temporary loss of one supply chain member makes him reluctant to stay at the supply chain collaboration. For the purpose of the sustainable collaboration program, it is important to identify any potential loss made by its participant and prepare proper countermeasures against that problem. Instead of measuring the final outcome of VMI, the new study should focus on the continuing performance over time to figure out whether VMI has distinct impacts on the supply chain performance in different times.

The key objective of this study is to figure out whether VMI has different impacts on its participant's performance in different times. While a majority of the past studies evaluate the overall performance of VMI, this study observes the change of its performance as time passes and find out how VMI affects the early and latter outputs of the supply chain system.

In order to evaluate the performance of VMI, this study develops the mathematical model of two-stage supply chain system with one manufacturer and one retailer. The

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proposed optimization model represents the supply chain system where the manufacturer and retailer decide the optimal prices, lot size, and production rate in a way to maximize their own profits. In the proposed supply chain models, VMI is defined as the supply chain system where the manufacturer determines the lot size and pays ordering and inventory holding costs on behalf of the retailer. The numerical examples are used to compare VMI with non-VMI, and the changes of profits are observed as time passes in the experiment.

The numerical examples show that the manufacturer under VMI experiences the certain profit loss early and then increases his profit latter. The retailer, however, increases his profit in the beginning of VMI implementation but his profit significantly decreases in the end. Overall, VMI allows the entire supply chain system to achieve the constantly increasing profit over time. This results implies that VMI requires the carefully designed scheme to provide the timely incentives to each supply chain member and maintain the member's willingness to participate in this collaboration program.

The additional numerical analysis is conducted to examine the impact of conditional factors on the performance of VMI and the outcome indicates that VMI is preferred for the supply chain system with high ordering and inventory holding costs. Meanwhile, VMI is not an appropriate choice when the market demand is large and the manufacturer requires high profit margin.

2. Literature Reviews

The supply chain collaboration emphasizes the active cooperation and collective goal sharing among different companies, and it is developed to overcome the certain inefficiency such as double marginalization and bullwhip effect, which commonly appears in most conventional supply chain systems (Lee, Padmanabhan, & Whang, 1997; Simchi-Levi, Kaminsky, & Simchi-Levi, 2000). VMI is one of the well-known programs that are designed to realize the supply chain collaboration and it has been proved to be the very effective program that allows many companies to achieve substantial operational improvement in diverse industries (Bookbinder, Gumus & Jewkes, 2010; Niranjan, Wagner & Nguyen, 2012). Under VMI, the downstream customer's inventories are managed by the upstream company (Bai, Gong, Jin & Xu, 2019; Yao, Dong & Dresner, 2012), and it is known to provide the supply chain system with significant improvements such as lower inventory level, better customer service level, shorter leadtime, and more accurate forecasted demand (Kuk, 2004; Waller, Johnson, & Davis, 1999). The widespread application of VMI leads many academic researchers to pay attentions to this particular collaboration program (Park & Shim, 2008). VMI has been researched regarding various issues including information sharing, inventory control, stock allocation, and contract (Disney & Towill, 2002; Fry et al., 2001; Yu, Chu, & Chen, 2009; Zhao, Chen, Leung, & Lai, 2010).

Although VMI would bring the overall performance improvement for the entire supply chain system, a group of researchers suspect that VMI may not be equally beneficial to its participants. According their claims, the buyer takes advantage of the vendor who has to hold the extra burden of controlling the buyer's inventory and paying his costs under VMI (Kannan et al., 2013; Yao et al., 2007). While a number of researchers conclude that VMI improves every participant's performance (Achabal, McIntyre, Smith, & Kalyanam, 2000; Ben-Daya, Hassini, Hariga, & AlDurgam, 2013; ElHafsi, Camus, & Craye, 2010), some studies still support that only the buyer receives a significant benefit from VMI.

In Dong and Xu (2002) 's study, the pricing and ordering policies are analyzed to find out how VMI affects the supply chain performance. In particular, they focus on the both short term and long term impacts of VMI on each member's profit. Their numerical analysis shows that VMI results in the supplier's loss in the short-term period, even though he can obtain the increased profit in the long run.

In Mishra and Raghunathan(2004)'s study, VMI is applied to the special situation where a retailer can switch between two different manufacturers' brands. According to their numerical analysis on comparison with Retailer-Managed Inventory (RMI), the retailer gets the benefit of increased profit from VMI. Meanwhile, the manufacturers experience profit loss, because VMI intensifies the brand competition between competing manufacturers.

Chen (2018) considers the two stage supply chain system with a manufacturer and multiple retailers under VMI with Just-In-Time shipment policy and evaluates its performance compared with the decentralized system with independent replenishment. The result of his study also supports that only the retailer gets the benefit from VMI and VMI can result in the manufacturer's profit loss. He claims that a certain incentive policy such as revenue sharing contract must be implemented to compensate the manufacturer's deficit under VMI. Table 1 summarizes the key contents of the selected past studies that assess the performance of VMI.

This study is inspired by a series of past studies that make the atypical conclusion that VMI may not be equally beneficial for its every participant. Instead of using the comprehensive performance measurement to examine VMI, this study keeps tracking how its members perform over time and compares the short term and long term impacts of VMI on their performances.

Table 1: Selected Studies on the Performance of VMI

Authors (Year)	Compared Systems	Supply Chain Structure (Participants)	Performance Measurement	Beneficiaries of VMI
Achabal, McIntyre, Smith and Kalyanam (2000)	Before and after VMI implementation	Two stages (a vender and multiple retailers)	Customer service, inventory turnover, and lost sales	Both
Dong & Xu (2002)	Pre-VMI, VMI and full channel coordination	Two stages (a supplier and a buyer)	Profit	Both (long-term) Only a buyer (short-term)
Mishra and Raghunathan (2004)	RMI and VMI	Two stages (two manufacturers and one retailer)	Profit	Only a retailer
Yao, Evers and Dresner (2007)	With and without VMI	Two stages (a supplier and a buyer)	Inventory holding cost	Only a buyer
Yao and Dresner (2008)	Without collaboration, information sharing, continuous replenishment, and VMI	Two stages (a manufacturer and a retailer)	Inventory level	Both
ElHafsi, Camus and Craye (2010)	With and without VMI contract	Two stages (a manufacturer and multiple retailers)	Cost	Both
Ben-Daya, Hassini, Hariga and AlDurgam (2013)	Traditional system, VMI with consignment, and centralized system	Two stages (a vendor and multiple buyers)	Cost	Both
Hariga and Al-Ahmari (2013)	VMI with consignment and independent policy	Two stages (a supplier and a retailer)	Profit	Both
Kannan, Grigore, Devika and Senthilkumar (2013)	Traditional system and VMI	Two stages (a vendor and multiple buyers)	Cost	Only buyers
Lee, Cho and Seung-Kuk (2016)	Traditional, integrated systems, and VMI	Two stages (a vendor and a retailer)	Cost	Both
Chen (2018)	Decentralized system and VMI	Two stages (a manufacturer and multiple retailers)	Profit	Only retailers

3. Data and Research Methodology

3.1. Research Model

This study develops the supply chain models of both VMI and non-VMI and compares the performances of these two supply chain systems over time. The formulated mathematical model represents a two-stage supply chain system where a manufacturer and a retailer deal with a single product item. The manufacturer makes products at the production cost (υ) and sell them to the retailer at the transfer price (p_t). The retailer purchases the products form the manufacturer according to his orders (q_0) and sell them to the retailer market at the retail price (r_t).

The following non-VMI represents the traditional supply chain system without VMI implementation and it becomes the basis for comparison with VMI. Equation (1) indicates the manufacturer's problem that maximizes his profit by determining the transfer price (p_t) and production rate (y_t). Manufacturer's profit (π_M) contains revenue, setup cost, inventory holding cost, production cost, and transportation cost. During the time between 0 and terminal time T, his total profit is discounted at the certain rate (γ). The extended Economic Order Quantity (EOQ) model as the optimal control problem is used to represent the inventory policy in the proposed model (Tungalag, Erdenebat, & Enkhbat, 2017). The first constraint denotes the manufacturer's demand change (Equation (2)), and the initial demand size is specified by the second constraint (Equation (3)). The demand function denotes the naive model of innovation diffusion, and the proposed model assumes that the sales is dependent on the price alone (Kalish, 1985).

In Equation (4), the retailer's problem is to maximize his own profit by deciding the retail price (r_t) and order quantity (q_0). The retailer' profit (π_R) consists of the sales revenue, purchasing payment to the manufacturer, ordering cost, and inventory holding cost. Two constraints represent the retail market demand change (Equation (5)) and the initial market size (Equation (6)).

$$\max_{p_t, y_t} \pi_M = \int_0^T e^{-\gamma \cdot t}\left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0\left(1 - q_0\left(1 - \frac{t}{T}\right) h_M \cdot \frac{x_t^M}{y_t} - \upsilon \cdot y_t - \delta \cdot x_t^M\right) dt \quad (1)$$

$$\text{subject to } x_t^{M\prime} = N_M \cdot e^{-d_M \cdot p_t} \quad (2)$$

$$x_0^M = B \quad (3)$$

$$\max_{r_t, q_0} \pi_R = \int_0^T e^{-\gamma \cdot t}\left(r_t \cdot x_t^R - p_t \cdot x_t^R - \frac{o_R \cdot x_t^R}{q_0} - q_0\left(1 - \frac{t}{T}\right) h_R\right) dt \quad (4)$$

$$\text{subject to } x_t^{R\prime} = N_R \cdot e^{-d_R \cdot r_t} \quad (5)$$

$$x_0^R = B \quad (6)$$

Under VMI, the manufacturer takes a full responsibility of managing the retailer's inventories. The following mathematical models represent the manufacturer's and retailer's problems once the supply chain adopts VMI. In Equation (7), the manufacturer makes decisions on the transfer price, production rate, and order quantity in a way to maximize his profit. The constraints in Equations (8) and (9) specify the demand change and initial demand size. The retailer's problem in Equation (10) describes the retailer's decision on the retail price to maximize his profit.

Different from non-VMI where the retailer places orders (Equation (4)), the manufacturer determines the order quantity under VMI (Equation (7)). In addition, VMI let the manufacturer take the responsibility of paying the ordering and inventory costs (Equation (7)), which are paid by the retailer in non-VMI (Equation (4)). Furthermore, the manufacturer and retailer share the demand information through the active information sharing under VMI (Equations (8) and (11)). Meanwhile, the proposed models assume that the manufacturer and retailer use their own distinct demand functions under non-VMI without any information sharing activities (Equations (2) and (5)).

$$\max_{p_t, y_t, q_0} \pi_M = \int_0^T e^{-\gamma \cdot t}\left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0\left(1 - \frac{t}{T}\right) h_M \cdot \frac{x_t^M}{y_t} - \frac{o_R \cdot x_t^M}{q_0} - q_0\left(1 - \frac{t}{T}\right) h_R - \upsilon \cdot y_t - \delta \cdot x_t^M\right) dt \quad (7)$$

$$\text{subject to } x_t^{M\prime} = N_R \cdot e^{-d_R \cdot \alpha \cdot p_t} \quad (8)$$

$$x_0^M = B \quad (9)$$

$$\max_{r_t} \pi_R = \int_0^T e^{-\gamma \cdot t}(r_t \cdot x_t^R - p_t \cdot x_t^R) dt \quad (10)$$

$$\text{subject to } x_t^{R\prime} = N_R \cdot e^{-d_R \cdot r_t} \quad (11)$$

$$x_0^R = B \quad (12)$$

Table 2 describes the notations of the proposed mathematical models. The closed forms of optimal solutions are obtained by using the optimal control theory (Sethi & Thompson, 1981). Appendix describes the solution procedure and the shooting procedure for the numerical optimal solutions.

Table 2: Notations used in the mathematical models

Manufacturer		Retailer	
π_M	Profit	π_R	Profit
p_t	Price	r_t	Price
y_t	Production rate	q_0	Lot size
o_M	Setup cost per order	o_R	Ordering cost
h_M	Unit inventory holding cost	h_R	Unit inventory holding cost
υ	Unit production cost	α	Profit margin
δ	Transportation cost per price		
x_t^M	Estimated market demand	x_t^R	Market demand
$x_t^{M\prime}$	Rate of demand change	$x_t^{R\prime}$	Rate of demand change
N_M	Potential market size	N_R	Potential market size
d_M	Price elasticity of demand	d_R	Price elasticity of demand
B	Initial market demand	γ	Discount rate
T	Time to leave the market		

3.2. Data

This study conducts the numerical analysis to compare VMI and non-VMI in terms of their performances over time. The numerical examples based on the proposed mathematical models are designed with the different levels of six parameters including the unit inventory holding costs, ordering (setup) costs, potential market size, and profit margin (o_M, o_R, h_M, h_R, N_R, α). Table 3 shows the values of parameters used in the base case and they are arbitrarily decided. With five distinct levels of each parameter, 15,625 cases are observed in the numeral analysis ($5^6 = 15{,}625$).

Table 3: Parameters of Base Case

N_M = 90	N_R = 100	B = 1,000
d_M = 0.0012	d_R = 0.001	υ = 3
o_M = 1,500	o_R = 1,000	δ = 10
h_M = 3	h_R = 5	α = 1.15
γ = 0.01	T = 30	

4. Results

4.1. Performance Comparison between VMI and Non-VMI

This study observes how VMI performs as time passes compared with non-VMI. Figure 1 shows how the averaged profit differences between VMI and non-VMI (profit of

VMI – profit of non-VMI) change over time. In the beginning of the entire period, VMI results in less supply chain profit than non-VMI. As time passes, VMI becomes to outperform non-VMI and achieves greater supply chain profit after all. While the manufacturer under VMI makes less profit than under non-VMI at the early stage of time, his profit significantly increases over time under VMI. Meanwhile, VMI brings more profit for the retailer than non-VMI early, but the retailer's profit becomes less under the VMI than non-VMI later.

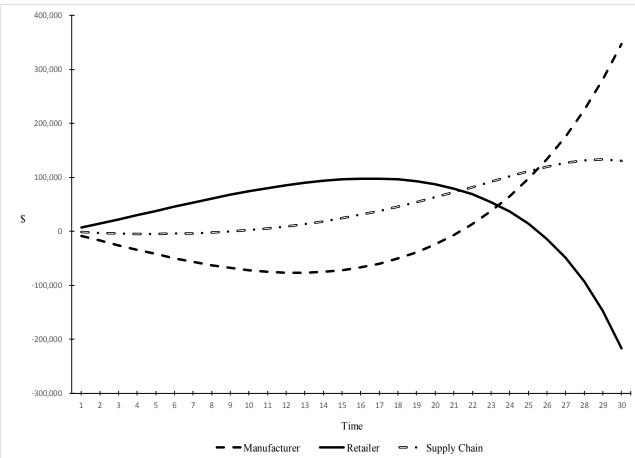

Figure 1: Profit Differences between VMI and Non-VMI on Timeline

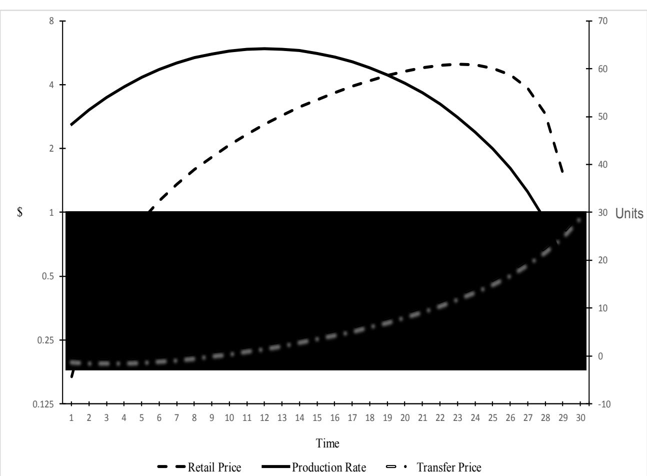

Figure 2: Differences between VMI and Non-VMI on Timeline

The differences in the retail price, transfer price, and production rate between VMI and non-VMI appeared in Figure 2 explain the reason that VMI affects each supply chain member's profit in the different way. According to Figure 2, the retailer price significantly increases at first and suddenly decreases in the end. The transfer price shows consistent increase over time. Meanwhile, the production rate increase in the beginning, and then it decreases significantly later.

After all, the manufacturer's early profit is smaller under VMI than non-VMI, because the additional burden of inventory holding and ordering costs that he has to pay under VMI. Meanwhile, since the manufacturer keeps increasing his transfer price over time, he can overcome the initial profit loss later and increase his profit. Furthermore, the significantly reduced production rate at the later point of time also lead to cost saving for the manufacturer.

The retailer can obtain greater profit under VMI than non-VMI in the beginning, mainly because he does not pay any inventory holding and ordering costs under VMI. However, the retailer's profit significantly deceases over time under VMI, since he has to reduce the retailer price to increase the market demand as Figure 2 shows.

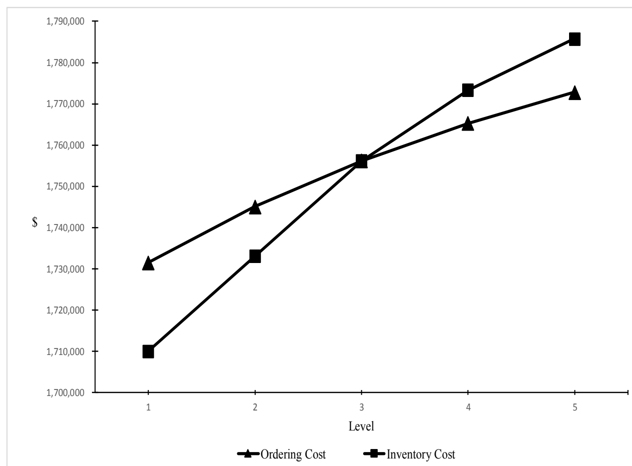

Figure 3: Impact of Costs on Profit Difference between VMI and Non-VMI

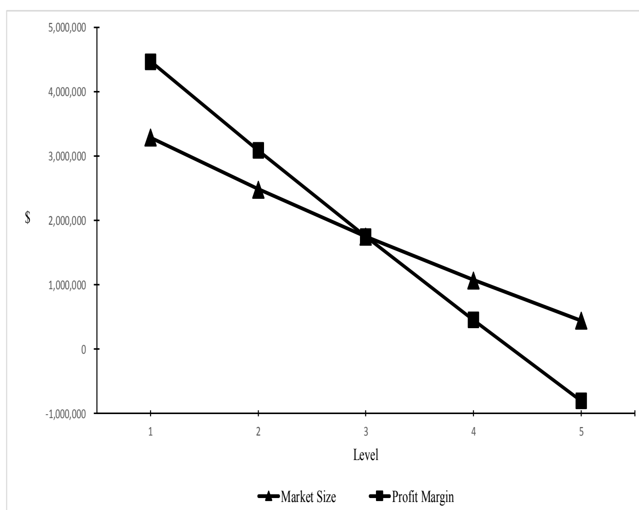

Figure 4: Impact of Market Size and Profit Margin on Profit Difference between VMI and Non-VMI

4.2. Impacts of Conditional Factors on Performance Difference

This study conducts additional experiments on the proposed supply chain models and examines how the conditional factors affect the performance of VMI. In the numerical examples, four factors inducing the ordering cost, inventory holding cost, market size, and profit margin are varied in five levels for the test.

Figure 3 shows the differences in the supply chain profit between VMI and non-VMI with different levels of the ordering cost and inventory holding cost. The outperformance of VMI over non-VMI increases as the ordering and inventory holding costs become higher. Meanwhile, the increased market size and profit margin have negative impacts on the performance of VMI. The profit difference between VMI and non-VMI decreases as the market size and profit margin increase as Figure 4 describes, and even non-VMI outperforms VMI when the manufacturer requires the significantly high profit margin.

4.3. Interpretation of Results

The superiority of VMI as the supply chain collaboration program is well known in both industries and academia. Meanwhile, the initial performance of VMI, particularly the vendor's, has been questionable due to the burden of extra works that the vender must conduct under VMI. To observe how VMI performs as time passes under VMI, this study examines the profit differences between VMI and non-VMI over time by using the numerical examples of the proposed supply chain models.

The numerical analysis shows that VMI achieves less supply chain profit than non-VMI at first and then overcomes the initial loss later. In particular, this pattern of the initial loss and latter gain is substantial for the manufacturer. This result indicates that VMI eventually bring the benefit to the manufacturer as well as the entire supply chain, but they have to suffer in the early stage of VMI implementation. The manufacturer loses profit in the beginning of VMI implementation, because relatively low transfer price and high production rate as well as the burden of extra costs of inventory holding and ordering costs. By implication, the VMI program requires the additional contract terms regarding the transfer price and production rate to mitigate the manufacturer's initial loss.

While the retailer obtains greater profit at first under VMI than non-VMI, his benefit from VMI significantly diminishes over time due to the reduced retailer price. Even though the reduction of the retailer price is necessary to maintain the sufficient market demand, substantially low price in the end seriously hurts the retailer's revenue under VMI. This result implies that the careful design of pricing plan is required with joint works of the manufacturer and retailer to achieve the best outcome from VMI.

This study also describes how the conditional factors affect the performance of VMI. The profit difference between VMI and non-VMI becomes larger when the ordering and inventory holding costs increases. This outcome points out that the implementation of VMI is preferred when the inventory holding and ordering costs are sufficiently high. Meanwhile, larger market size and higher profit margin make VMI perform worse. By implication, it is not desirable to apply VMI to the supply chain system when the market has a huge demand size or when the manufacturer insists the considerably high profit margin.

5. Conclusions

While VMI is known to be the effective supply chain collaboration program that ultimately brings benefits to the entire supply chain system, its early effect on the vendor's earnings has been questionable due to his burden of extra costs under VMI. This study examines the performance of VMI over time to figure out how each supply chain member's profit changes as time passes.

The mathematical optimization models are developed to represent two stage supply chain system where the manufacturer and retailer determines the lot size, price, and production rate to maximize their own profits. To compare the performance of VMI with one of non-VMI, this study conducts analysis on the numerical examples of the proposed models.

The numerical analysis provides the following noteworthy results and useful managerial implications. First, VMI requires the special scheme that compensates for each supply chain member's temporary loss. The numerical analysis of this study still supports that VMI definitely leads to greater supply chain profit than non-VMI. However, the manufacturer and retailer under VMI experience certain losses during the particular time periods. Therefore, VMI should be redesigned by having additional functions that support the consistent profits for every supply chain member in order to urge them to willingly participate in this collaboration program.

Second, it is necessary to consider the conditional factors carefully, when VMI is implemented to the supply chain system. In the numerical examples, this study shows that the relative performance of VMI compared with non-VMI is sensitive to the conditional factors such as the inventory holding and ordering costs. This result implies that the detailed research on the supply chain system before the implementation of VMI is essential to obtain the best possible outcome.

Finally, VMI is not for every type of supply chain systems, and it should not be used in certain special situations. According to the numerical examples, VMI performs poorly when the retail market has a huge demand size. VMI even results in less profit than non-VMI if the manufacturer holds the substantially high profit margin. By implication, VMI must be merely one of various collaboration programs, which bring the expected benefits to the supply chain system under limited circumstances.

The contributions that this study makes for academia and industries are two-fold. First, this study discloses the true nature of VMI and provides the theoretical background to explain that each supply chain member can take a different position on VMI. While most past studies support the VMI implementation by showing its benefits for every member, this study provide the distinct idea that each supply chain member can have different preference for VMI because the benefit from VMI may not be always equitable for every member over time.

Second, the outcomes of this study provide the important managerial implication that allows the sustainable VMI application. Since both manufacturer and retailer under VMI can experience the certain loss in different times, VMI requires to equip with the special scheme that provides the timely incentives to each supply chain member to compensate his loss. Otherwise, the supply chain system may fail to maintain VMI, because the supply chain member would not stay with VMI once he realizes certain loss in any time.

Some issues are pointed out as the limitations of the current study and they would be new research subjects for future studies. First, the numerical examples used in this study are based on the arbitrary parameter setting that may not fully represent the real industry. The future researchers can conduct more realistic numerical analysis on the real data obtained from case studies.

Second, the supply chain models proposed by this study adopts mere the naive demand model that may fail to apprehend the complex market response to the price. The forthcoming studies are expected to obtain more generalizable results by applying more sophisticated innovation diffusion models such as the multiplicative separable form to their models (Kalish, 1985).

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Appendix

1. Non-VMI

First of all, the Hamiltonian equation of the manufacturer's problem in Equations (1), (2), and (3) is obtained as Equation (A1). In the given Hamiltonian equations, the transfer price (p_t) and production rate (y_t) are the decision variables, and the market demand (x_t^M) is the state variable. The value of λ_t^M serves as the adjoint variable.

$$H_M = e^{-\gamma \cdot t}\left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_M \cdot \frac{x_t^M}{y_t} - \upsilon \cdot y_t - \delta \cdot x_t^M\right) + \lambda_t^M \cdot N_M \cdot e^{-d_M \cdot p_t} \quad \text{(A1)}$$

According to the optimal control theory, the following equations (Equations (A2) through (A5)) indicate the necessary conditions for optimality.

$$\frac{\partial H_M}{\partial p_t} = 0 \quad \text{(A2)}$$
$$\frac{\partial H_M}{\partial y_t} = 0 \quad \text{(A3)}$$
$$\frac{\partial H_M}{\partial x_t^M} = -\lambda_t^{M\prime} \quad \text{(A4)}$$
$$\frac{\partial H_M}{\partial \lambda_t^M} = x_t^{M\prime} \quad \text{(A5)}$$

The identical process in the preceding procedure is applied to for the retailer's problem as shown in Equations (A6) through (A9). In this Hamiltonian equation (Equation (A6)), the retail price (r_t) is the decision variable, and the market demand (x_t^R) is the state variable. The adjoint variable of λ_t^R is also included in the Hamiltonian equation.

$$H_R = e^{-\gamma \cdot t}\left(r_t \cdot x_t^R - p_t \cdot x_t^R - \frac{o_R \cdot x_t^R}{q_0} - q_0\left(1-\frac{t}{T}\right)h_R\right) + \lambda_t^R \cdot N_R \cdot e^{-d_R \cdot r_t} \quad \text{(A6)}$$
$$\frac{\partial H_R}{\partial r_t} = 0 \quad \text{(A7)}$$
$$\frac{\partial H_R}{\partial x_t^R} = -\lambda_t^{R\prime} \quad \text{(A8)}$$
$$\frac{\partial H_R}{\partial \lambda_t^R} = x_t^{R\prime} \quad \text{(A9)}$$

After all, the following solutions of all variables are obtained as they appear in Equations (A10) through (A17)

$$q_0 = \sqrt{\frac{2 \cdot O_R \cdot x_T^r}{h_R}} \quad \text{(A10)}$$
$$p_t = \frac{\gamma \cdot t}{d_M} \cdot ln\left(\frac{x_t^M}{d_M \cdot \lambda_t^M \cdot N_M}\right) \quad \text{(A11)}$$
$$y_t = \sqrt{\frac{q_0\left(1-\frac{t}{T}\right) \cdot h_M \cdot x_t^M}{\upsilon}} \quad \text{(A12)}$$
$$\lambda_t^M = \lambda_T^M + \int_t^T e^{-\gamma \cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1-\frac{t}{T}\right)\frac{h_M}{y_t} - \delta\right)dt \quad \text{(A13)}$$
$$x_t^M = B + \int_0^T N_M \cdot e^{-d_M \cdot p_t}\,dt \quad \text{(A14)}$$
$$r_t = \frac{\gamma \cdot t}{d_R} ln\left(\frac{x_t^R}{d_R \cdot \lambda_t^R \cdot N_R}\right) \quad \text{(A15)}$$
$$\lambda_t^R = \lambda_T^R + \int_t^T e^{-\gamma \cdot t}\left(r_t - p_t - \frac{o_R}{q_0}\right)dt \quad \text{(A16)}$$
$$x_t^R = B + \int_0^T N_R \cdot e^{-d_R \cdot r_t}\,dt \quad \text{(A17)}$$

2. VMI

The optimization problems under VMI can be solved in the same way to solve the problem under non-VMI. Equations (A18) and (A23) represent the Hamiltonian equations of the given optimization problems, and Equations (A19) - (A22) and (A24) - (A26) are the necessary conditions for optimality.

$$H_M = e^{-\gamma \cdot t}\left(p_t \cdot x_t^M - \frac{o_M \cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_M \cdot \frac{x_t^M}{y_t} - \frac{o_R \cdot x_t^M}{q_0} - q_0\left(1-\frac{t}{T}\right)h_R - \upsilon \cdot y_t - \delta \cdot x_t^M\right) + \lambda_t^M \cdot N_R \cdot e^{-d_R \cdot \alpha \cdot p_t} \quad \text{(A18)}$$
$$\frac{\partial H_M}{\partial p_t} = 0 \quad \text{(A19)}$$
$$\frac{\partial H_M}{\partial y_t} = 0 \quad \text{(A20)}$$
$$\frac{\partial H_M}{\partial x_t^M} = -\lambda_t^{M\prime} \quad \text{(A21)}$$
$$\frac{\partial H_M}{\partial \lambda_t^M} = x_t^{M\prime} \quad \text{(A22)}$$
$$H_R = e^{-\gamma \cdot t}(r_t \cdot x_t^R - p_t \cdot x_t^R) + \lambda_t^R \cdot N_R \cdot e^{-d_R \cdot r_t} \quad \text{(A23)}$$
$$\frac{\partial H_R}{\partial r_t} = 0 \quad \text{(A24)}$$
$$\frac{\partial H_R}{\partial x_t^R} = -\lambda_t^{R\prime} \quad \text{(A25)}$$
$$\frac{\partial H_R}{\partial \lambda_t^R} = x_t^{R\prime} \quad \text{(A26)}$$

The following equations (Equations (A27) through (A34)) indicate the optimal solutions of the variables.

$$q_0 = \sqrt{\frac{2 \cdot (o_M + o_R) \cdot x_T^r}{(h_M + h_R)}} \quad \text{(A27)}$$
$$p_t = \frac{\gamma \cdot t}{d_R \cdot \alpha} \cdot ln\left(\frac{x_t^M}{d_R \cdot \alpha \cdot \lambda_t^M \cdot N_R}\right) \quad \text{(A28)}$$
$$y_t = \sqrt{\frac{q_0\left(1-\frac{t}{T}\right) \cdot h_M \cdot x_t^M}{\upsilon}} \quad \text{(A29)}$$
$$\lambda_t^M = \lambda_T^M + \int_t^T e^{-\gamma \cdot t}\left(p_t - \frac{o_M + o_R}{q_0} - q_0\left(1-\frac{t}{T}\right)\frac{h_M}{y_t} - \delta\right)dt \quad \text{(A30)}$$
$$x_t^M = B + \int_0^T N_R \cdot e^{-d_R \cdot \alpha \cdot p_t}\,dt \quad \text{(A31)}$$

$$r_t = \frac{\gamma \cdot t}{d_R} ln\left(\frac{x_t^R}{d_R \cdot \lambda_t^R \cdot N_R}\right) \quad (A32)$$

$$\lambda_t^R = \lambda_T^R + \int_t^T e^{-\gamma \cdot t}(r_t - p_t)dt \quad (A33)$$

$$x_t^R = B + \int_0^T N_R \cdot e^{-d_R \cdot r_t} dt \quad (A34)$$

In the non-VMI case, the detailed steps of the shooting procedure for the numerical optimal solutions are followings:

Step 1. Set arbitrary values for old_sum_x1, old_sum_x2, old_sum_x3, and old_sum_x4.

In the optimal solutions of λ_t^M, x_t^M, λ_t^R, x_t^R, the integral portions of Equations (A13), (A14), (A16), and (A17) are approximated as the following discrete summation forms:

$$old\_sum\_x1 = \sum_{t=\tau+1}^{T} e^{-\gamma \cdot t}\left(p_t - \frac{o_M}{q_0} - q_0\left(1 - \frac{t}{T}\right)\frac{h_M}{y_t} - \delta\right)$$

$$old\_sum\_x1 = \sum_{t=1}^{T} N_M \cdot e^{-d_M \cdot p_t}$$

$$old\_sum\_x1 = \sum_{t=\tau+1}^{T} e^{-\gamma \cdot t}\left(r_t - p_t - \frac{o_R}{q_0}\right)$$

$$old\_sum\_x1 = \sum_{t=1}^{t} N_R \cdot e^{-d_R \cdot r_t}$$

Step 2. Obtain the optimal solutions of λ_t^M, x_t^M, λ_t^R, x_t^R in Equations (A13), (A14), (A16), and (A17) with the values of old_sum_x1, old_sum_x2, old_sum_x3, and old_sum_x4 from Step 1. Compute the values of q_0, p_t, y_t, and r_t according to Equations (A10), (A11), (A12), and (A15).

Step 3. Calculate new values of old_sum_x1, old_sum_x2, old_sum_x3, and old_sum_x4 (new_sum_x1, new_sum_x2, new_sum_x3, and new_sum_x4) with the values of λ_t^M, x_t^M, λ_t^R, x_t^R, q_0, p_t, y_t, and r_t that are obtained in Step 2.

Step 4. If |new_sum_x1 - old_sum_x1| > tol_x1, |new_sum_x2 - old_sum_x2| > tol_x2, |new_sum_x3 - old_sum_x3| > tol_x3, |new_sum_x4 - old_sum_x4| > tol_x4, with the small values of the tolerances for tol_x1, tol_x2, tol_x3, and tol_x4

set old_sum_x1 = new_sum_x1*ch+old_sum_x1 × (1-ch)
old_sum_x2 = new_sum_x2*ch+old_sum_x2 × (1-ch)
old_sum_x3 = new_sum_x3*ch+old_sum_x3 × (1-ch)
old_sum_x4 = new_sum_x4*ch+old_sum_x4 × (1-ch)
where 0 < ch < 1
and then go to Step 2
else exit.

The shooting procedure to obtain the numerical optimal solutions for VMI is same with the above prcess and only difference is the optimal solution eqaution for each variable.