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통합 베이즈 티코노프 정규화 방법의 확장과 영상복원에 대한 응용

류재홍*

An Extension of Unified Bayesian Tikhonov Regularization Method and Application to Image Restoration

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요 약

본 논문은 통합 베이즈 티코노프 정규화 방법을 확장하는 것을 제시한다. 통합된 방법은 티코노프 정규화 모수와 베이즈 하이퍼 모수들의 관계를 정립하고 최대 사후 확률과 근거 프레임워크를 사용한 정규화 모수를 구하는 공식을 제시한다. 데이터 행렬의 차원이 m by n (m >= n)일 때, total misfit는 기존의 m에서 m ± n로 확장된다. 따라서 탐색 범위도 1에서 2n+1개의 정수로 확장된다. 선형 탐색보다는 황금분할 탐색으로 시간을 줄인다. 상대오차를 최적화하는 새로운 벤치마크를 제안하고 이를 목표로 하는 새 모델 선택 판정기준을 소개한다. 실험결과는 영상 복원 문제에 대하여 제안하는 방법의 효능을 보여준다.

ABSTRACT

This paper suggests an extension of the unified Bayesian Tikhonov regularization method. The unified method establishes the relationship between Tikhonov regularization parameter and Bayesian hyper-parameters, and presents a formula for obtaining the regularization parameter using the maximum posterior probability and the evidence framework. When the dimension of the data matrix is m by n (m >= n), we derive that the total misfit has the range of m ± n instead of m. Thus the search range is extended from one to 2n + 1 integer points. Golden section search rather than linear one is applied to reduce the time. A new benchmark for optimizing relative error and new model selection criteria to target it are suggested. The experimental results show the effectiveness of the proposed method in the image restoration problem.

키워드

Golden Section Search, Image Restoration, Model Selection Criteria, Unified Bayesian Tikhonov Regularization
황금 분할 탐색, 영상 복원, 모델 선택 판정 기준, 통합 베이즈 티코노프 정규화

Ⅰ. Introduction

Image restoration is an example of inversion problems in the ill-posed systems[1-2]. Image restoration is the process to recover an original image from distorted one by using an appropriate

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degradation model[3-4]. In the linear degradation model, we assume that a given input image f is blurred by a Point Spread Function(: PSF) h and further distorted by a Gaussian noise η. This can be written in the form

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y) \tag{1}$$

where symbol ★ denotes convolution.
In the Fourier transform, we have

$$G(u,v) = H(u,v)F(u,v) + N(u,v). \tag{2}$$

In the inverse filtering, we have

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}. \tag{3}$$

This formula shows that if $H(u,v)$ is zero or very small in the high frequency region and $N(u,v)$ is still not vanished in the corresponding region, the second term $N(u,v)/H(u,v)$ is amplified. Thus we need a remedy solving ill-posed inverse problem.

In this paper, the unified Bayesian Tikhonov regularization is extended and applied to the image restoration problems. In section II, square residual and smoothing term in the frequency domain are introduced. In section III, unified Bayesian Tikhonov regularization method is reviewed. In section IV, an extension of the method is suggested. In section V, experimental results show the effectiveness of the proposed method followed by the conclusion and reference sections[5-16].

II. Regularization Parameter Selection in the Frequency Domain

Regularized estimation vector is defined as

$$\hat{\boldsymbol{f}}_\lambda = (\boldsymbol{H}^T\boldsymbol{H} + \lambda \boldsymbol{C}^T\boldsymbol{C})^{-1}\boldsymbol{H}^T\boldsymbol{g}. \tag{4}$$

Here, $\boldsymbol{H}$ and $\boldsymbol{C}$ are the block circulant matrices of a PSF h and a smoothing operator p respectively. λ is the Tikhonov regularization parameter. $\boldsymbol{g}$ and $\hat{\boldsymbol{f}}_\lambda$ are the column vectors stacking columns of a degraded image g and a recovered image $\widehat{f_\lambda}$ respectively. Both $\boldsymbol{H}$ and $\boldsymbol{C}$ matrices have the dimension, MN by MN. That is, $m = n = MN$ with image dimension M by N.

In the frequency domain, block circulant matrices $\boldsymbol{H}$ and $\boldsymbol{C}$ are diagonalized. We denote them $\boldsymbol{S}$ and $\boldsymbol{T}$ respectively. Let $\boldsymbol{z}$ and $\boldsymbol{b}$ be the column vectors of the Fourier transform of $\boldsymbol{f}$ and $\boldsymbol{g}$ respectively.

Regularized estimation vector is defined as

$$\hat{z}_\lambda = (\boldsymbol{S}^H\boldsymbol{S} + \lambda \boldsymbol{T}^H\boldsymbol{T})^{-1}\boldsymbol{S}^H\boldsymbol{b}, \tag{5}$$

where superscript H denotes the Hermitian or conjugate transpose.

Square residual is defined as

$$\| \xi_\lambda \|^2 = \sum_{i=1}^{n} \frac{\lambda^2 |t_i|^4}{(|s_i|^2 + \lambda |t_i|^2)^2} |b_i|^2. \tag{6}$$

Here, s_i and t_i are the diagonal elements of the matrices $\boldsymbol{S}$ and $\boldsymbol{T}$ respectively. b_i is the element of column vector $\boldsymbol{b}$ stacking columns of 2D Fourier transform of the degraded image g.

Smoothing term is defined as

$$\| \boldsymbol{C}\hat{\boldsymbol{f}}_\lambda \|^2 = \sum_{i=1}^{n} \frac{|s_i|^2 |t_i|^2}{(|s_i|^2 + \lambda |t_i|^2)^2} |b_i|^2. \tag{7}$$

III. Unified Bayesian Interpretation of Tikhonov Regularization

MAP estimator maximizes the posterior pdf $p(\boldsymbol{f}|\boldsymbol{g})$ which can be expressed using Bayes' law[5] as follows

$$p(\boldsymbol{f}|\boldsymbol{g}) = \frac{p(\boldsymbol{g}|\boldsymbol{f})\, p(\boldsymbol{f})}{p(\boldsymbol{g})}. \tag{8}$$

Assume that both error and original image are Gaussian random vectors, we have

$$p(\boldsymbol{g}|\boldsymbol{f}) = \frac{1}{(2\pi\sigma_\xi^2)^{m/2}} exp\left[-\frac{\| \boldsymbol{Hf}-\boldsymbol{g} \|^2}{2\sigma_\xi^2}\right], \tag{9}$$

$$p(\boldsymbol{f}) = \frac{1}{(2\pi\sigma_{Cf}^2)^{n/2}} exp\left[-\frac{\| \boldsymbol{Cf} \|^2}{2\sigma_{Cf}^2}\right]. \tag{10}$$

By taking the negative log for the Bayes′ law, we have the MAP interpretation of Tikhonov regularization as

$$\lambda = \frac{\sigma_\xi^2}{\sigma_{Cf}^2}. \tag{11}$$

In the evidence framework[6], we have fixed-point iteration known as the MacKay update

$$\sigma_{Cf}^2 = \frac{\| \boldsymbol{Cf} \|^2}{\gamma} = \frac{2E_F}{\gamma} = \frac{2E_F}{\chi_F^2} = 1/\alpha, \tag{12}$$

$$\sigma_\xi^2 = \frac{\| \boldsymbol{\xi} \|^2}{m-\gamma} = \frac{2E_D}{m-\gamma} = \frac{2E_D}{\chi_D^2} = 1/\beta. \tag{13}$$

In these equations, α and β are unknown hyper-parameters. E_F and E_D are regularization and cost terms respectively and γ is the number of effective parameters,

$$\gamma = \sum_{i=1}^{n} \frac{\beta\mu_i}{\beta\mu_i + \alpha\nu_i}. \tag{14}$$

Here, μ_i and ν_i denote singular value of $\boldsymbol{H}^T\boldsymbol{H}$ and $\boldsymbol{C}^T\boldsymbol{C}$ respectively.

Then we obtain a fixed point iteration method for the regularization parameter.

$$\lambda = \frac{\alpha}{\beta} = \frac{\gamma}{m-\gamma}\frac{E_D}{E_F} = \frac{\gamma}{m-\gamma}\frac{\| \boldsymbol{\xi} \|^2}{\| \boldsymbol{Cf} \|^2}, \tag{15}$$

with

$$\gamma = \sum_{i=1}^{n} \frac{\mu_i}{\mu_i + \lambda\nu_i}. \tag{16}$$

Ⅳ. An Extension of the Unified Bayesian Tikhonov Regularization Method

In the Tikhonov regularization for an ill-posed problem $\boldsymbol{Hf} = \boldsymbol{g}$, we seek a limiting vector $\hat{\boldsymbol{f}}_\lambda$ to fit data $\boldsymbol{g}$ in least squares sense with penalty term for large normed solution in the cost function as

$$J(\boldsymbol{f}) = \frac{1}{2}(\| \boldsymbol{Hf}-\boldsymbol{g} \|^2 + \lambda \| \boldsymbol{Cf} \|^2). \tag{17}$$

Here, we use Laplacian or identity smoother. Laplacian is positive definite so the dimension is n by n as in the identity matrix.

Equation(15) can be set up as proportionality

$$\frac{\| \boldsymbol{Hf}-\boldsymbol{g} \|^2}{m-\gamma} = \frac{\lambda \| \boldsymbol{Cf} \|^2}{\gamma}. \tag{18}$$

Here, the total misfit[6] is m. Left side of the equation is the unbiased estimate of variance(: UEV). Then general cross validation(: GCV) is defined as follows[7].

$$GCV = \frac{m \| \boldsymbol{Hf}-\boldsymbol{g} \|^2}{(m-\gamma)^2} = UEV\frac{m}{m-\gamma}. \tag{19}$$

Final prediction error(: FPE) is defined as follows[8].

$$FPE=\frac{(m+\gamma)\ \|\ \boldsymbol{Hf}-\boldsymbol{g}\ \|^{2}}{m(m-\gamma)}=UEV\frac{m+\gamma}{m}. \quad (20)$$

The cost function can be converted to the normal equation formulation with the additional n equations,

$$J(\boldsymbol{f})=\frac{1}{2}\left\|\begin{bmatrix}\boldsymbol{H}\\ \sqrt{\lambda}\,\boldsymbol{C}\end{bmatrix}\boldsymbol{f}-\begin{bmatrix}\boldsymbol{g}\\ 0\end{bmatrix}\right\|^{2}. \quad (21)$$

The corresponding proportionality is written as

$$\frac{\|\ \boldsymbol{Hf}-\boldsymbol{g}\ \|^{2}+\lambda\ \|\ \boldsymbol{Cf}\ \|^{2}}{m+n}=\frac{0}{0}. \quad (22)$$

Here, the total misfit is m+n.

From equation (18,) we have the following,

$$\frac{\|\ \boldsymbol{Hf}-\boldsymbol{g}\ \|^{2}+\lambda\ \|\ \boldsymbol{Cf}\ \|^{2}}{m}=\frac{\lambda\ \|\ \boldsymbol{Cf}\ \|^{2}}{\gamma}. \quad (23)$$

Applying the gamma factor m/(m – λ), we have Bayesian update for GCV variance,

$$\frac{\|\ \boldsymbol{Hf}-\boldsymbol{g}\ \|^{2}+\lambda\ \|\ \boldsymbol{Cf}\ \|^{2}}{m-\gamma}=\frac{\lambda\ \|\ \boldsymbol{Cf}\ \|^{2}}{\gamma}. \quad (24)$$

Combing equation (22) and (24), we have

$$\frac{\|\ \boldsymbol{Hf}-\boldsymbol{g}\ \|^{2}+\lambda\ \|\ \boldsymbol{Cf}\ \|^{2}}{(m+k)-\gamma}=\lambda\frac{\|\ \boldsymbol{Cf}\ \|^{2}}{\gamma}, \quad k=0,\pm 1,\ldots,\pm n. \quad (25)$$

Here, the total misfit has the range of m±n.

The Tikhonov regularization parameter is set up as a fixed point iteration method,

$$\lambda=\frac{\gamma}{m+k-\gamma}\frac{2J(\boldsymbol{f})}{\|\ \boldsymbol{Cf}\ \|^{2}},\ k=0,\pm 1,\ldots,\pm n. \quad (26)$$

In the frequency domain, we use equation (6) and (7) with the number of effective parameters,

$$\gamma=\sum_{i=1}^{n}\frac{\mu_i}{\mu_i+\lambda\nu_i}=\sum_{i=1}^{n}\frac{|s_i|^2}{|s_i|^2+\lambda|t_i|^2}. \quad (27)$$

Range of k has 2n + 1 points being searched for the global point of the relative error as new benchmark and the model selection criteria to target it. Golden section search can reduce time[9].

We suggest the new criteria raising gamma factor to the power of p,

$$GCVp==UEV\left[\frac{m}{m-\gamma}\right]^{p}, \quad (28)$$

$$FPEp=UEV\left[\frac{m+\gamma}{m}\right]^{p}. \quad (29)$$

From these equations we define the corresponding geometric average criterion(: GAC) as

$$GACp=UEV\left[\frac{m+\gamma}{m-\gamma}\right]^{0.5p}. \quad (30)$$

We find that the p = 0.7 and 2 show the best results for the Laplacian and identity smoother respectively to target the relative error benchmark.

V. Experimental Results

We report the experiments with the extended method proposed in the previous section. The method has two results of relative error(: EUB_RE) and GACp criterion(: EUB). Figure 1 shows the satellite image data[10]. We compare the extended method with the previous one(:UB) and the conventional techniques MDP, GCV, L-curve and Wiener filter[7, 11-13]. Results are depicted in the figures 2 and 3. The UB method shows comparable results with conventional methods using smoothing operator. However, the UB filter depicts under smoothing with identity matrix that is more severe compared to the GCV method. The EUB_RE shows the benchmark results and the EUB has the best performance under the new benchmark.

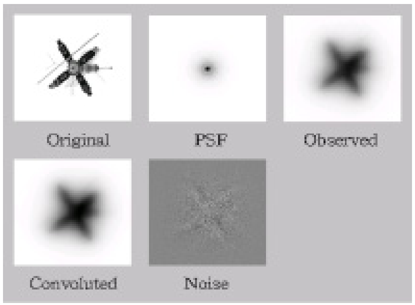

Fig. 1 Satellite image data

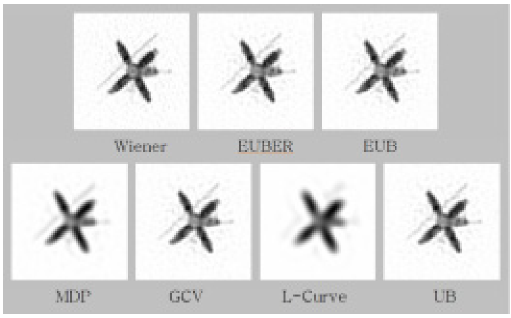

Fig. 2 Restored images with Laplacian smoother

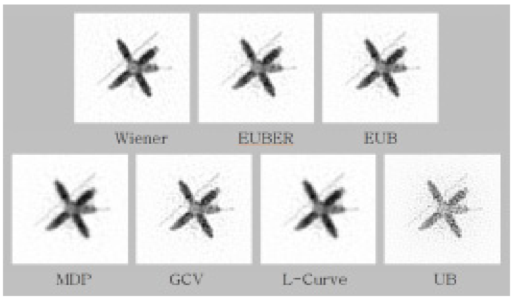

Fig. 3 Restored images with identity smoother

Image restoration performance is measured by the figure-of-merit functions such as relative error (RE), signal to noise ratio (SNR), peak SNR (PSNR) and improvement of SNR (ISNR)[13-16].

Table 1. Performance Results using Laplacian smoother

Measure / Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
MDP	0.3803	8.397	22.03	5.356	8.29e-2
GCV	0.3449	9.247	22.87	6.206	4.45e-3
L-curve	0.4509	6.917	20.55	3.876	1.00e-0
UB	0.3489	9.146	22.77	6.106	7.91e-3
EUB	0.3433	9.287	22.92	6.247	2.54e-3
EUB_RE	0.3433	9.287	22.92	6.247	2.47e-3
Wiener	0.3243	9.781	23.41	6.740	N/A

Table 2. Performance Results with identity smoothing operator

Measure / Method	RE ↓	SNR ↑	PSNR	ISNR	Remark λ
MDP	0.3648	8.758	22.39	5.717	4.75e-4
GCV	0.3575	8.935	22.56	5.895	9.80e-5
L-curve	0.3623	8.818	22.45	5.777	4.38e-4
UB	0.5694	4.891	18.52	1.850	1.72e-5
EUB	0.3496	9.129	22.76	6.088	1.59e-4
EUB_RE	0.3492	9.139	22.77	6.098	1.84e-4
Wiener	0.3243	9.781	23.41	6.740	N/A

Table 1 and 2 show the image restoration performance with remarking value of regularization parameter λ. The EUB has the best performance.

Ⅵ. Conclusions

The unified Bayesian Tikhonov regularization is extended. When the dimension of the data matrix is m by n (m >= n), total misfit can be m ± n instead of m. Thus the search range is extended from one to 2n + 1 integer points. Golden section search rather than linear one is applied to reduce the time. A new benchmark for optimizing relative error and the new model selection criteria to target it are introduced and successfully applied to image restoration problems. We suggest L1 Smoother instead of L2 one for the further research direction.

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