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FUZZY JOIN AND MEET PRESERVING MAPS ON ALEXANDROV L-PRETOPOLOGIES†

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Abstract. We introduce the concepts of fuzzy join-complete lattices and Alexandrov L-pre-topologies in complete residuated lattices. We investigate the properties of fuzzy join-complete lattices on Alexandrov L-pre-topologies and fuzzy meet-complete lattices on Alexandrov L-pre-cotopologies. Moreover, we give their examples.

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1. Introduction

Ward et al.[13] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool as algebraic structures for many valued logics [1-11,14].

Kim [6-8] studied the relations between L-fuzzy upper and lower approximation spaces and Alexandrov L-topologies in complete residuated lattices. Moreover, categories of fuzzy preorders, approximating operators and Alexandrov topologies are isomorphic [8]. In particular, fuzzy powerset operators are investigated in [9].

For a complete Heyting algebra(or a frame) as the base category, Zhang[16-18] introduced fuzzy complete lattices and the Dedekind-MacNeille completions for fuzzy posets in complete lattices. Moreover, he investigate the properties of completeness for fuzzy powerset operators on fuzzy poset (L^X, e_{L^X}).

In this paper, we introduce the concept of fuzzy join(resp. meet) -complete lattice on Alexandrov L-pretopologies (resp. cotopologies). Zhang [14] only use

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the complete for a fuzzy poset (L^X, e_{L^X}). We diversify Zhang's definition by using completeness for a fuzzy poset (τ, e_τ) on Alexandrov L-pretopology. We investigate the properties of join (meet)-preserving maps. The maps between topological structures are easily handle by using join (meet)-preserving maps. For examples, we study the relations among open maps, continuous maps and join (meet)-preserving maps. We give their examples.

2. Preliminaries

Definition 2.1. [1,3-5,11] An algebra $(L, \wedge, \vee, \odot, \to, \bot, \top)$ is called a *complete residuated lattice* if it satisfies the following conditions:

(L1) $(L, \leq, \vee, \wedge, \bot, \top)$ is a complete lattice with the greatest element $\top$ and the least element $\bot$;

(L2) $(L, \odot, \top)$ is a commutative monoid;

(L3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x, y, z \in L$.

In this paper, we always assume that $(L, \leq, \odot, \to^*)$ is complete residuated lattice.

For $\alpha \in L, A \in L^X$, we denote $(\alpha \to A), (\alpha \odot A), \alpha_X \in L^X$ as $(\alpha \to A)(x) = \alpha \to A(x)$, $(\alpha \odot A)(x) = \alpha \odot A(x)$, $\alpha_X(x) = \alpha$ and $x^* = x \to \bot$.

Lemma 2.2. [1,3-5,11] *For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.*

(1) $\top \to x = x$, $\bot \odot x = \bot$,

(2) If $y \leq z$, *then* $x \odot y \leq x \odot z$, $x \to y \leq x \to z$ *and* $z \to x \leq y \to x$,

(3) $x \leq y$ *iff* $x \to y = \top$.

(4) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(5) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(6) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(7) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(8) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$ *and* $x \to y \leq (x \odot z) \to (y \odot z)$,

(9) $(x \to y) \odot (y \to z) \leq x \to z$,

(10) $\bigvee_{i \in \Gamma} x_i \to \bigvee_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$ *and* $\bigwedge_{i \in \Gamma} x_i \to \bigwedge_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$,

(11) $x \to y \leq (y \to z) \to (x \to z)$ *and* $x \to y \leq (z \to x) \to (z \to y)$.

(12) If $(x^*)^* = x$ *for each* $x \in X$, *then* $(x \odot y^*)^* = x \to y$ *and* $x \to y = y^* \to x^*$.

Definition 2.3. [1,3-5,10] Let X be a set. A function $e_X : X \times X \to L$ is called:

(E1) *reflexive* if $e_X(x, x) = \top$ for all $x \in X$,

(E2) *transitive* if $e_X(x, y) \odot e_X(y, z) \leq e_X(x, z)$, for all $x, y, z \in X$,

(E3) *antisymmetric* if $e_X(x, y) = e_X(y, x) = \top$, then $x = y$.

If e satisfies (E1) and (E2), (X, e_X) is called a *fuzzy preordered set*. If e satisfies (E1), (E2) and (E3), (X, e_X) is called a *fuzzy partially ordered set* (simply, fuzzy poset).

Definition 2.4. [2,14-18] Let (X, e_X) be a fuzzy poset and $A \in L^X$.

(1) A point x_0 is called a *fuzzy join* of A, denoted by $x_0 = \sqcup_X A$ on (X, e_X), if it satisfies

(J1) $A(x) \le e_X(x, x_0)$,

(J2) $\bigwedge_{x\in X}(A(x) \to e_X(x, y)) \le e_X(x_0, y)$.

The pair (X, e_X) is called *fuzzy join complete* if $\sqcup_X A$ exists for each $A \in L^X$.

A point x_1 is called a *fuzzy meet* of A, denoted by $x_1 = \sqcap_X A$ on (X, e_X), if it satisfies

(M1) $A(x) \le e_X(x_1, x)$,

(M2) $\bigwedge_{x\in X}(A(x) \to e_X(y, x)) \le e_X(y, x_1)$.

The pair (X, e_X) is called *fuzzy meet complete* if $\sqcap_X A$ exists for each $A \in L^X$.

The pair (X, e_X) is called *fuzzy complete* if $\sqcap_X A$ and $\sqcup_X A$ exists for each $A \in L^X$.

Remark 2.1. Let (X, e_X) be a fuzzy poset and $A \in L^X$.

(1) $x_0 = \sqcup_X A$ on (X, e_X) iff $\bigwedge_{x\in X}(A(x) \to e_X(x, y)) = e_X(x_0, y)$.

(2) $x_1 = \sqcap_X A$ on (X, e_X) iff $\bigwedge_{x\in X}(A(x) \to e_X(y, x)) = e_X(y, x_1)$.

(3) If $e_X(x, y) = e_X(z, y)$ for all $y \in X$, then $1 = e_X(x, x) = e_X(z, x)$ and $e_X(x, z) = e_X(z, z) = 1$ implies $x = z$.

Definition 2.5. [9,18] Let (L^X, e_{L^X}) and (L^Y, e_{L^Y}) be fuzzy posets and $\mathcal{F} : L^X \to L^Y$ a map.

(1) $\mathcal{F}$ is called a *join preserving map* if $\mathcal{F}(\sqcup_{L^X}\Phi) = \sqcup_{L^Y}\mathcal{F}^{\to}(\Phi)$ for all $\Phi \in L^{L^X}$, where $\mathcal{F}^{\to}(\Phi)(B) = \bigvee_{\mathcal{F}(A)=B}\Phi(A)$.

(2) $\mathcal{F}$ is is called a *meet preserving map* if $\mathcal{F}(\sqcap_{L^X}\Phi) = \sqcap_{L^Y}\mathcal{F}^{\to}(\Phi)$ for all $\Phi \in L^{L^X}$.

3. Fuzzy join and meet preserving maps on Alexandrov L-pretopologies

Definition 3.1. (1) A subset $\tau \subset L^X$ is called an *Alexandrov L-pretopology* on X iff it satisfies the following conditions:

(O1) $\alpha_X \in \tau$.

(O2) If $A_i \in \tau$ for all $i \in I$, then $\bigvee_{i\in I} A_i \in \tau$.

(O3) If $A \in \tau$ and $\alpha \in L$, then $\alpha \odot A \in \tau$.

(2) A subset $\eta \subset L^X$ is called an *Alexandrov L-precotopology* on X iff it satisfies the following conditions:

(CO1) $\alpha \to \perp_X \in \eta$.

(CO2) If $A_i \in \eta$ for all $i \in I$, then $\bigwedge_{i\in I} A_i \in \eta$.

(CO3) If $A \in \eta$ and $\alpha \in L$, then $\alpha \to A \in \eta$.

A subset $\tau \subset L^X$ is called an *Alexandrov L-topology* on X iff it is both Alexandrov L-pretopology and Alexandrov L-precotopology on X.

Lemma 3.2. Let $\tau \subset L^X$. Define $e_\tau : \tau \times \tau \to L$ as $e_\tau(A, B) = \bigwedge_{x\in X}(A(x) \to B(x))$. Then the following statements hold.

(1) (τ, e_τ) is a fuzzy poset.

(2) $\sqcup_\tau \Phi$ is a fuzzy join of $\Phi \in L^\tau$ iff $\bigwedge_{A\in\tau}(\Phi(A) \to e_\tau(A, B)) = e_\tau(\sqcup_\tau \Phi, B)$.

(3) $\sqcap_\tau \Phi$ is a fuzzy meet of $\Phi \in L^\tau$ iff $\bigwedge_{A\in\tau}(\Phi(A) \to e_\tau(B, A)) = e_\tau(B, \sqcap_\tau \Phi)$.

(4) If $\sqcup_\tau \Phi$ is a fuzzy join of $\Phi \in L^\tau$, then it is unique. Moreover, if $\sqcap_\tau \Phi$ is a fuzzy meet of $\Phi \in L^\tau$, then it is unique.

Proof (1) (E1) $e_\tau(A, A) = \bigwedge_{x\in X}(A(x) \to A(x)) = \top$ for all $A \in \tau$,

(E2) By Lemma 2.2(9), $e_\tau(A, B) \odot e_\tau(B, C) = \bigwedge_{x\in X}(A(x) \to B(x)) \odot \bigwedge_{x\in X}(B(x) \to C(x)) \leq \bigwedge_{x\in X}((A(x) \to B(x)) \odot (B(x) \to C(x))) \leq e_\tau(A, C)$, for all $A, B, C \in \tau$,

(E3) If $e_\tau(A, B) = e_\tau(B, A) = \top$, By Lemma 2.2(3), $A = B$. Hence (τ, e_τ) is a fuzzy poset.

(2) Let $\sqcup_\tau \Phi$ be a fuzzy join of $\Phi \in L^\tau$. By (J1), since $\Phi(A) \leq e_\tau(A, \sqcup_\tau \Phi)$, we have $\Phi(A) \odot e_\tau(\sqcup_\tau \Phi, B) \leq e_\tau(A, \sqcup_\tau \Phi) \odot e_\tau(\sqcup_\tau \Phi, B) \leq e_\tau(A, B)$.

Hence $e_\tau(\sqcup_\tau \Phi, B) \leq \bigwedge_{A\in\tau}(\Phi(A) \to e_\tau(A, B))$. By (J2), $e_\tau(\sqcup_\tau \Phi, B) = \bigwedge_{A\in\tau}(\Phi(A) \to e_\tau(A, B))$

(3) It is similarly proved as (2).

(4) Let A_1, A_2 be fuzzy joins of $\Phi \in L^\tau$. Then , for all $B \in \tau$,

$$\bigwedge_{A\in\tau}(\Phi(A) \to e_\tau(A, B)) = e_\tau(A_1, B) = e_\tau(A_2, B).$$

Put $B = A_1$. Then $\top = e_\tau(A_1, A_1) = e_\tau(A_2, A_1)$ iff $A_2 \leq A_1$. Put $B = A_2$. Then $\top = e_\tau(A_1, A_2) = e_\tau(A_2, A_2)$ iff $A_1 \leq A_2$. Hence $A_1 = A_2$.

Theorem 3.3. *Let (X, τ_X) and (Y, τ_Y) be Alexandrov L-pretopological spaces. Then the following statements are equivalent:*

(1) *$\mathcal{F} : (\tau_X, e_{\tau_X}) \to (\tau_Y, e_{\tau_Y})$ is a join preserving map, that is, $\mathcal{F}(\sqcup_{\tau_X}\Phi) = \sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi)$.*

(2) *For all $\alpha \in L, A, A_i \in \tau_X$, we have $\mathcal{F}(\alpha \odot A) = \alpha \odot \mathcal{F}(A) \in \tau_Y$ and $\mathcal{F}(\bigvee_{i\in\Gamma} A_i) = \bigvee_{i\in\Gamma}\mathcal{F}(A_i) \in \tau_Y$*

Proof (1) $\Rightarrow$ (2) Since $\mathcal{F}$ is a join preserving map, we have $\mathcal{F}(\sqcup_{\tau_X}\Phi) = \sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi)$ where $\mathcal{F}^{\to}(\Phi)(B) = \bigvee_{B=\mathcal{F}(A)}\Phi(A)$ for all $\Phi \in L^{\tau_X}$. Moreover,

$$\begin{aligned} e_{\tau_X}(\sqcup_{\tau_X}\Phi, B) &= \textstyle\bigwedge_{A\in\tau_X}(\Phi(A) \to e_\tau(A, B)) \\ &= \textstyle\bigwedge_{A\in\tau} e_{\tau_X}(\Phi(A) \odot A, B) \\ &= e_{\tau_X}(\textstyle\bigvee_{A\in\tau_X}\Phi(A) \odot A, B), \end{aligned}$$

$$\begin{aligned} &e_{\tau_Y}(\sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi), B) \\ &= \textstyle\bigwedge_{C\in\tau_Y}(\mathcal{F}^{\to}(\Phi)(C) \to e_{\tau_Y}(C, B)) \\ &= \textstyle\bigwedge_{C\in\tau_Y} e_{\tau_Y}(\mathcal{F}^{\to}(\Phi)(C) \odot C, B) \\ &= e_{\tau_Y}(\textstyle\bigvee_{C\in\tau_Y}\mathcal{F}^{\to}(\Phi)(C) \odot C, B). \end{aligned}$$

By Remark 2.1(3),
$\sqcup_{\tau_X}\Phi = \bigvee_{A\in\tau_X}(\Phi(A) \odot A) \in \tau_X$ and $\sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi) = \bigvee_{C\in\tau_Y}(\mathcal{F}^{\to}(\Phi)(C) \odot C) \in \tau_Y$.

Define $\Phi_1 : \tau_X \to L$ as $\Phi_1(A) = \alpha$ and $\Phi_1(B) = \bot$, otherwise. Then

$$(\sqcup_{\tau_X}\Phi_1)(x) = \bigvee_{D\in\tau_X}(\Phi_1(D)\odot D(x)) = \alpha\odot A(x).$$

Since $\mathcal{F}^{\to}(\Phi_1)(B) = \bigvee_{B=\mathcal{F}(A)}\Phi_1(A)$ and $\mathcal{F}(\sqcup_{\tau_X}\Phi_1) = \sqcup\mathcal{F}^{\to}(\Phi_1)$ for all $\Phi_1 \in L^{\tau_X}$, we have

$$\begin{aligned}\sqcup\mathcal{F}^{\to}(\Phi_1)(y) &= \textstyle\bigvee_{C=\mathcal{F}(A)\in\tau_Y}(\mathcal{F}^{\to}(\Phi_1)(C)\odot C(y))\\ &= \Phi_1(A)\odot\mathcal{F}(A)(y) = \alpha\odot\mathcal{F}(A)(y)\\ &= \mathcal{F}(\sqcup_{\tau_X}\Phi_1)(y) = \mathcal{F}(\alpha\odot A)(y).\end{aligned}$$

Hence $\mathcal{F}(\alpha\odot A) = \alpha\odot\mathcal{F}(A)\in\tau_Y$.

Let $\{A_i\in\tau_X \mid i\in\Gamma\}$ be given. Define $\Phi_2 : \tau_X\to L$ as $\Phi_2(A_i) = \top$ for $i\in\Gamma$ and $\Phi_2(B) = \bot$, otherwise. Then

$$(\sqcup_{\tau_X}\Phi_2)(x) = \bigvee_{A\in\tau_X}(\Phi_2(A)\odot A(x)) = \bigvee_{i\in\Gamma}A_i(x).$$

Since $\mathcal{F}^{\to}(\Phi_2)(B) = \bigvee_{B=\mathcal{F}(A)}\Phi_2(A)$ and $\mathcal{F}(\sqcup_{\tau_X}\Phi_2) = \sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi_2)$ for $\Phi_2 \in L^{\tau_X}$, we have

$$\begin{aligned}\mathcal{F}(\sqcup\Phi_2)(y) &= \mathcal{F}(\textstyle\bigvee_{i\in\Gamma}A_i)(y),\\ \sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi_2)(y) &= \textstyle\bigvee_{B\in L^Y}(\mathcal{F}^{\to}(\Phi_2)(B)\odot B(y))\\ &= \textstyle\bigvee_{B\in\tau_Y}((\bigvee_{B=\mathcal{F}(A)}\Phi_2(A))\odot B(y))\\ &= \textstyle\bigvee_{A\in\tau_X}(\Phi_2(A)\odot\mathcal{F}(A)(y))\\ &= \textstyle\bigvee_{i\in\Gamma}\mathcal{F}(A_i)(y).\end{aligned}$$

Hence $\mathcal{F}(\bigvee_{i\in\Gamma}A_i) = \bigvee_{i\in\Gamma}\mathcal{F}(A_i)\in\tau_Y$.

(2) $\Rightarrow$ (1) Put $B_0 = \sqcup\mathcal{F}^{\to}(\Phi)$ for all $\Phi\in L^{\tau_X}$. Then $\bigvee_{A\in\tau_X}\Phi(A)\odot\mathcal{F}(A)\in\tau_Y$ from (2). Thus,

$$\begin{aligned}&e_{\tau_Y}(B_0,B)\\ &= \textstyle\bigwedge_{C\in\tau_Y}(\mathcal{F}^{\to}(\Phi)(C)\to e_{\tau_Y}(C,B))\\ &= \textstyle\bigwedge_{C\in\tau_Y}((\bigvee_{\mathcal{F}(A)=C}\Phi(A)\to e_{\tau_Y}(\mathcal{F}(A),B))\\ &= \textstyle\bigwedge_{A\in\tau_X}(\Phi(A)\to e_{\tau_Y}(\mathcal{F}(A),B))\\ &= \textstyle\bigwedge_{A\in\tau_X}e_{\tau_Y}(\Phi(A)\odot\mathcal{F}(A),B)\\ &= e_{\tau_Y}(\textstyle\bigvee_{A\in\tau_X}\Phi(A)\odot\mathcal{F}(A),B).\end{aligned}$$

Hence $\mathcal{F}(\sqcup_{\tau_X}\Phi) = \sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi)$ from:

$$\begin{aligned}&\sqcup_{\tau_Y}\mathcal{F}^{\to}(\Phi)(y) = B_0(y) = \textstyle\bigvee_{A\in\tau_X}\Phi(A)\odot\mathcal{F}(A)(y)\\ &= \mathcal{F}(\textstyle\bigvee_{A\in\tau_X}(\Phi(A)\odot A)(y) \text{ (by (2))}\\ &= \mathcal{F}(\sqcup_{\tau_X}\Phi)(y).\end{aligned}$$

Corollary 3.4. *Let (X,τ_X) and (Y,τ_Y) be Alexandrov L-pretopological spaces. Let $f : X\to Y$ be a map and $f^{\to} : L^X\to L^Y$ defined as $f^{\to}(A)(y) = \bigvee_{x\in f^{-1}(\{y\})}A(x)$. Then the following statements are equivalent:*

(1) $f^{\to} : (\tau_X, e_{\tau_X})\to(\tau_Y, e_{\tau_Y})$ is a join preserving map.

(2) *For all* $\alpha \in L, A, A_i \in \tau_X$*, we have* $f^{\to}(\alpha \odot A) = \alpha \odot f^{\to}(A) \in \tau_Y$ *and* $f^{\to}(\bigvee_{i\in\Gamma} A_i) = \bigvee_{i\in\Gamma} f^{\to}(A_i) \in \tau_Y$

(3) $f^{\to} : (X, \tau_X) \to (Y, \tau_Y)$ *is an open map, that is, for each* $A \in \tau_X$*,* $f^{\to}(A) \in \tau_Y$*.*

We can prove the above corollary from Theorem 3.3 and $f^{\to}(\alpha \odot A) = \alpha \odot f^{\to}(A)$ and $f^{\to}(\bigvee_{i\in\Gamma} A_i) = \bigvee_{i\in\Gamma} f^{\to}(A_i)$.

Corollary 3.5. *Let* (X, τ_X) *and* (Y, τ_Y) *be an Alexandrov L-pretopological spaces. Let* $f : X \to Y$ *be a map and* $f^{\leftarrow} : L^Y \to L^X$ *defined as* $f^{\leftarrow}(B)(x) = B(f(x))$*.*

Then the following statements are equivalent:

(1) $f^{\leftarrow} : (\tau_Y, e_{\tau_Y}) \to (\tau_X, e_{\tau_X})$ *is a join preserving map.*

(2) *For all* $\alpha \in L, B, B_i \in \tau_Y$*, we have* $f^{\leftarrow}(\alpha \odot B) = \alpha \odot f^{\leftarrow}(B) \in \tau_X$ *and* $f^{\leftarrow}(\bigvee_{i\in\Gamma} B_i) = \bigvee_{i\in\Gamma} f^{\leftarrow}(B_i) \in \tau_X$*.*

(3) $f : (X, \tau_X) \to (X, \tau_Y)$ *is continuous, that is, for each* $B \in \tau_Y$*,* $f^{\leftarrow}(B) \in \tau_X$*.*

It follows from $f^{\leftarrow}(\alpha \odot A) = \alpha \odot f^{\leftarrow}(A)$ and $f^{\leftarrow}(\bigvee_{i\in\Gamma} A_i) = \bigvee_{i\in\Gamma} f^{\leftarrow}(A_i)$.

Theorem 3.6. *Let* (X, η_X) *and* (Y, η_Y) *be an Alexandrov L-precotopological spaces. Then the following statements are equivalent:*

(1) $\mathcal{G} : (\eta_X, e_{\eta_X}) \to (\eta_Y, e_{\eta_Y})$ *is a meet preserving map.*

(2) *For all* $\alpha \in L, A, A_i \in \eta_X$*, we have* $\mathcal{G}(\alpha \to A) = \alpha \to \mathcal{G}(A) \in \eta_Y$ *and* $\mathcal{G}(\bigwedge_{i\in\Gamma} A_i) = \bigwedge_{i\in\Gamma} \mathcal{G}(A_i) \in \eta_Y$*.*

Proof (1) $\Rightarrow$ (2) Since $\mathcal{G}$ is a meet preserving map, $\mathcal{G}(\sqcap_{\eta_X}\Phi) = \sqcap_{\eta_Y}\mathcal{G}^{\to}(\Phi)$ for all $\Phi \in L^{\eta_X}$. Then

$$\begin{aligned} e_{\eta_X}(B, \sqcap_{\eta_X}\Phi) &= \textstyle\bigwedge_{A\in\eta_X}(\Phi(A) \to e_{\eta_X}(B, A)) \\ &= \textstyle\bigwedge_{A\in\eta_X} e_{\eta_X}(B, \Phi(A) \to A) \\ &= e_{\eta_X}(B, \textstyle\bigwedge_{A\in\eta_X} \Phi(A) \to A), \end{aligned}$$

$$\begin{aligned} e_{\eta_Y}(B, \sqcap_{\eta_Y}\mathcal{G}^{\to}(\Phi)) &= \textstyle\bigwedge_{C\in\eta_Y}(\mathcal{G}^{\to}(\Phi)(C) \to e_{\eta_Y}(B, C)) \\ &= \textstyle\bigwedge_{C\in\eta_Y}((\bigvee_{\mathcal{G}(A)=C} \Phi(A) \to e_{\eta_Y}(B, C))) \\ &= \textstyle\bigwedge_{A\in\eta_X}(\Phi(A) \to e_{\eta_Y}(B, \mathcal{G}(A)) \\ &= \textstyle\bigwedge_{A\in\eta_X} e_{\eta_Y}(B, \Phi(A) \to \mathcal{G}(A)) \\ &= e_{\eta_Y}(B, \textstyle\bigwedge_{A\in\eta_X} \Phi(A) \to \mathcal{G}(A)). \end{aligned}$$

By Remark 2.1(3), $\sqcap_{\eta_X}\Phi = \bigwedge_{A\in\tau_X}(\Phi(A) \to A)$ and $\sqcap_{\eta_Y}\mathcal{G}^{\to}(\Phi) = \bigwedge_{A\in\eta_X}(\Phi(A) \to \mathcal{G}(A)) \in \eta_Y$.

Define $\Phi_1 : \eta_X \to L$ as $\Phi_1(A) = \alpha$ and $\Phi_1(B) = \bot$, otherwise. Then

$$(\sqcap_{\eta_X}\Phi_1)(x) = \bigwedge_{A\in\eta_X}(\Phi_1(A) \to A(x)) = \alpha \to A(x)$$

Since $\mathcal{G}^{\to}(\Phi_1)(B) = \bigvee_{B=\mathcal{G}(A)} \Phi_1(A)$ and $\mathcal{G}(\sqcap_{\eta_X}\Phi_1) = \sqcap_{\eta_Y}\mathcal{G}^{\to}(\Phi_1)$ for $\Phi_1 \in L^{\eta_X}$,

$$\begin{aligned} &\sqcap_{\eta_Y}\mathcal{G}^{\to}(\Phi_1)(y) = \textstyle\bigwedge_{B\in\eta_X}(\Phi_1(A) \to \mathcal{G}(A)(y)) \\ &= \alpha \to \mathcal{G}(A)(y) = \mathcal{G}(\sqcap_{\eta_X}\Phi_1)(y) = \mathcal{G}(\alpha \to A)(y). \end{aligned}$$

Hence $\mathcal{G}(\alpha \to A) = \alpha \to \mathcal{G}(A) \in \eta_Y$.

(J2) Let $\{A_i \in \eta_X \mid i \in \Gamma\}$ be given. Define $\Phi_2 : \eta_X \to L$ as $\Phi_2(A_i) = \top$ for $i \in \Gamma$ and $\Phi_2(B) = \bot$ otherwise. Then

$$\sqcap_{\eta_X} \Phi_2(x) = \bigwedge_{A \in \eta_X} (\Phi_2(A) \to A(x)) = \bigwedge_{i \in \Gamma} A_i(x).$$

Since $\mathcal{G}^{\to}(\Phi_2)(B) = \bigvee_{B=\mathcal{G}(A)} \Phi_2(A)$ and $\mathcal{G}(\sqcap_{\eta_X} \Phi_2) = \sqcap_{\eta_Y} \mathcal{G}^{\to}(\Phi_2)$ for $\Phi_2 \in L^{\eta_X}$, we have

$$\begin{aligned} &\sqcap_{\eta_Y} \mathcal{G}^{\to}(\Phi_2)(y) = \bigwedge_{A \in \eta_X} (\Phi_2(A) \to \mathcal{G}(A)(y)) \\ &= \bigwedge_{i \in \Gamma} \mathcal{G}(A_i)(y) = \mathcal{G}(\sqcap_{\eta_X} \Phi_2)(y) = \mathcal{G}(\bigwedge_{i \in \Gamma} A_i)(y). \end{aligned}$$

Hence $\mathcal{G}(\bigwedge_{i \in \Gamma} A_i) = \bigwedge_{i \in \Gamma} \mathcal{G}(A_i) \in \eta_Y$.

(2) $\Rightarrow$ (1) Since $\bigwedge_{A \in \eta_X} \Phi(A) \to \mathcal{G}(A) \in \eta_Y$,

$$\begin{aligned} &e_{\eta_Y}(B, \sqcap_{\eta_Y} \mathcal{G}^{\to}(\Phi)) \\ &= \bigwedge_{C \in \eta_Y} (\mathcal{G}^{\to}(\Phi)(C) \to e_{\eta_Y}(B, C)) \\ &= \bigwedge_{C \in \eta_Y} ((\bigvee_{\mathcal{G}(A)=C} \Phi(A) \to e_{\eta_Y}(B, C))) \\ &= \bigwedge_{A \in \eta_X} (\Phi(A) \to e_{\eta_Y}(B, \mathcal{G}(A)) \\ &= \bigwedge_{A \in \eta_X} e_{\eta_Y}(B, \Phi(A) \to \mathcal{G}(A)) \\ &= e_{\eta_Y}(B, \bigwedge_{A \in \eta_X} \Phi(A) \to \mathcal{G}(A)). \end{aligned}$$

Hence $\mathcal{G}(\sqcap_{\eta_X} \Phi) = \sqcap_{\eta_Y} \mathcal{G}^{\to}(\Phi)$ from:

$$\begin{aligned} &\sqcap_{\eta_Y} \mathcal{G}^{\to}(\Phi)(y) = \bigwedge_{A \in \eta_X} (\Phi(A) \to \mathcal{G}(A)(y)) \\ &= \bigwedge_{A \in \eta_X} \mathcal{G}(\Phi(A) \to A)(y) \\ &= \mathcal{G}(\bigwedge_{A \in \eta_X} (\Phi(A) \to A))(y) \\ &= \mathcal{G}(\sqcap_{\eta_X} \Phi)(y). \end{aligned}$$

Theorem 3.7. *Let $(x^*)^* = x$ for each $x \in L$. Let (X, η_X) and (Y, η_Y) be an Alexandrov L-precotopological spaces. Then the following statements are equivalent:*

(1) *$\mathcal{G} : (\eta_X, e_{\eta_X}) \to (\eta_Y, e_{\eta_Y})$ is a meet preserving map.*

(2) *Define $\mathcal{F} : (\tau_X, e_{\tau_X}) \to (\tau_Y, e_{\tau_Y})$ as $\mathcal{F}(A) = \mathcal{G}^*(A^*)$ where $\tau_X = \{A^* \in L^X \mid A \in \eta_X\}$ and $\tau_Y = \{B^* \in L^Y \mid B \in \eta_Y\}$. Then $\mathcal{F} : \tau_X \to \tau_Y$ is a join preserving map.*

Proof (1) $\Rightarrow$ (2) Put $B_0 = \sqcup \mathcal{F}^{\to}(\Phi)$ for each $\Phi \in L^{\tau_X}$. Then, for $\Psi \in L^{\eta_X}$ with $\Psi(A^*) = \Phi(A)$, since $\mathcal{G}(\sqcap_{\eta_X} \Psi) = \mathcal{G}(\bigwedge_{A^* \in \eta_X} (\Psi(A^*) \to A^*)) = \sqcap_{\eta_Y} \mathcal{G}^{\to}(\Psi) = \bigwedge_{A^* \in \eta_X} (\Psi(A^*) \to \mathcal{G}(A^*))$, we have

$$
\begin{aligned}
& e_{\tau_Y}(B_0, B) \\
& = \bigwedge_{C\in\tau_Y}(\mathcal{F}^{\rightarrow}(\Phi)(C) \rightarrow e_{\tau_Y}(C, B)) \\
& = \bigwedge_{C\in\tau_Y}((\bigvee_{\mathcal{F}(A)=C}\Phi(A) \rightarrow e_{\tau_Y}(\mathcal{F}(A), B)) \\
& = \bigwedge_{A\in\tau_X}(\Phi(A) \rightarrow e_{\tau_Y}(\mathcal{F}(A), B)) \\
& = \bigwedge_{A^*\in\tau_X^*}(\Phi(A) \rightarrow e_{\eta_Y}(B^*, \mathcal{F}^*(A))) \\
& = e_{\eta_Y}(B^*, \bigwedge_{A^*\in\tau_X^*}(\Phi(A) \rightarrow \mathcal{F}^*(A))) \\
& = e_{\eta_Y}(B^*, \bigwedge_{A^*\in\tau_X^*}(\Psi(A^*) \rightarrow \mathcal{G}(A^*))) \\
& = e_{\tau_Y}(\bigvee_{A\in\tau_X}(\Psi(A^*) \odot \mathcal{G}^*(A^*)), B) \\
& = e_{\tau_Y}(\bigvee_{A\in\tau_X}\Phi(A) \odot \mathcal{F}(A), B).
\end{aligned}
$$

$$
\begin{aligned}
e_{\tau_Y}(B_0, B) & = e_{\eta_Y}(B^*, \bigwedge_{A^*\in\tau_X^*}(\Psi(A^*) \rightarrow \mathcal{G}(A^*))) \\
& = e_{\eta_Y}(B^*, \mathcal{G}(\bigwedge_{A^*\in\tau_X^*}(\Psi(A^*) \rightarrow A^*))) \\
& = e_{\eta_Y}(\mathcal{G}^*(\bigwedge_{A^*\in\tau_X^*}(\Psi(A^*) \rightarrow A^*)), B) \\
& = e_{\tau_Y}(\mathcal{F}(\bigvee_{A\in\tau_X}\Phi(A) \odot A), B). \\
& = e_{\tau_Y}(\mathcal{F}(\sqcup\Phi), B).
\end{aligned}
$$

Hence $\mathcal{F}(\sqcup\Phi) = \sqcup\mathcal{F}^{\rightarrow}(\Phi)$.

(2) $\Rightarrow$ (1) It is similarly proved as (1) $\Rightarrow$ (2).

From above theorems, we can obtain the following corollaries.

Corollary 3.8. *Let $(x^*)^* = x$ for each $x \in L$. Let (X, η_X) and (Y, η_Y) be an Alexandrov L-precotopological spaces. Let $f : X \rightarrow Y$ be a map. Then the following statements are equivalent:*

(1) *$f^{\rightarrow} : (\eta_X, e_{\eta_X}) \rightarrow (\eta_Y, e_{\eta_Y})$ is a meet preserving map.*

(2) *For all $\alpha \in L, A, A_i \in \eta_X$, we have $f^{\rightarrow}(\alpha \rightarrow A) = \alpha \rightarrow f^{\rightarrow}(A) \in \eta_Y$ and $f^{\rightarrow}(\bigwedge_{i\in\Gamma} A_i) = \bigwedge_{i\in\Gamma} f^{\rightarrow}(A_i) \in \eta_X$*

(3) *Define $h : \tau_X \rightarrow \tau_Y$ as $h(A) = (f^{\rightarrow}(A^*))^*$ where $\tau_X = \{A^* \in L^X \mid A \in \eta_X\}$ and $\tau_Y = \{B^* \in L^Y \mid B \in \eta_Y\}$. Then h is a join preserving map.*

Since $f^{\leftarrow}(\alpha \rightarrow A) = \alpha \rightarrow f^{\leftarrow}(A)$, $f^{\leftarrow}(\bigwedge_{i\in\Gamma} A_i) = \bigwedge_{i\in\Gamma} f^{\leftarrow}(A_i)$ and $f^{\leftarrow}(A) = (f^{\leftarrow}(A^*))^*$, the following corollary holds.

Corollary 3.9. *Let $(x^*)^* = x$ for each $x \in L$. Let (X, η_X) and (Y, η_Y) be an Alexandrov L-precotopological spaces with $\tau_X = \{A^* \in L^X \mid A \in \eta_X\}$ and $\tau_Y = \{B^* \in L^Y \mid B \in \eta_Y\}$.. Let $f : X \rightarrow Y$ be a map. Then the following statements are equivalent:*

(1) *$f^{\leftarrow} : (\eta_Y, e_{\eta_Y}) \rightarrow (\eta_X, e_{\eta_X})$ is a meet preserving map.*

(2) *For all $\alpha \in L, A, A_i \in \eta_Y$, we have $f^{\leftarrow}(\alpha \rightarrow A) = \alpha \rightarrow f^{\leftarrow}(A) \in \eta_X$ and $f^{\leftarrow}(\bigwedge_{i\in\Gamma} A_i) = \bigwedge_{i\in\Gamma} f^{\leftarrow}(A_i) \in \eta_X$*

(3) *$f : (X, \eta_X) \rightarrow (X, \eta_Y)$ is continuous;i.e., for each $A \in \tau_Y$, $f^{\leftarrow}(A) \in \tau_X$.*

(4) *$f^{\leftarrow} : (\tau_Y, e_{\tau_Y}) \rightarrow (\tau_X, e_{\tau_X})$ is a join preserving map.*

(5) *$f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ is continuous;i.e., for each $A \in \tau_Y$, $f^{\leftarrow}(A) \in \tau_X$.*

Example 3.10. Let $([0,1], \odot, \to, 0, 1)$ be a complete residuated lattice (ref.[1-4]) as

$$x \odot y = \max\{0, x+y-1\},\ x \to y = \min\{1-x+y, 1\}.$$

Define $x^* = x \to 0 = 1-x$. Then $(x^*)^* = x$. Let $X = \{x, y, z\}$ and $A \in [0,1]^X$ with $A(x) = 0.6, A(y) = 0.7, A(z) = 0.4$.

(1) Define an Alexandrov $[0,1]$-pretopology

$$\tau_X = \{(\alpha \odot A) \vee \beta_X) \mid \alpha, \beta \in [0,1]\}.$$

Since $0.8 \to A = (0.8, 0.9, 0.6) \notin \tau_X$, τ_X is not an Alexandrov $[0,1]$-precotopology. Moreover, (τ_X, e_{τ_X}) is a fuzzy poset . For each $\Phi : \tau_X \to [0,1]$, since $\bigvee_{C \in \tau_X}(\Phi(C) \odot C) \in \tau_X$ for $C \in \tau_X$, it follows that

$$\begin{aligned} e_\tau(\sqcup_{\tau_X}\Phi, B) &= \bigwedge_{C \in \tau_X}(\Phi(C) \to e_{\tau_X}(C, B)) \\ &= \bigwedge_{C \in \tau_X} e_{\tau_X}(\Phi(C) \odot C, B) \\ &= e_{\tau_X}(\bigvee_{C \in \tau_X}(\Phi(C) \odot C), B) \end{aligned}$$

By Lemma 2.9(2), (τ_X, e_{τ_X}) is a fuzzy join-complete lattice.

We obtain an Alexandrov $[0,1]$-precotopology $\eta = \{(\alpha \to A) \wedge \beta_X \mid \alpha, \beta \in [0,1]\}$ and an Alexandrov $[0,1]$-topology $\tau = \{((\alpha \odot A) \vee \beta_X), (\alpha \to A) \wedge \beta_X \mid \alpha, \beta \in [0,1]\}$. Similarly, (η, e_η) is a fuzzy meet-complete lattice and (η, e_τ) is a fuzzy complete lattice.

(2) From (1), we obtain an Alexandrov $[0,1]$-precotopology

$$\eta_X = \{A^* \mid A \in \tau_X\} = \{(\alpha \to A^*) \wedge \beta_X \mid \alpha, \beta \in [0,1]\}.$$

Then (η_X, e_{η_X}) is a fuzzy poset. For each $\Psi : \eta_X \to [0,1]$ such that $\Psi(A) = \Phi(A^*)$, since $\bigvee_{C^* \in \tau_X}(\Phi(C^*) \odot C^*) \in \tau_X$, we have

$$\begin{aligned} e_{\eta_X}(B, \sqcap_{\eta_X}\Psi) &= \bigwedge_{C \in \eta_X}(\Psi(C) \to e_{\eta_X}(B, C)) \\ &= \bigwedge_{C^* \in \tau_X}(\Psi(C) \to e_{\tau_X}(C^*, B^*)) \\ &= \bigwedge_{C^* \in \tau_X} e_{\tau_X}(\Psi(C) \odot C^*, B^*) \\ &= e_{\tau_X}(\bigvee_{C^* \in \tau_X}(\Phi(C^*) \odot C^*), B^*) \\ &= e_{\eta_X}(B, \bigwedge_{C^* \in \tau_X}(\Phi(C^*) \to C)) \\ &= e_{\eta_X}(B, \bigwedge_{C \in \tau_X^*}(\Psi(C) \to C)) \end{aligned}$$

By Lemma 2.9(3), (η_X, e_{η_X}) is a fuzzy meet-complete lattice.

(3) Let $Y = \{u, v\}$ and $f : X \to Y$ be a map defined as $f(x) = f(y) = u, f(z) = v$. Then we obtain $f^\to(A)(u) = 0.7, f^\to(A)(v) = 0.4$. We obtain an Alexandrov $[0,1]$-pretopology

$$\tau_Y = \{(\alpha \odot f^\to(A)) \vee \beta_Y \mid \alpha, \beta \in [0,1]\}.$$

Then (τ_Y, e_{τ_Y}) is a fuzzy poset and a join-complete lattice. Since $f^\to(\alpha \odot A) = \alpha \odot f^\to(A)$ and $f^\to(\bigvee_{i \in \Gamma} A_i) = \bigvee_{i \in \Gamma} f^\to(A_i)$, by Corollary 3.4(2) , $f^\to : (\tau_X, e_{\tau_X}) \to (\tau_Y, e_{\tau_Y})$ is a join-preserving map. Since $f^\leftarrow(f^\to(A)) = (0.7, 0.7, 0.4) \notin \tau_X$ for $f^\to(A) \in \tau_Y$, by Corollary 3.5(3), $f^\leftarrow : (\tau_Y, e_{\tau_Y}) \to (\tau_X, e_{\tau_X})$ is not a join-preserving map.

We obtain an Alexandrov $[0,1]$-precotopology

$$\eta_Y = \{(\alpha \to f^{\to}(A)^*) \wedge \beta_Y \mid \alpha, \beta \in [0,1]\}.$$

For $A^* \in \eta_X$, $f^{\to}(A^*) = (0.4, 0.6) = (0.9 \to f^{\to}(A)^*) \wedge 0.6_X \in \eta_Y$. By Corollary 3.8(2), $f^{\to} : (\eta_X, e_{\eta_X}) \to (\eta_Y, e_{\eta_Y})$ is a meet-preserving map.

We obtain an Alexandrov $[0,1]$-precotopology

$$\eta^1_Y = \{(\alpha \to f^{\to}(A^*)) \wedge \beta_Y) \mid \alpha, \beta \in [0,1]\}.$$

Since $\odot$ is continuous;i.e; $x \odot \bigwedge_{i\in\Gamma} y_i^* = \bigwedge_{i\in\Gamma}(x \odot y_i^*)$, $(x^*)^*(x) = x$ and $(\bigvee_{i\in\Gamma} x_i)^* = \bigwedge_{i\in\Gamma} x_i^*$,

$$x \to \bigvee_{i\in\Gamma} y_i = (x \odot \bigwedge_{i\in\Gamma} y_i^*)^* = (\bigwedge_{i\in\Gamma}(x \odot y_i^*))^* = \bigvee_{i\in\Gamma}(x \to y_i).$$

Since f is onto, $f^{\to}((\alpha \to A^*) \wedge \beta_X)(y) = \bigvee_{x\in f^{-1}(\{y\})}((\alpha \to A^*) \wedge \beta_X)(x) = (\alpha \to \bigvee_{x\in f^{-1}(\{y\})} A^*(x)) \wedge \beta_Y = ((\alpha \to f^{\to}(A^*)(y)) \wedge \beta_Y) \in \eta^1_Y$.

Since $\bigwedge_{i\in\Gamma} B_i = (\alpha \to A^*) \wedge \beta_X$ for $B_i \in \eta_X$, $f^{\to}(\bigwedge_{i\in\Gamma} B_i) = \bigwedge_{i\in\Gamma} f^{\to}(B_i) \in \eta^1_Y$. By Corollary 3.8(2), $f^{\to} : (\eta_X, e_{\eta_X}) \to (\eta^1_Y, e_{\eta^1_Y})$ is a meet-preserving map.

(4) Let $Z = \{u, v, w\}$ and $h : X \to Z$ be a map defined as $h(x) = h(y) = u, h(z) = v$. We obtain an Alexandrov $[0,1]$-pretopology

$$\tau_Z = \{(\alpha \odot h^{\to}(A)) \vee \beta_Z) \mid \alpha, \beta \in [0,1]\}.$$

Then (τ_Z, e_{τ_Z}) is a fuzzy poset. Since $h^{\to}(\alpha \odot A) = \alpha \odot h^{\to}(A)$ and $h^{\to}(\bigvee_{i\in\Gamma} A_i) = \bigvee_{i\in\Gamma} h^{\to}(A_i)$, by Corollary 3.4(2), $h^{\to} : (\tau_X, e_{\tau_X}) \to (\tau_Z, e_{\tau_Z})$ is a join-preserving map.

We obtain an Alexandrov $[0,1]$-precotopology

$$\eta_Z = \{(\alpha \to h^{\to}(A^*)) \wedge \beta_Z) \mid \alpha, \beta \in [0,1]\}.$$

Since

$$\begin{aligned} &h^{\to}(A^*)(u) = 0.4, h^{\to}(A^*)(v) = 0.6, h^{\to}(A^*)(w) = 0,\\ &h^{\to}(0.7 \to A^*) = (0.7, 0.9, 0)\\ &\neq 0.7 \to h^{\to}(A^*) = (0.7, 0.9, 0.3). \end{aligned}$$

By Corollary 3.8(2), $h^{\to} : (\eta_X, e_{\eta_X}) \to (\eta_Z, e_{\eta_Z})$ is not a meet-preserving map.

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