

New Design Method of Stable Lens System Against Chromatic Variation Based on Paraxial Ray Tracing

Jong-Ung Lee*

Department of Laser and Optical Information Engineering, Cheongju University, Cheongju 28503, Korea

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This paper presents a new method for designing a lens system stable against chromatic variation at a specified wavelength. Conventional lenses are corrected for chromatic aberration, but the new method suppresses chromatic changes of the marginal ray in the image-side. By doing so, paraxial properties of the lens system are stabilized against chromatic variation. Since the new method is based on paraxial ray tracing, the stabilizing conditions against chromatic variation are given by recurrence formulas. However, there is an analytic solution for the case of a cemented doublet in the air. A stable doublet at 405 nm wavelength is designed and analyzed.

Keywords : Lens system design, Chromatic variation, Cemented doublet, Achromatization

OCIS codes : (080.1010) Aberrations; (220.1000) Aberration compensation; (220.2740) Geometric optical design; (220.3620) Lens system design; (220.3630) Lenses

I. INTRODUCTION

Achromatization is a necessary process to design a lens system which operates at various wavelengths or wavelength bands. Many achromatization methods based on the thin lens approximation and dispersion constants have been introduced in optical texts and monographs dealing with optical system design [1-3]. In addition to the chromatic variation, the change in ambient temperature affects the first order properties and aberrations of optical imaging system. Hence, various design methods which correct the chromatic variation and the thermal change at the same time have been presented [4-6]. In either case, the main purpose of the achromatization is to minimize image degradation due to chromatic variation within a specified wavelength range [7-9]. Since the refractive index of optical material varies rapidly in the blue region around 400 nm, expensive abnormal glasses are used to correct the chromatic variations in the blue region. For some applications, one wants a lens system which has good imaging performance at the blue region but is not necessary for the entire range of visible light. In that case,

a lens which is stable against chromatic variation at a specified wavelength may be very suitable for the purpose. In 2017, a collimator lens that was very stable against chromatic variation was presented [10]. To minimize chromatic variation of the effective focal length, the first- and second-order wavelength derivatives of the refractive power were corrected in the paper. However, the chromatic variation of image height was not considered.

This paper presents a new method to design a stable lens system at a specified wavelength based on paraxial ray tracing. In the conventional achromatization, the chromatic aberrations such as axial and lateral color are corrected. Unlike the conventional method, the new method suppresses chromatic changes of the marginal ray in the image-side. By doing so, chromatic shift of the paraxial focus and chromatic change of the image height are minimized at the same time. The conditions for stabilizing against chromatic variation are derived from the first order wavelength derivatives of incident heights and paraxial angles. Hence, they are given by recurrence formulas. However, there is an analytic solution for the case of a cemented doublet in the air. For a design example, a stable doublet at 405 nm

*Corresponding author: julee@cju.ac.kr, ORCID 0000-0001-8245-2278

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wavelength is designed. The paraxial properties and RMS wavefront errors of the lens are very stable against chromatic variations around 405 nm wavelength, as expected.

II. STABILIZATION OF LENS SYSTEM AGAINST CHROMATIC VARIATION

Figure 1 shows a lens system and its marginal ray. Let us begin with the first order properties of the lens system and use the following notations:

- n_j , refractive index of the medium after refraction on the j -th surface
- h_j , incident height of the marginal ray on the j -th surface
- u_j , paraxial angle of the marginal ray after refraction on the j -th surface
- η_j , image height formed by the j -th surface
- r_j , curvature radius of the j -th surface
- d_j , axial distance from the j -th surface to the next surface
- $\psi_j \equiv \frac{n_j - n_{j-1}}{r_j}$, refractive power of the j -th surface
- $t_j \equiv \frac{d_j}{n_j}$, converted thickness of d_j in the air
- $\alpha_j \equiv n_j u_j$, numerical aperture of the marginal ray after refraction
- $a_{j-1} \equiv \frac{h_j}{h_{j-1}}$, relative height of the marginal ray
- $m_j \equiv \frac{n_{j-1} u_{j-1}}{n_j u_j}$, transverse magnification of the j -th surface

In Fig. 1, surface 0 is the object, and η_0 is the object height. In the image-side, surface k means the last surface of the lens system. Hence, η_k is image height of lens system. In optical imaging, Lagrange's invariant and its chromatic variation in the object-side are given as follows:

$$H = n_0 u_0 \eta_0,$$

$$\frac{dH}{d\lambda} = \left(\frac{dn_0}{d\lambda}\right) u_0 \eta_0 + n_0 \left(\frac{du_0}{d\lambda}\right) \eta_0 + n_0 u_0 \left(\frac{d\eta_0}{d\lambda}\right)$$

$$= H \left\{ \frac{1}{n_0} \left(\frac{dn_0}{d\lambda}\right) + \frac{1}{u_0} \left(\frac{du_0}{d\lambda}\right) + \frac{1}{\eta_0} \left(\frac{d\eta_0}{d\lambda}\right) \right\}. \quad (1)$$

By the same way, Lagrange's invariant and its chromatic variation in the image-side are given as follows:

$$H = n_k u_k \eta_k,$$

$$\frac{dH}{d\lambda} = H \left\{ \frac{1}{n_k} \left(\frac{dn_k}{d\lambda}\right) + \frac{1}{u_k} \left(\frac{du_k}{d\lambda}\right) + \frac{1}{\eta_k} \left(\frac{d\eta_k}{d\lambda}\right) \right\}. \quad (2)$$

Even though the wavelength of imaging light varies, the object height η_0 and the incident angle u_0 are not changed:

$$\frac{du_0}{d\lambda} = 0, \quad \frac{d\eta_0}{d\lambda} = 0. \quad (3)$$

Since Lagrange's invariant and its chromatic variation should be same on both sides, the chromatic variation of H always satisfies the following relation:

$$\frac{1}{H} \left(\frac{dH}{d\lambda}\right) = \frac{1}{n_0} \left(\frac{dn_0}{d\lambda}\right) = \frac{1}{n_k} \left(\frac{dn_k}{d\lambda}\right) + \frac{1}{u_k} \left(\frac{du_k}{d\lambda}\right) + \frac{1}{\eta_k} \left(\frac{d\eta_k}{d\lambda}\right). \quad (4)$$

Consider only the lens system whose object and image are in the same medium,

$$\frac{1}{n_0} \left(\frac{dn_0}{d\lambda}\right) = \frac{1}{n_k} \left(\frac{dn_k}{d\lambda}\right), \quad (5)$$

then the chromatic variation of image height is given by Eq. (6):

$$\frac{d\eta_k}{d\lambda} = -\frac{\eta_k}{u_k} \left(\frac{du_k}{d\lambda}\right). \quad (6)$$

Eq. (6) means that if the chromatic variation of u_k is suppressed, then the chromatic variation of image height is suppressed also. This is the first condition to design a lens system stable against chromatic variation:

$$\frac{du_k}{d\lambda} = 0, \quad \text{the condition for stabilizing image height.} \quad (7)$$

Let's return to Fig. 1. The position of paraxial image and its wavelength derivative are given as follows:

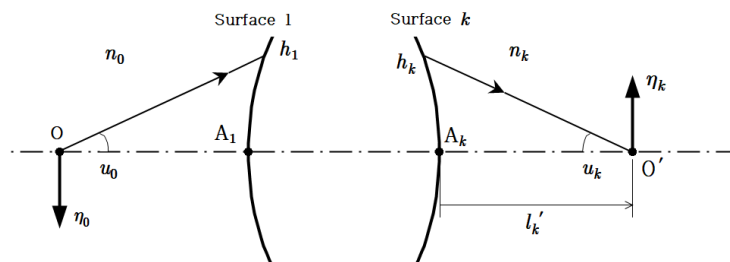


FIG. 1. Layout of an optical imaging system.

$$l'_k = -\frac{h_k}{u_k}, \quad (8)$$

$$\frac{dl'_k}{d\lambda} = -\frac{1}{u_k} \left(\frac{dh_k}{d\lambda} \right) + \frac{h_k}{u_k^2} \left(\frac{du_k}{d\lambda} \right) = l'_k \left\{ \frac{1}{h_k} \left(\frac{dh_k}{d\lambda} \right) - \frac{1}{u_k} \left(\frac{du_k}{d\lambda} \right) \right\}. \quad (9)$$

Even though a lens system has been stabilized for the chromatic variation of image height, the chromatic variation of h_k affects image position. Therefore, the chromatic variation of h_k should be suppressed to fix image position.

$$\frac{dh_k}{d\lambda} = 0, \quad \text{the additional condition for stabilizing image position.} \quad (10)$$

Eqs. (7) and (10) are the stabilizing conditions presented in this paper. In conventional design, chromatic aberrations are corrected to minimize chromatic variation, but we suppress the chromatic variations of the marginal ray in the image-side. By doing so, the image position and the image height are stabilized against chromatic variation at the same time.

Figure 2 shows a paraxial ray transferring from the $(j-1)$ -th surface to the j -th surface and refracting on the j -th surface. Eq. (11) is the transfer equation of the ray, and Eq. (12) is the refraction equation:

$$h_j = h_{j-1} + d_{j-1}u_{j-1}, \quad (11)$$

$$n_j u_j = n_{j-1} u_{j-1} - h_j \psi_j. \quad (12)$$

Let's rewrite Eqs. (11) and (12) by using the converted thickness in the air t_j and the paraxial numerical aperture α_j as follows:

$$h_j = h_{j-1} + t_{j-1} \alpha_{j-1}, \quad (13)$$

$$\alpha_j = \alpha_{j-1} - h_j \psi_j. \quad (14)$$

From the definitions of t_j and ψ_j , the chromatic variations of t_j and ψ_j are given as follows:

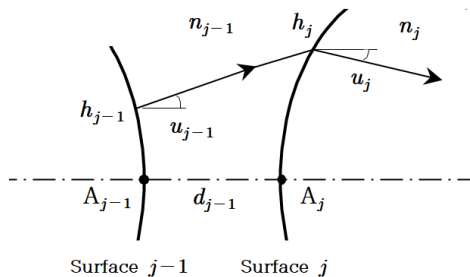


FIG. 2. Ray transfer between surfaces.

$$\frac{dt_j}{d\lambda} = -\frac{d_j}{n_j^2} \left(\frac{dn_j}{d\lambda} \right) = -\left(\frac{1}{n_j} \frac{dn_j}{d\lambda} \right) t_j, \quad (15)$$

$$\frac{d\psi_j}{d\lambda} = \frac{1}{r_j} \left(\frac{dn_j}{d\lambda} - \frac{dn_{j-1}}{d\lambda} \right) = \frac{1}{(n_j - n_{j-1})} \left(\frac{dn_j}{d\lambda} - \frac{dn_{j-1}}{d\lambda} \right) \psi_j. \quad (16)$$

By using Eqs. (13) and (14), the above equations are rewritten as the functions of h and α :

$$\frac{dt_{j-1}}{d\lambda} = -\left(\frac{1}{n_{j-1}} \frac{dn_{j-1}}{d\lambda} \right) \left(\frac{h_j - h_{j-1}}{\alpha_{j-1}} \right), \quad (17)$$

$$\frac{d\psi_j}{d\lambda} = -\frac{1}{(n_j - n_{j-1})h_j} \left(\frac{dn_j}{d\lambda} - \frac{dn_{j-1}}{d\lambda} \right) (\alpha_j - \alpha_{j-1}). \quad (18)$$

From Eqs. (13), (17), (14) and (18), the chromatic variation of ray height and numerical aperture are given as follows:

$$\frac{dh_j}{d\lambda} = \frac{dh_{j-1}}{d\lambda} + \left\{ \frac{d\alpha_{j-1}}{d\lambda} - \frac{\alpha_{j-1}}{n_{j-1}} \left(\frac{dn_{j-1}}{d\lambda} \right) \right\} \left(\frac{h_j - h_{j-1}}{\alpha_{j-1}} \right), \quad (19)$$

$$\frac{d\alpha_j}{d\lambda} = \frac{d\alpha_{j-1}}{d\lambda} + \left\{ \frac{1}{h_j} \left(\frac{dh_j}{d\lambda} \right) + \frac{1}{n_j - n_{j-1}} \left(\frac{dn_j}{d\lambda} - \frac{dn_{j-1}}{d\lambda} \right) \right\} (\alpha_j - \alpha_{j-1}). \quad (20)$$

For convenience, let's rewrite Eqs. (19) and (20) by using the dimensionless design parameter a_j and m_j [11, 12]:

$$\frac{dh_j}{d\lambda} = \frac{dh_{j-1}}{d\lambda} + \left\{ \frac{1}{\alpha_{j-1}} \left(\frac{d\alpha_{j-1}}{d\lambda} \right) - \frac{1}{n_{j-1}} \left(\frac{dn_{j-1}}{d\lambda} \right) \right\} \cdot (a_{j-1} - 1) h_{j-1}, \quad (21)$$

$$\frac{d\alpha_j}{d\lambda} = \frac{d\alpha_{j-1}}{d\lambda} + \left\{ \frac{1}{h_j} \left(\frac{dh_j}{d\lambda} \right) + \frac{1}{n_j - n_{j-1}} \left(\frac{dn_j}{d\lambda} - \frac{dn_{j-1}}{d\lambda} \right) \right\} \cdot (1 - m_j) \alpha_j. \quad (22)$$

Let's consider Eqs. (7) and (10), the conditions for stabilizing against chromatic variation. Since the image is in the air and the k -th surface is the last surface of the lens system, the conditions can be expressed by the following recurrence formulas:

$$\frac{dh_k}{d\lambda} = \frac{dh_{k-1}}{d\lambda} + \left\{ \frac{1}{\alpha_{k-1}} \left(\frac{d\alpha_{k-1}}{d\lambda} \right) - \frac{1}{n_{k-1}} \left(\frac{dn_{k-1}}{d\lambda} \right) \right\} \cdot (a_{k-1} - 1) h_{k-1} = 0, \quad (23)$$

$$\begin{aligned} \frac{du_k}{d\lambda} &= \frac{d\alpha_k}{d\lambda} = \frac{d\alpha_{k-1}}{d\lambda} \\ &+ \left\{ \frac{1}{h_k} \left(\frac{dh_k}{d\lambda} \right) + \frac{1}{n_k - n_{k-1}} \left(\frac{dn_k}{d\lambda} - \frac{dn_{k-1}}{d\lambda} \right) \right\} \\ &\cdot (1 - m_k) \alpha_k = 0. \end{aligned} \quad (24)$$

III. STABILIZATION OF CEMENTED DOUBLET IN THE AIR

The stabilizing conditions could be applied all of lens system which use two or more kinds of optical materials. But it may not be easy to find the solution of Eqs. (23) and (24) if the lens system is consisted of many elements. However, there is an analytic solution for the case of cemented doublet in the air.

For the case of cemented doublet in the air, the following conditions are always satisfied.

$$n_0 = 1, \quad \frac{dn_0}{d\lambda} = 0,$$

$$n_3 = 1, \quad \frac{dn_3}{d\lambda} = 0,$$

$$\frac{dh_1}{d\lambda} = 0, \quad \frac{d\alpha_0}{d\lambda} = 0.$$

Since the 3rd surface is the last surface of the lens, the stabilizing conditions are given by

$$\frac{dh_3}{d\lambda} = \frac{dh_2}{d\lambda} + \left\{ \frac{1}{\alpha_2} \left(\frac{d\alpha_2}{d\lambda} \right) - \frac{1}{n_2} \left(\frac{dn_2}{d\lambda} \right) \right\} (a_2 - 1) h_2 = 0, \quad (25)$$

$$\frac{d\alpha_3}{d\lambda} = \frac{d\alpha_2}{d\lambda} + \left\{ \frac{1}{h_3} \left(\frac{dh_3}{d\lambda} \right) + \frac{1}{n_2 - 1} \left(\frac{dn_2}{d\lambda} \right) \right\} (1 - m_3) \alpha_3 = 0. \quad (26)$$

From the recurrence relations of Eqs. (21) and (22), we can get chromatic variations of the marginal ray at the first and the second surfaces:

$$\frac{d\alpha_1}{d\lambda} = \frac{1 - m_1}{n_1 - 1} \left(\frac{dn_1}{d\lambda} \right) \alpha_1, \quad (27)$$

$$\frac{dh_2}{d\lambda} = \frac{1 - n_1 m_1}{n_1 (n_1 - 1)} \left(\frac{dn_1}{d\lambda} \right) (a_1 - 1) h_1, \quad (28)$$

$$\begin{aligned} \frac{d\alpha_2}{d\lambda} &= \frac{d\alpha_1}{d\lambda} + \left\{ \frac{1}{h_2} \left(\frac{dh_2}{d\lambda} \right) + \frac{1}{n_2 - n_1} \left(\frac{dn_2}{d\lambda} - \frac{dn_1}{d\lambda} \right) \right\} \\ &\cdot (1 - m_2) \alpha_2. \end{aligned} \quad (29)$$

A cemented doublet has five structural parameters (a_1 , a_2 , m_1 , m_2 and m_3), and two scaling parameters (h_1 , u_3). The structural parameters determine the shape of the lens system, and scale parameters are given by specifications, EFL and f-number of the system. Since there are two conditions for stabilizing a lens system, three structural parameters should be assigned to determine the shape after selecting two glasses. For convenience, let's take a_1 , a_2 and m_1 as the known parameters. By doing so, Eqs. (25) and (26) can be expressed as linear functions of (m_2 , m_3). Let's define A_0 , A_1 and A_2 as follows:

$$A_0 \equiv \frac{1}{h_2} \left(\frac{dh_2}{d\lambda} \right) = \frac{(1 - n_1 m_1)(a_1 - 1)}{n_1 (n_1 - 1) a_1} \left(\frac{dn_1}{d\lambda} \right), \quad (30)$$

$$A_1 \equiv \frac{(1 - m_1)}{n_1 - 1} \left(\frac{dn_1}{d\lambda} \right), \quad (31)$$

$$A_2 \equiv A_0 + \frac{1}{n_2 - n_1} \left(\frac{dn_2}{d\lambda} - \frac{dn_1}{d\lambda} \right). \quad (32)$$

Then, Eq. (29) becomes

$$\frac{1}{\alpha_2} \left(\frac{d\alpha_2}{d\lambda} \right) = (A_1 - A_2) m_2 + A_2. \quad (33)$$

By using Eqs. (33) and (30), we obtain a linear equation of m_2 from Eq. (25):

$$A_0 + \left\{ (A_1 - A_2) m_2 + A_2 - \frac{1}{n_2} \left(\frac{dn_2}{d\lambda} \right) \right\} (a_2 - 1) = 0. \quad (34)$$

Eq. (34) is the solution of Eq. (25), one of the stabilizing conditions. Let's consider another condition. Eq. (26) can be rewritten as follows:

$$\begin{aligned} \frac{1}{\alpha_3} \left(\frac{d\alpha_3}{d\lambda} \right) &= \frac{1}{\alpha_2} \left(\frac{d\alpha_2}{d\lambda} \right) m_3 \\ &+ \left\{ \frac{1}{h_3} \left(\frac{dh_3}{d\lambda} \right) + \frac{1}{n_2 - 1} \left(\frac{dn_2}{d\lambda} \right) \right\} (1 - m_3) = 0. \end{aligned}$$

Now, Eq. (25) is already satisfied by Eq. (34). By using Eq. (33), the above equation becomes

$$\frac{1}{\alpha_3} \left(\frac{d\alpha_3}{d\lambda} \right) = \{ (A_1 - A_2) m_2 + A_2 \} m_3 + \frac{1 - m_3}{n_2 - 1} \left(\frac{dn_2}{d\lambda} \right) = 0. \quad (35)$$

Since m_2 is given by Eq. (34), Eq. (35) is a linear function of m_3 . Eq. (35) is the solution of Eq. (26).

When all of the structural and scaling parameters are determined, design data of the doublet is given as follows:

$$q_3 \equiv -\frac{h_1}{u_3}, \quad (36)$$

$$d_1 = \frac{n_1(1-a_1)}{m_2 m_3} q_3, \quad (37)$$

$$d_2 = \frac{n_2(1-a_2)a_1}{m_3} q_3, \quad (38)$$

$$r_1 = \frac{n_1 - n_0}{(1-m_1)m_2 m_3} q_3, \quad (39)$$

$$r_2 = \frac{(n_2 - n_1)a_1}{(1-m_2)m_3} q_3, \quad (40)$$

$$r_3 = \frac{(n_3 - n_2)a_1 a_2}{1-m_3} q_3. \quad (41)$$

parameters and two scaling parameters as mentioned in the previous section. From Tables 1 and 2, some of the design parameters are given as follows:

$$h_1 = 12.5 \text{ mm},$$

$$\alpha_3 = u_3 = -0.01,$$

$$q_3 = f' = -\frac{h_1}{u_3} = 125 \text{ mm},$$

$$n_0 = 1, \quad \frac{dn_0}{d\lambda} = 0,$$

$$n_1 = 1.530196, \quad \frac{dn_1}{d\lambda} = -1.279284 \times 10^{-4},$$

$$n_2 = 1.650759, \quad \frac{dn_2}{d\lambda} = -3.251850 \times 10^{-4},$$

$$n_3 = 1, \quad \frac{dn_3}{d\lambda} = 0,$$

$$m_1 = 0.$$

IV. DESIGN OF A STABLE CEMENTED DOUBLET

Conventional doublets are designed for the entire visible range. But they have relatively poor imaging performance in the blue region around 405 nm wavelength because of rapid change of refractive index. In order to reduce the performance degradation in the blue region, expensive abnormal glasses are used for designing super-achromats [13, 14].

In this study, a stable doublet at 405 nm wavelength is designed by using popular glasses only, not the expensive glasses. Table 1 shows basic specifications of the stable doublet. The glasses and their refractive indices are listed in Table 2. A cemented doublet has five structural

parameters (a_1 , a_2 , m_2 and m_3). When (a_1 , a_2) are chosen, then (m_2 , m_3) can be obtained by Eqs. (34) and (35). Since the stabilizing conditions correct the chromatic variations only, (a_1 , a_2) should be taken to minimize spherical aberration and coma. To get the best solution, we search the best combination of (a_1 , a_2) in the range of 0.899 to 0.999. Outside of the search range, axial thickness of the lens becomes too thick or thin so that the lens cannot be used for practical purpose.

Figure 3 shows the third order spherical aberration (SA) of the stable doublet as a function of (a_1 , a_2). SA has negative value only and makes a narrow valley along the dotted line in Fig. 3. Around A ($a_1 = 0.899$, $a_2 = 0.969$) in Fig. 3 might be the best combination if we consider SA only. But axial thicknesses of the lenses are too thick as shown in Figs. 5 and 6. The third order tangential coma (TCO) is shown in Fig. 4. There are zero coma solutions along the dotted line in Fig. 4. Axial thickness d_1 and d_2 are shown in Figs. 5 and 6 respectively.

Considering SA, TCO and axial thicknesses, the stable

TABLE 1. Design specification of the stabilized doublet at wavelength of 405 nm

Effective focal length	125 mm
Entrance pupil diameter (f-number)	25 mm (F/5.0)
Half field angle	1°
Paraxial numerical aperture	0.1
Paraxial image height	2.1819 mm
Optical glasses	NBK7, NF2

TABLE 2. Refractive indices and the first order derivatives

Glass (Schott)	Wavelength (nm)					$\frac{dn}{d\lambda}$ @ 405 nm
	385	395	405	415	425	
NBK7	1.532974	1.531528	1.530196	1.528966	1.527827	-1.279284E-04
NF2	1.657996	1.654185	1.650759	1.647664	1.644854	-3.251850E-04

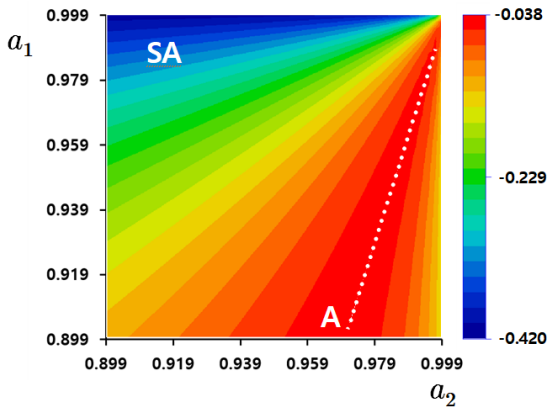


FIG. 3. Distribution of spherical aberration as a function of a_1 & a_2 .

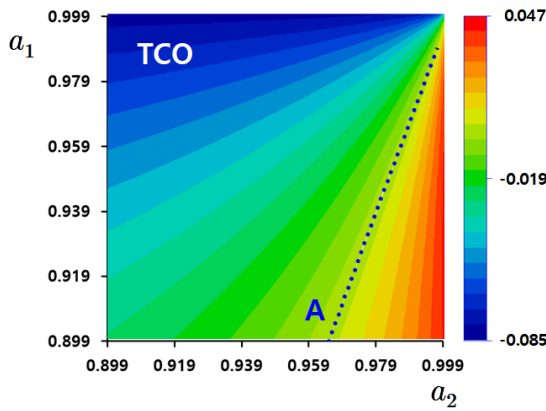


FIG. 4. Distribution of tangential coma as a function of a_1 & a_2 .

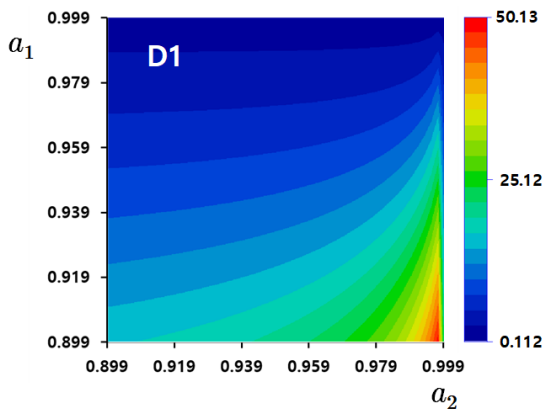


FIG. 5. Distribution of the thickness d_1 as a function of a_1 & a_2 .

doublet was taken at $a_1 = 0.961$ and $a_2 = 0.988$. The design data are listed in Table 3. Optical layout of the system is shown in Fig. 7. Marginal ray data and their chromatic variations are listed in Table 4. Table 4 shows that the marginal ray meets the stabilizing conditions of Eqs. (25) and (26). Table 5 shows the marginal ray data and paraxial properties of the stable doublet. Within wavelength 385 nm

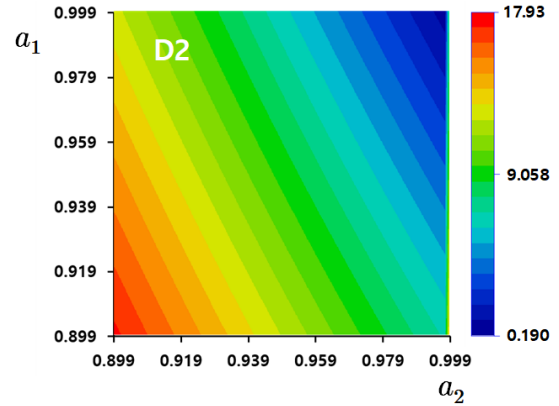


FIG. 6. Distribution of the thickness d_2 as a function of a_1 & a_2 .

TABLE 3. Design data of the stabilized doublet

#	r (mm)	d (mm)	Glass	Remark
0		Infinity		Object
1	78.30217	8.813513	NBK7	Stop
2	-58.24740	3.980852	NF2	
3	-192.00774	118.68350		
4		-0.19574		Image

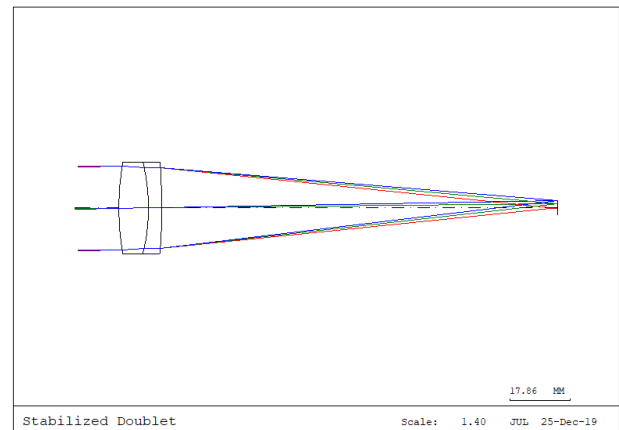


FIG. 7. Optical layout of the stabilized doublet ($a_1 = 0.961$, $a_2 = 0.988$).

to 425 nm, chromatic variations of paraxial properties are very small as expected. Figure 8 shows chromatic variations of EFL. The third order aberrations of the stable doublet are listed in Table 6. EFL has the minimum value at 405 nm wavelength in Fig. 8 and the doublet has zero LAT (lateral color). They are results of the stabilizing conditions of Eqs. (25) and (26). Figure 9 shows chromatic variations of rms wavefront errors. Since the doublet is stabilized against chromatic variations, rms wavefront errors are quite small within wavelength 390 nm to 440 nm. It may be a good characteristic of the doublet for applications in the blue region.

TABLE 4. Marginal ray data and its chromatic variations

#	Ray data		Design parameters		Chromatic variations	
	h (mm)	u	a	m	$dh/d\lambda$	$du/d\lambda$
0		0.000000			0.00000E+00	0.00000E+00
1	12.500000	-0.055313	0.961000	0.000000	0.00000E+00	8.72186E-06
2	12.012500	-0.036211	0.988000	1.415960	7.68702E-05	-1.93090E-05
3	11.868400	-0.100000		0.597754	3.82220E-09	9.31194E-10
4	0.000000					

TABLE 5. Chromatic variations of the first order parameters

Wave length (nm)	h_3 (mm)	u_3	EFL (mm)	BFL (mm)	BFL-EFL	Image height (mm)
385	11.86845	-0.099975	125.03090	118.713800	-6.317100	2.181902
390	11.86840	-0.099987	125.01640	118.699600	-6.316800	2.181893
395	11.86837	-0.099994	125.00690	118.690300	-6.316600	2.181887
400	11.86836	-0.099999	125.00160	118.685100	-6.316500	2.181884
405	11.86835	-0.100000	125.00000	118.683500	-6.316500	2.181883
410	11.86835	-0.099999	125.00150	118.685000	-6.316500	2.181884
415	11.86837	-0.099995	125.00570	118.689100	-6.316600	2.181887
420	11.86839	-0.099990	125.01220	118.695500	-6.316700	2.181891
425	11.86842	-0.099983	125.02080	118.703900	-6.316900	2.181896
Max.	11.86845	-0.099975	125.03090	118.713800	-6.316500	2.181902
Min.	11.86835	-0.100000	125.00000	118.683500	-6.317100	2.181883
Max-Min	0.00010	0.000025	0.03090	0.030300	0.000600	0.000019

TABLE 6. The third order aberrations of the stabilized doublet (units in mm, calculated by Code V)

#	SA	TCO	TAS	SAS	PTB	DST	AX	LAT	PTZ
1	-0.0576	-0.0189	-0.0031	-0.0017	-0.0011	-0.0002	-0.0671	-0.0073	-0.0044
2	0.1367	-0.0152	0.0008	0.0004	0.0002	0.0000	0.2210	-0.0082	0.0008
3	-0.1213	0.0370	-0.0042	-0.0017	-0.0005	0.0002	-0.1529	0.0155	-0.0021
SUM	-0.0421	0.0029	-0.0066	-0.0031	-0.0013	0.0000	0.0010	0.0000	-0.0057

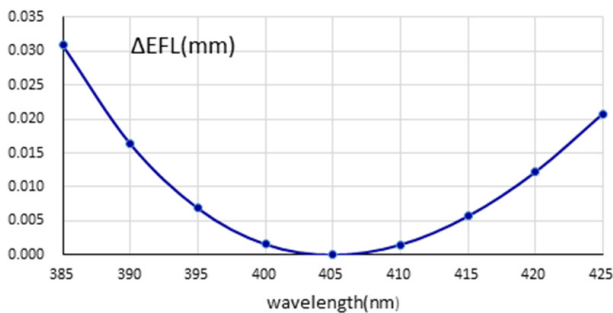


FIG. 8. Chromatic variation of effective focal length.

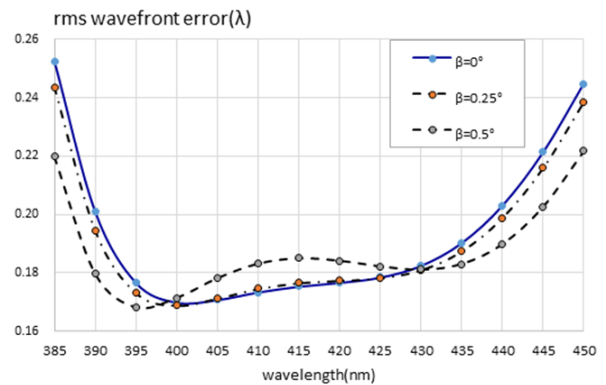


FIG. 9. Chromatic variation of rms wavefront errors.

V. CONCLUSION

A new design method to obtain a stable lens system against chromatic variation is presented. In the conventional design of a lens system, chromatic aberrations such as axial and lateral color are corrected. In contrast, the new method suppresses chromatic variations of the marginal ray in the image-side. By doing so, the position and the height of image are stabilized against chromatic variation at the same time.

Since the new method is based on paraxial ray tracing, the stabilizing conditions are given by recurrence formulas. However, there is an analytic solution for the case of cemented doublet in the air. By using the analytic solution, a stable cemented doublet at 405 nm wavelength is designed and analyzed. Paraxial properties such as EFL, BFL and image height are very stable as expected. Even though the stable doublet is designed by using popular glasses, the doublet has quite good imaging performance in the blue region. In conclusion, the new design method presented in this paper is expected to be a very useful way to design a stable lens system against chromatic variation at a desired wavelength.

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