

기반 기술

J. Adv. Navig. Technol. 24(6): 549-556, Dec. 2020

# 마코프 지연을 갖는 네트워크 제어 시스템을 위한 상태 궤환 제어기 설계 A State Feedback Controller Design for a Networked Control System with a Markov Delay

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#### [**요**] 약

이 논문에서는 마코프 프로세스로 모델링되는 전송 오류나 전송 지연이 있는 네트워크 제어 시스템을 위한 제어기 설계 방법 들을 제안한다. 마코프 지연을 갖는 제어 시스템을 위한 안정화 조건을 지연 의존적인 리아프노프-크라소프스키 범함수가 증가된 제어 시스템의 리아프노프 함수와 동일한 형태를 갖는다는 점을 이용하여 찾는다. 유도된 안정화 조건으로부터 복잡도를 줄이기 위하 수 개의 제어 설계기 방법을 제안하다. 모의 실험을 통하여 제안된 방법 중 행렬 변수의 탐색 공간을 블록 대각 행렬로 제하하 는 제한 부분 공간 방법이 성능과 복잡도 사이에서 가장 좋은 트레이드오프를 제공함을 확인되었다.

# [Abstract]

This paper proposes several suboptimal methods of designing a controller for a networked control system with state feedback where delay due to transmission error and transmission delay is modeled as a Markov process. A stability condition for a control system with Markov delay is found through an equivalent relationship that corresponding delay-dependent Lyapunov-Krasovskii functional has the same form of the Lyapunov function of an augmented control system. Several suboptimal methods of designing a controller from the stability condition are proposed to reduce complexity. A simple numerical experiment shows that a restricted subspace method which limits the search space of a matrix variable to a block diagonal form provides the best tradeoff between the complexity and performance

Key word : Controller design, Delay, Markov process, Stability, State feedback.

### https://doi.org/10.12673/jant.2020.24.6.549

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Received 4 October 2020; Revised 23 November 2020 Accepted (Publication) 16 December 2020 (30 December 2020)

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549

www.koni.or.kr pISSN: 1226-9026 eISSN: 2288-842X

# 1. Introduction

Evolution of the internet and mobile technology has accelerated ubiquitous connections to various devices and systems. Widespread use of networks does not only result in much more information exchange but also a networked control of a variety of devices. With this trend, the importance of cyber physical systems (CPS) and associated control methods grows significantly [1]. The control of a group of drones and autonomous driving cars assisted by a network shows how a network controlled system will influence the future technology and industry significantly. A 5G network is also expected to provide seamless communication with a negligible delay to enable CPS more efficiently [2].

A system inter-connected by networks is bound to have delays due to the intrinsic nature of the network. Associated delay can be broadly classified into transmission delay and delay due to transmission errors. Controlling a networked controlled system (NCS) can be considered as controlling a system with delay, which has been studied over the past decades [3]-[5]. Stability condition or controller designs for a system with delay is often based on a Lyapunov method. Many existing studies have been exploiting two methods; the Lyapunov- Krasovskii method and the Lyapunov Razumikhin method, which often result in feasibility or optimization problem with bilinear matrix inequalities (BMIs) or linear matrix inequalities (LMIs).

Controller design in NCS depends on a delay model.  $H_{\infty}$ stability was also studied for NCS with time varying delays in consideration of packet dropouts over both sensor-to-controller channel and controller-to-actuator channel [6] or varying network delay and random data packet dropout [7]. Random delay in a networked controlled system is often modeled as a Markov process in a discrete-time domain [8], since its operation depends on the communication protocol which has a fixed time slot or frame structure. An optimal linear quadratic gaussian (LQG) control for a system with delay due to erasures in a link between a sensor and a controller which is modeled as Markov process was shown to be the combination of linear quadratic regulator (LQR) state feedback and optimal estimator that estimates the state over a communication link [9]. The necessary and sufficient condition for the stochastic stability of a network control system with a bounded random delay was provided in [10]. An iterative algorithm for designing a controller to stabilize stochastically with state feedback was proposed to deal with uncertainty due to quantization error and communication delays modeled by a finite Markov process [11]. The stabilization of a delayed system was extended to the stochastic stabilization of discrete-time Markovian jumping neural network, which resulted in a LMI

approach based on Lyapunov-Krasovskii functional [12]. The sufficient condition for the stochastic stabilization of a system with Markov delay where some elements in a transition matrix were unknown was given as linear matrix inequality for a network with both the sensor to controller channel and controller to actuator channel [13],[14]. A stabilization condition for the boolean network with Markov delays of which states are either 0 or 1 was formulated into a convex problem [15]. However, most of the existing approaches take system augmentation to be transformed to a system with no delay, which results in complexity growing significantly with the maximum delay.

In this paper, we consider a method for deterministic stabilization of a network controlled system where a delay is modeled as a Markov process while existing approaches address this problem stochastically [10]-[12]. A deterministic stabilization is likely to be more robust to modeling error in the transition probability of the Markov process. In addition, we proposed three different non-iterative methods of designing a state feedback controller for a system with the delay so that they can reduce the complexity of the existing approach. We exploit equivalent relationship between the stability condition for a system with delay based on Lyapunov Karsovskii functional and Lyapunov stability condition for a switched system [16]. A sufficient condition for a networked control system with Markov delay is presented from the equivalent stabilization of the switched system. The proposed deterministic stability condition for a system with Markov delay provides tradeoff between the conservatism from the most general stability condition in [16] and specificity from transition probabilities in stochastic stabilization [10]. Several methods to develop a stabilizing controller from the sufficient conditions are proposed. This paper is organized as follows. In section-2, we provide a considered system model and problem formulation. A sufficient condition for stabilizing a system with random delay deterministically is developed in a form of LMIs in section-3. Section-4 presents four different methods to design a stabilizing controller from the sufficient condition. The performance and processing time of the proposed design methods are compared through numerical experiments in section-5. Some concluding remarks are made in section-6.

#### II. A System Model

We consider a linear time invariant control system in a discrete time domain. It can be expressed as

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where  $x(k) \in \mathbb{R}^{n \times 1}$  is a state vector,  $u(k) \in \mathbb{R}^{m \times 1}$  is a control input, A and B are constant matrices. When the control input is determined from the state information of the system, which is available on network, transmission error can happen. In this case, two main approaches are either to use zero or the latest available information in synthesizing a control input. Since it is not known which one is better theoretically or experimentally for sure [17], we take the latter approach for simplicity. The corresponding networked control system of interest is shown in fig-1. At the k th time, u(k) will be determined from delayed state information as follows.

$$u(k) = K(d_k)x(k-d_k)$$
<sup>(2)</sup>

where  $d_k$  is the delay at the kth time, and  $K(d_k)$  is a delay dependent controller gain.

Delay associated with transmission is assumed to take one time unit at least where associated delay in real time totally depends on the communication protocol. In many communication systems, a packet is transmitted in a frame of the fixed length, and transmission delay is fixed between point to point at least. However, when data transmission occurs over multi-nodes, it can take several time units. Since zero-order-hold approach is assumed,  $d_k = d_{k-1} + 1$  when transmission error happens or transmission is delayed. When a new information is received successfully,  $d_{k+1} \le d_k$ . Thus,  $d_k$  takes values in  $\{1,2,\cdots d_{k-1}+1\}$ . Corresponding transition probability matrix T of delay between consecutive times can be defined using  $t_{ij}$ 

$$t_{ij} = \Pr(d_{k+1} = j | d_k = i) \tag{3}$$

It is implicitly assumed that probability transition is stationary. Due to the assumption,  $t_{ij} = 0$  for j > i+1. In the subsequent sections, delay dependent condition considering the characteristics of the delay and corresponding stabilizing controller will be developed throughout this paper.

# III. Stability Condition

Deriving a stability condition from using Lyapunov-Krasovskii approach often results in complex conditions or conservative results due to associated inequalities even though it has been successfully exploited in dealing with stability conditions in control systems with various types of delays. To overcome these issues, we exploits the equivalence between a delay dependent



Fig. 1. A system model.

Lyapunov-Krasovskii functional and the Lyapunov method for a switched system with state augmentation [16].

The corresponding switched system can be expressed as

$$\begin{split} \overline{x}(k+1) &= \widetilde{A}(d_k)\overline{x}(k) \tag{4} \\ \widetilde{A}(d_k) &= \begin{bmatrix} A \ 0 \ \cdots \ 0 \\ I_n \ 0 \ \cdots \ \vdots \\ 0 \ \cdots \ \ddots \ \vdots \\ 0 \ \cdots \ I_n \ 0 \end{bmatrix} + B\overline{K} \end{split}$$

where 
$$\overline{x}(k) = \begin{bmatrix} x(k)^T x(k-1)^T \cdots x(k-d_{\max})^T \end{bmatrix}^T,$$
$$\widetilde{B} = \begin{bmatrix} B^T 0 \cdots 0 \end{bmatrix}^T, \quad \widetilde{E}(d_k) = \begin{bmatrix} 0 \ \widetilde{E}^0(d_k) \end{bmatrix}, \quad \widetilde{E}^0(d_k) = \begin{bmatrix} 0 \cdots 0 \\ d_{k-1} \end{bmatrix}^T, \quad 0 \cdots 0 \end{bmatrix},$$

 $d_{\max}$  is maximum delay, and 0 is a matrix having all zero elements of which dimension varies accordingly with slight abuse of notation. Using this equivalent switched system, the stability condition can be given in the following.

**Theorem-1** : A networked control system, (1) with state feedback and random Markov delay of which maximum is  $d_{\max}$  is stable if there exist K(i) and Y(i) for all  $i \in D = \{1, 2, \dots, d_{\max}\}$  which satisfy LMIs

$$\begin{bmatrix} Y(i) & Y(i)\tilde{A}(i)^T\\ \tilde{A}(i)Y(i) & Y(j) \end{bmatrix} > 0$$
(5)

for  $\{(i,j)|i \in D, j \in \{1,2,\dots,i+1\}\}$ .

**Proof**: We dene a Lyapunov function V(k) as

$$V(k) = \overline{x}(k)^T P(d_k) \overline{x}(k) \tag{6}$$

From the definition of V(k), Lyapunov difference function  $\Delta_k$  can be expressed as

$$\Delta_k = \overline{x}(k)^T \left( \widetilde{A}(i)^T P(d_{k+1}) \widetilde{A}(i) - P(d_k) \right) \overline{x}(k) \tag{7}$$

Since delay follows a Markov process with a transition

probability defined in (3), when  $d_k = i$ 

$$\Delta_k = \overline{x}(k)^T \bigl( \widetilde{A}(i)^T P(d_{k+1}) \widetilde{A}(i) - P(i) \bigr) \overline{x}(k) < 0 \quad (8)$$

for all  $d_{k+1} \in \{1, 2, \dots, i+1\}$ . These are equivalent to the following matrix inequality conditions.

$$\widetilde{A}(i)^T P(j) \widetilde{A}(i) - P(i) \tag{9}$$

for all  $j \in \{1, 2, \dots, i+1\}$ . After applying Schur complement and congruence transformation associated with  $Y(i) = P(i)^{-1}$ , (5) follows.

It is noted that the theorem-1 is dierent from the theorem-1 in [10] in the sense that it is not a statistical stability condition but a conventional deterministic stability condition. It can be considered as a special case of the theorem-1 in [16] in the sense that delay follows the Markov process with a particular structure. The theorem-1 can be further simplified for a particular network protocol in the following way.

**Proposition-1** : When state information is transmitted consecutively without automatic request queuing (ARQ) with fixed delay of a unit time, a networked control system (1) with state feedback and random Markov delay of which maximum is dmax is stable if there exist K(i) and Y(i) for all  $i \in D$  which satisfy LMIs in (5) for  $\{(i,j)|i\in D, j\in\{1,i+1\}\}$ 

**Proof**: When  $d_k = i$ ,  $d_{k+1}$  is 1 or i+1 due to the assumed network transmission protocol. Remaining part of proof follows the same arguments in the proof of the theorem-1.

# IV. Stabilizing Controller Design

In a conventional control system, the design of a state feedback controller is usually straight-forward from stability conditions. However, it is not the case due to the structure made by transformation to an equivalent switched system. Let us rewrite  $\tilde{A}(i)$  in (4) as  $\tilde{A}(i) = \tilde{A} + \tilde{B}K(i)\tilde{E}(i)$  for notational simplicity. With this, (5) can be expressed into a conventional LMI form to derive a state feedback gain.

$$\begin{vmatrix} Y(i) & Y(i)\widetilde{A}(i)^T + L(i)^T B^T \\ \widetilde{A}(i) & Y(i) + \widetilde{B}L(i) & Y(j) \end{vmatrix} > 0 \quad (10)$$

where  $L(i) = K(i)\tilde{E}(i)Y(i)$ . It is noted that due to the rank deficiency in  $\tilde{E}(i)Y(i)$ , K(i) can not be directly solved from L(i) and Y(i). Thus, we propose three different suboptimal methods to find K(i) and assess their performance. For clarity, we first

provide the description of the existing P-K iterative method used for establishing baseline performance.

# 4-1 P-K iterative method

It is noted that (10) is bi-linear matrix inequality (BMI). One simple locally converging iterative algorithm called P-K iteration was proposed in [18], which has been applied to many different types of optimization with BMI constraints [19][20]. The main idea is that (10) is turned into a LMI by fixing a variable. We reproduce the P-K iterative (PKI) method for clarity as follows.

### Algorithm-1 : PKI method

**Step-1** : Initialize l = 0 and  $K_l(i) = 0$ ,  $\forall i \in D$ **Step-2** : Set  $K(i) = K_l(i), \forall i \in D$ ; and nd  $Y(i) = \arg\max\beta$  with respect to  $Y(i), \forall i \in D$  while being subject to constraints that

$$\begin{bmatrix} Y(i) & Y(i)\widetilde{A}(i)^T + L(i)^T \widetilde{B}^T \\ \widetilde{A}(i) & Y(i) + \widetilde{B}L(i) & Y(j) \end{bmatrix} > \beta,$$
  
$$\{(i, j)|_i \in D, j \in \{1, i+1\}\}$$

**Step-3**: Set  $Y(i) = Y_l(i)$ ,  $\forall i \in D$ , and find  $K_l(i) = \arg \max \beta$  with respect to K(i),  $\forall i \in D$  while being subject to the same constraint as in the step-2.

**Step-4** : If  $\beta > 0$  or  $l > l_{max}$ , stop, else l = l+1, and go to the step-2.

#### 4-2 Restricted subspace (RS) method

While the PKI method solves the BMI through two types of associated LMIs, the space of Y(i) can be restricted to a particular space of symmetric matrices which enables to find K(i) directly from L(i) and Y(i). Let the space of Y(i),  $S_Y$  be as follows.

$$S_{Y} = \left\{ \begin{bmatrix} Y_{0} \ 0 & \cdots & 0 \\ 0 & Y_{1} & \vdots \\ \vdots & \ddots & 0 \\ 0 & \cdots & 0 & Y_{d_{\max}} \end{bmatrix} | Y_{l} \in S_{+}^{n}, \forall l \in 0 \cup D \right\}$$
(11)

where  $S^n_+$  is the space of a positive definite matrix of size  $n \times n$ . For Y(i) belonging to  $S_Y$ , K(i) can be solved as

$$K(i) = L(i) Y_{i+1}(i)^{-1}$$
(12)

where  $Y_{i+1}(i)$  is the *i*+1th diagonal block of Y(i). The reduced space method can be summarized in the following.

#### Algorithm-2: Restricted subspace method

**Step-1** : Find L(i),  $\forall i \in D$  and  $Y(i) \in S_Y$ ,  $\forall i \in D$  satisfying LMIs in (10) **Step-2** : Calculate K(i),  $\forall i \in D$  with (12)

#### 4-3 Progressive method

One of necessary conditions for positive definite matrices is that all leading principle minors are positive definite. We propose a heuristic method from exploiting this necessary condition which is called a progressive method. To develop further, A(i) and Y(i)can be represented as  $2 \times 2$  block matrices.

$$\widetilde{A}(i) = \begin{bmatrix} A & BK(i)\widetilde{E}^{0}(d_{k}) \\ A_{21} & A_{22} \end{bmatrix}, \quad Y(i) = \begin{bmatrix} Y_{11}(i) & Y_{21}(i)^{T} \\ Y_{21}(i) & Y_{22}(i) \end{bmatrix}$$
(13)

where  $Y_{11}(i) \in S_{+}^{n}$ ,  $Y_{21}(i) \in R^{nd_{mx} \times n}$ ,  $Y_{22}(i) \in S_{+}^{nd_{mx}}$ . (5) can be rearranged with inserting (13) into (5) as

$$\begin{bmatrix} Y_{11}(i) \ Y_{21}(i)^T \ \chi_{13}(i) \ \chi_{14}(i) \\ * \ Y_{22}(i) \ \chi_{23}(i) \ \chi_{24}(i) \\ * \ * \ Y_{11}(j) \ Y_{21}(j)^T \\ * \ * \ * \ Y_{22}(j) \end{bmatrix}$$
(14)

where  $\chi_{13}(i) = Y_{11}(i)A^T + Y_{21}(i)(BK(i)\widetilde{E}^0(d_k))^T$ ,  $\chi_{14}(i)$   $= Y_{11}(i)A_{21}^T + Y_{21}(i)^TA_{22}^T$ ,  $\chi_{23}(i) = Y_{21}(i)A^T + Y_{22}(i)(BK(i)\widetilde{E}^0(d_k))^T$ ,  $\chi_{24}(i) = Y_{21}(i)A_{21}^T + Y_{22}(i)^TA_{22}^T$ , and \* denotes matrix entries from symmetricity. From the positive definiteness of the first and the second block leading principle minor, any constant positive definite matrix can be assigned to Y(i). From the third block leading principle minor, controller gains can be found from linear matrix inequalities applied on the upper  $3 \times 3$  sub-blocks of matrix given in (14). This procedure can be summarized as follows.

Algorithm-3 : Progressive method

Step-1 : Set 
$$\begin{bmatrix} Y_{11}(i) & Y_{21}(i)^T \\ Y_{21}(i) & Y_{22}(i) \end{bmatrix} = I_{2n}$$
  
Step-2 : Calculate  $K(i), \forall i \in D$  with constraints  
 $\begin{bmatrix} Y_{11}(i) & Y_{21}(i)^T & \chi_{13}(i) \\ * & Y_{22}(i) & \chi_{23}(i) \\ * & * & Y_{11}(j) \end{bmatrix} > 0$   
Step-3 : Calculate  $Y(i), \forall i \in D$  with constraints  
 $\begin{bmatrix} Y_{-1}(i) & Y_{-1}(i)^T & Y_{-1}(i) \end{bmatrix}$ 

$$\begin{bmatrix} Y_{11}(i) & Y_{21}(i) & \chi_{13}(i) \\ * & Y_{22}(i) & \chi_{23}(i) \\ * & * & Y_{11}(j) \end{bmatrix} > 0$$

#### 4-4 Opportunistic method

While the restricted subspace method tries to find a solution in some subspace of solution space, one can find a solution opportunistically. It is noted that (10) can be satisfied if the following two matrix inequalities hold for a pair (i, j)

$$\begin{bmatrix} Y(i)/2 \ Y(i)\widetilde{A}(i)^{T}/2 + \widetilde{L}_{l}(i)^{T}\widetilde{B}^{T} \\ * Y(j)/2 \end{bmatrix} > 0, \, l \in 1,2$$
(15)

where  $\widetilde{L}_1(i) = [L_1(i) \ 0], \widetilde{L}_2(i) = [0 \ L_2(i)], L_1(i) \in \mathbb{R}^{m \times n}$ , and  $L_2(i) \in \mathbb{R}^{m \times nd_{ms}}$ . It is noted that  $L_1(i)$  and  $L_2(i)$  are none other than  $K(i)\widetilde{E}^0(i) Y_{21}(i)$  and  $K(i)\widetilde{E}^0(i) Y_{22}(i) \ L_1(i)$ . Since the feasibility problem with the constraints (15) are solved independently from K(i) while  $L_1(i)$  and  $L_2(i)$  are dependent on each other, finding  $L_1(i)$  and  $L_2(i)$  satisfying (15) does not guarantee the feasibility of K(i) satisfying (10). Even though there can be many ways to determine K(i) from given  $L_1(i)$  and  $L_2(i)$ , the least square method is considered so that K(i) can be chosen to minimize the sum of squared error. The associated procedures can be summarized in the following way.

#### Algorithm-4 : Opportunistic method

**Step-1**: Find  $L_1(i)$  and  $L_2(i)$  from the feasibility problem of (15). **Step-2**: Calculate K(i),  $\forall i \in D$  with the least square method from  $L_1(i)$  and  $L_2(i)$ .

# **V. Numerical Experiment**

We consider a simple numerical experiment to assess the characteristics of the proposed methods. PKI method will be considered as a baseline reference for comparison since it is just direct application of an existing method to this problem. Both system parameters m and n were set to be 2.  $d_{max}$  was set to be ranged from 2 to 7. To evaluate the statistical performance of the proposed methods, 500 pairs of A and B were independently generated from a standard normal distribution for a given  $d_{max}$ . The maximum number of iterations for the PKI method was set to be 10.

The success rate of finding a stabilizing controller was compared in table-1. The success of finding a stabilizing controller was judged from verifying whether the given controller satisfied all LMI constraints. It is observed that all considered methods except the opportunistic one provide similar performance. The degradation with increasing  $d_{max}$  in the opportunistic method can be attributed to increasing error in the least square due to the increase in the dimension of vector space. It is also observed that the performance does not seem to have

method).									
	dmax	2	3	4	5	6	7		
	PKI	0.320	0.394	0.348	0.328	0.318	0.376		
	RS	0.326	0.394	0.350	0.332	0.324	0.376		

0.350

0.348

0.394

0.394

0.316

0.326

Pro.

Progressive method, Opp : Opportunistic

0.332

0.266

0.324

0.188

0.376

0.176

Table 1. (PKI; PKI method, RS; RS method, Pro;

Opp. Table. 2. Average processing time in seconds for calculating the controller gain. (PKI; PKI method,

				,		
dmax	2	3	4	5	6	7
PKI	3.33	5.04	11.21	24.08	46.16	85.51
RS	0.32	0.57	1.07	1.93	3.39	5.78
Pro.	0.44	0.86	1.83	3.72	7.32	13.95
Opp.	0.44	0.96	2.33	5.16	10.73	21.23

RS; RS method, Pro; Progressive method, Opp; Opportunistic method).

much dependency on the maximum delay. In table-2, the average processing times are compared. The PKI method shows the largest processing time as expected due to iterations. Even though the processing times of other methods are comparable, the smallest processing time is achieved by the RS method due to the reduction of the number of variables from the block diagonal structure of a variable matrix. It is also observed that the progressive method has larger processing time due to solving a feasibility problem twice despite the reduction of the dimension of a variable matrix. From these results, the RS method is observed to provide the best tradeoff between performance and complexity among the proposed methods. The RS method provides the almost same performance as PKI while its complexity is about 13-17% of PKI depending on delay, which shows the advantage of the RS method.

As a more practical example, a problem of inverted pendulum on a cart was considered[6][8]. Four elements in the state vector in this problem corresponds to position of the cart, velocity of the cart, angle of the inverted pendulum and angle velocity of the inverted pendulum respectively. For the same physical configuration as in [10], matrices A and B are given as

$$A = \frac{1}{2} \begin{bmatrix} 1.0000 \ 1.0000 \ -0.0166 \ -0.0005 \\ 0 \ 1.0000 \ -0.3374 \ -0.0166 \\ 0 \ 0 \ 1.0996 \ 0.1033 \\ 0 \ 0 \ 2.0247 \ 1.0996 \end{bmatrix},$$
(16)  
$$B = \begin{bmatrix} 0.0045 \\ 0.0896 \\ -0.0068 \\ -0.1377 \end{bmatrix}$$

 $\frac{1}{2}$  in A was artificially multiplied so that PKI and RS could have a solution in the design problem.  $d_{max}$  was set to 2. Processing



**Fig. 2.** Trajectory of position of the cart  $(x_1(k))$  and angle of the inverted pendulum  $(x_3(k))$ .

times for PKI and RS were 10.5596 and 0.5181 seconds. The states corresponding to the position of the cart and the angle of the inverted pendulum which were controlled through the controllers developed from each algorithm were shown in fig-2. This result verifies that the RS method can be used for developing controller with much less complexity than the PKI method while performing as good as the PKI method.

# VI. Conclusions

In this paper, a sufficient condition for deterministic stability of networked control system having delay incurred by transmission error and the bounded transmission delay was proposed from the Lyapunov condition for an equivalent augmented switched system. Several heuristic methods for designing a state feedback controller were proposed and compared through numerical experiments. The RS method was found to provide a comparable performance to the baseline method with much less processing time.

Nonetheless, there is some limitation of this research, since the system setup is rather simple for the characterization of the problem and associated controller design. The proposed algorithm can be extended to a system with multiple state delays and multiple input days [21]. Further works need to be done to deal with system model uncertainty, or disturbance and measurement error. It can be also further extended to incorporate digitization and various network issues such as malicious false control information to be applicable to a practical networked control system.

# Acknowledgement

This work was supported by the Basic Science Research Program through the National Research Foundation of Korea Funded by the Ministry of Science and ICT underGrant NRF-2017R1A2B4007398.

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