Joint latent class analysis for longitudinal data: an application on adolescent emotional well-being

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Abstract

This study proposes generalized models of joint latent class analysis (JLCA) for longitudinal data in two approaches, a JLCA with latent profile (JLCPA) and a JLCA with latent transition (JLTA). Our models reflect cross-sectional as well as longitudinal dependence among multiple latent classes and track multiple class-sequences over time. For the identifiability and meaningful inference, EM algorithm produces maximum-likelihood estimates under local independence assumptions. As an empirical analysis, we apply our models to track the joint patterns of adolescent depression and anxiety among US adolescents and show that both JLCPA and JLTA identify three adolescent emotional well-being subgroups. In addition, JLCPA classifies two representative profiles for these emotional well-being subgroups across time, and these profiles have different tendencies according to the parent-adolescent-relationship subgroups.

Keywords: joint latent class profile analysis, joint latent transition analysis, EM algorithm, adolescent depression

1. Introduction

According to the 2017 National Survey on Drug Use and Health (NSDUH), about 13.3% of adolescents aged 12 to 17 in the United States have experienced a depressive episode (Center for Behavioral Health Statistics and Quality, 2018). Adolescent depression is a major concern for youth development that relates to physical development and cognitive development as well as success in school, work, and society. Several researchers have suggested that depression in adolescence is strongly linked with anxiety, but knowledge of the mechanisms underlying this link remains inconclusive (Garber and Weersing, 2010; Horn and Wuyek, 2010; Schleider *et al.*, 2014). The sequentially comorbid relationship between anxiety and depression likely involves a wide variety of cognitive, social, environmental, and biological factors (Schleider *et al.*, 2014). For example, mutual trust and harmonious relationships between parents and adolescents have been found to have a positive impact on youth health (Gutman *et al.*, 2005; White and Roosa, 2012).

In many studies, adolescent mental health have been inferred from responses to several related questionnaires rather than directly measured as a single scale, but it has been rarely considered as a latent variable. Some research has written recently used latent class analysis (LCA) to categorically classify subpopulations for depressive and anxiety symptoms (van Lang *et al.*, 2006; Kreuter *et al.*, 2008). LCA is a person-centered approach that can capture heterogeneity within and between subgroups, unlike factor analysis, which is a variable-centered approach (Nurius and Macy, 2008). There

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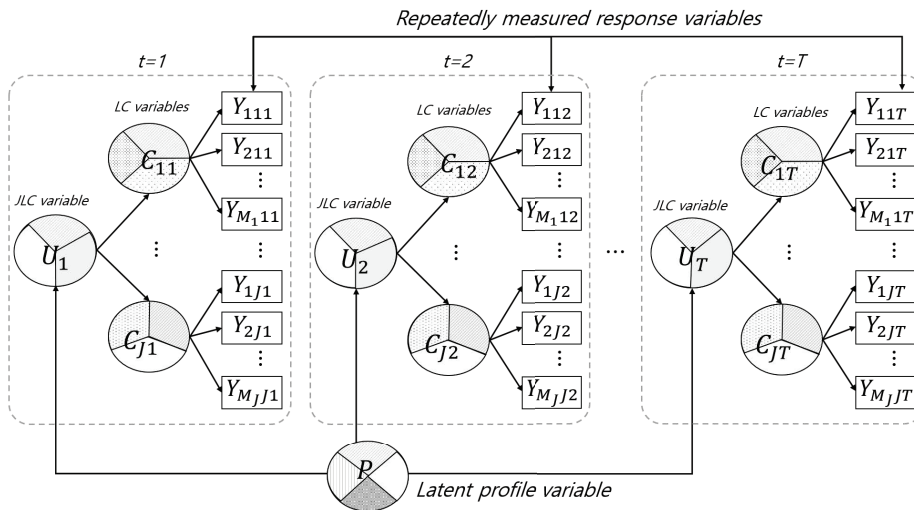
Figure 1: *Model concept of joint latent class profile analysis.*

are LCA variations for longitudinal data including latent transition analysis (LTA) and latent class profile analysis (LCPA) (Chung *et al.*, 2008, 2011; Collins and Lanza, 2010). LTA and LCPA use a set of repeatedly measured manifest items to track latent transition or the path of a latent classes; however, and these are not suitable for two or more latent class variables. Joint LCA (JLCA) is another LCA variation for cross-sectional data that allows several latent variables simultaneously under multiple manifest item sets of different attributes (Jeon *et al.*, 2017). For example, JLCA can consider multi-dimensional information of youth behavioral risk (joint latent variable) based on responses to substance use, violence, depression and suicide (each 4 latent variable). JLCA allows us to identify the association of different latent variables; however, it is limited to cross-sectional dependencies.

In this regard, this paper develops generalized latent class models that reflect cross-sectional dependence as well as longitudinal dependence between latent variables. In order to explain the dynamic changes or developments of multiple latent variables over time, we propose two types of new LC models of JLCA combined with the LTA concept and JLCA combined with the LCPA concept. For the parameter estimation, maximum likelihood estimation using an EM algorithm are used. As an empirical analysis, we apply our models to examine stage-sequential patterns of adolescent emotional well-being considering parent-adolescent relationships, using the data from Angel *et al.* (2009).

The rest of the paper is organized as follows. In Section 2, the model concepts and descriptions of the proposed JLCA with latent transition (JLTA) and JLCA with latent profile (JLCPA) are presented. Section 3 describes estimation methods for the parameters and model diagnosis. In Section 4, empirical analysis results are represented, and Section 5 concludes the paper.

2. Joint latent class analysis for longitudinal data

JLCA explores the joint action or association of multiple latent variables only for a static phenomena; therefore, we develop two JLC variation models by incorporating the JLCA with two typical latent class (LC) models for longitudinal data of LCPA and LTA. These two models allow the investigation of changes in multiple latent class variables over time that reflect longitudinal dependence.

2.1. Joint latent class profile analysis

Joint LCPA (JLCPA) is an extended LCPA where multiple latent variables are allowed to be associated and their latent paths can be tracked using sets of repeatedly measured manifest items. JLCPA also postulates a joint latent variable composed of several latent variables at each measurement time point. Figure 1 illustrats JLCA at each time t and its profile identified by sequences of joint latent class memberships across time.

Suppose that there are J latent class variables $C_{1t}, \ldots, C_{Jt}$ and the variable C_{jt} consists of K_j categories at time t. For a given t, the LC variable C_{jt} was identified from the jth item set $\mathbf{Y}_{jt} = (Y_{1jt}, \ldots, Y_{M_jjt})'$ and the manifest item Y_{m_jjt} ranges from 1 to r_{m_j} for $j = 1, \ldots, J$. Let a joint class variable U_t at time t has S nominal categories to describe the common joint patterns of J latent class memberships, and a latent profile variable P has L nominal profiles to describe the representative joint class sequences over time. Let $\mathbf{y}_{ijt} = (y_{i1jt}, \ldots, y_{iM_jjt})'$ be the ith individual's responses to a set of M_j manifest items measuring a jth latent feature at time t, then the complete-data likelihood of the ith individual over time can be presented as

$$
\begin{aligned}
L^*_{(\text{JLCPA})i} &= P(P = l, \mathbf{U} = \mathbf{s}, \mathbf{C} = \mathbf{c}, \mathbf{Y}_i = \mathbf{y}_i) \\
&= P(P = l)P(\mathbf{U} = \mathbf{s} \mid P = l)P(\mathbf{C} = \mathbf{c} \mid \mathbf{U} = \mathbf{s})P(\mathbf{Y}_i = \mathbf{y}_i \mid \mathbf{C} = \mathbf{c}) \\
&= \delta_l \prod_{t=1}^{T} \left\{ \gamma_{s_t|l}^{(t)} \prod_{j=1}^{J} \left[\eta_{c_j|s_t}^{(j,t)} \prod_{m_j=1}^{M_j} \prod_{k=1}^{r_{m_j}} \rho_{m_jkj|c_j}^{(t)}{}^{I(y_{im_jjt}=k)} \right] \right\},
\end{aligned}
\tag{2.1}
$$

where $I(y_{im_jjt} = k)$ is an indicator function that is equal to 1 when $y_{im_jjt} = k$ and 0 otherwise. The primary measurement parameter $\rho_{m_jkj|c_j}^{(t)}$ denotes the probability that an individual responds k to the m_j^{th} item of the jth latent variable, for a given latent class c_{jt} at time t. The secondary measurement parameter $\eta_{c_j|s_t}^{(j,t)}$ represents the conditional probability that an individual belongs to a class c_{jt} of jth latent variable at time t, for a given joint latent class s_t at time t. The tertiary measurement parameter $\gamma_{s_t|l}^{(t)}$ is the conditional probability that an individual belongs to a joint class s at time t, for a given profile l. Finally, δ_l represents the probability that an individual belongs to a profile l.

The likelihood of JLCPA is based on the following assumptions: (a) both the joint class membership and the joint class profile membership are related to the manifest items only through the class membership; (b) the manifest items are conditionally independent given a class membership; (c) given a joint class membership, each class membership is unrelated; and (d) the joint class membership is unrelated within a joint class profile. Assumptions (b), (c), and (d) indicate local independence which allows the inference of the unobserved classes, joint class, and joint class profile.

The observed-data likelihood, the likelihood of manifest items, can be derived by marginalizing the complete-data likelihood over all the considered latent variables $\mathbf{C}, \mathbf{U}, P$:

$$
L_{(\text{JLCPA})i} = P(\mathbf{Y}_i = \mathbf{y}_i) = \sum_{l=1}^{L} \sum_{s_1=1}^{S} \cdots \sum_{s_T=1}^{S} \sum_{c_1=1}^{K_1} \cdots \sum_{c_J=1}^{K_J} L^*_{(\text{JLCPA})i}.
\tag{2.2}
$$

2.2. Joint latent transition analysis

The second model is a joint LTA (JLTA) that identifies the marginal joint class prevalence at the initial time and latent transitions over two consecutive time points, instead of a latent profile or common pathways of joint class. We assume that the sequence of U_{it} constitutes a first-order Markov chain for

$t = 1, \ldots, T$. Let $\tau_{s_t|s_{t-1}}$ be a $S \times S$ transition matrix representing the probability that an individual belongs to a joint class s_t at time t given s_{t-1} at time $t - 1$, for $s_{t-1} = 1, \ldots, S$ and $s_t = 1, \ldots, S$. The complete-data likelihood of JLTA can be derived by substituting the prevalence of joint class profile δ_l and the tertiary measurement parameter $\gamma_{s_t|l}^{(t)}$ into the prevalence of joint latent class at initial time δ_{s_1} and the transition probability $\tau_{s_t|s_{t-1}}^{(t)}$. Then, the complete-data likelihood is

$$L_{(\text{JLTA})i}^* = P(\mathbf{U} = \mathbf{s}, \mathbf{C} = \mathbf{c}, \mathbf{Y}_i = \mathbf{y}_i)$$

$$= \delta_{s_1} \prod_{t=2}^{T} \tau_{s_t|s_{t-1}}^{(t)} \prod_{t=1}^{T} \prod_{j=1}^{J} \left[\eta_{c_j|s_t}^{(j,t)} \prod_{m_j=1}^{M_j} \prod_{k=1}^{r_{m_j}} \rho_{m_j k j|c_j}^{(t)}{}^{I(y_{im_j jt}=k)} \right], \quad (2.3)$$

where $I(y_{im_j jt} = k)$ is an indicator function that is equal to 1 when $y_{im_j jt} = k$ and 0 otherwise. The primary measurement parameter $\rho_{m_j k j|c_j}^{(t)}$ represents the probability that an individual responds k to the m_j^{th} item of the jth latent variable, for a given latent class c_{jt} at time t, and the secondary measurement parameter $\eta_{c_j|s_t}^{(j,t)}$ is the conditional probability that an individual belongs to a class c_{jt} of the jth latent variable at time t for a given joint latent class s_t at time t. Finally, δ_{s_1} is the prevalence of joint latent class at initial time, and $\tau_{s_t|s_{t-1}}^{(t)}$ is the transition probability.

JLTA also requires the local independence assumptions for meaningful inferences about all latent variables. The observed-data likelihood can be obtained as

$$L_{(\text{JLTA})i} = P(\mathbf{Y}_i = \mathbf{y}_i) = \sum_{s_1=1}^{S} \cdots \sum_{s_T=1}^{S} \sum_{c_1=1}^{K_1} \cdots \sum_{c_J=1}^{K_J} L_{(\text{JLTA})i}^*. \quad (2.4)$$

3. Parameter estimation and model diagnosis

Expectation-Maximization (EM) algorithm is the most widely used estimation method for LC-type models (Mooijaart and Van der Heijden, 1992; Jeon *et al.*, 2017; Lee and Chung, 2017). EM provides ML estimates even for missing data under missing at random assumption, and can yield stable estimates regardless of boundary solutions. The EM algorithm consists of the E-step and M-step, and once given with appropriate initial value, the two steps are repeated until the estimates converge to fixed points.

3.1. Estimation of JLCPA

E-step calculates the expectation of the log-complete data likelihood under the current estimates of the parameters. The expectation is expressed as a function of posterior probabilities that individual i belongs to profile l, the joint class sequence $\mathbf{s} = (s_1, \ldots, s_T)$ and the class $\mathbf{c} = (\mathbf{c}_1, \ldots, \mathbf{c}_T)$, given a set of manifest item responses $\mathbf{y}_i = (\mathbf{y}_{i1}, \ldots, \mathbf{y}_{iT})$, and calculated using the tentative parameter estimates from the previous iteration. The posterior probabilities can be obtained by $\theta_{i(l,\mathbf{s},\mathbf{c})} = P(P = l, \mathbf{U} = \mathbf{s}, \mathbf{C} = \mathbf{c} \mid \mathbf{Y}_i = \mathbf{y}_i) = L_{(\text{JLCPA})i}^*/L_{(\text{JLCPA})i}$. As the complete-data likelihood $L_{(\text{JLCPA})i}^*$ in equation (2.1) can be expressed with indicator functions as

$$L_{(\text{JLCPA})i}^* = \delta_l^{I(P_i=l)} \prod_{t=1}^{T} \left\{ \gamma_{s_t|l}^{(t)}{}^{I(U_{it}=s_t,P_i=l)} \prod_{j=1}^{J} \left[\eta_{c_j|s_t}^{(j,t)}{}^{I(C_{ij}=c_j,U_{it}=s_t)} \prod_{m_j=1}^{M_j} \prod_{k=1}^{r_{m_j}} \rho_{m_j k j|c_j}^{(t)}{}^{I(y_{im_j jt}=k,C_{ij}^{(t)}=c_j)} \right] \right\}, \quad (3.1)$$

and the expectation of log-complete-data likelihood can be derived as follows.

$$
E\left(\sum_{i=1}^{n} \log L^*_{(\text{JLCPA})i}\right) = \sum_{i=1}^{n} \sum_{l=1}^{L} \theta_{i(l)} \log \delta_l
$$

$$
+ \sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{s_t=1}^{S} \sum_{l=1}^{L} \theta_{i(l,s_t)}^{(t)} \log \gamma_{s_t|l}^{(t)}
$$

$$
+ \sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{c_j=1}^{C_j} \sum_{s_t=1}^{S} \theta_{i(s_t,c_j)}^{(t)} \log \eta_{c_j|s_t}^{(j,t)}
$$

$$
+ \sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ \sum_{c_j=1}^{C_j} \theta_{i(c_j)}^{(t)} \left[\sum_{m_j=1}^{M_j} \sum_{k=1}^{r_{m_j}} I\left(y_{im_jj}^{(t)} = k\right) \log \rho_{m_jkj|c_j}^{(t)} \right] \right\}. \tag{3.2}
$$

where $\theta_{i(l)} = \sum_{s_t} \theta_{i(l,s_t)}^{(t)}$, $\theta_{i(l,s_t)}^{(t)} = \sum_{c_1} \cdots \sum_{c_J} \theta_{i(l,s_t,\mathbf{c})}^{(t)}$, $\theta_{i(s_t,c_j)}^{(t)} = \sum_{l} \sum_{c_1} \cdots \sum_{c_{j-1}} \sum_{c_{j+1}} \cdots \sum_{c_J} \theta_{i(l,s_t,\mathbf{c})}^{(t)}$, and $\theta_{i(c_j)}^{(t)} = \sum_{s_t} \theta_{i(s_t,c_j)}^{(t)}$. Then, ML estimates for JLCPA can be obtained by taking partial derivatives and solving normal equations. The model parameters can therefor be derived by

$$
\hat{\delta}_l = \frac{\sum_i \theta_{il}}{n}, \quad \hat{\gamma}_{s_t|l}^{(t)} = \frac{\sum_i \theta_{i(l,s_t)}^{(t)}}{\sum_i \theta_{il}}, \quad \hat{\eta}_{c_j|s_t}^{(j,t)} = \frac{\sum_i \theta_{i(s_t,c_j)}^{(t)}}{\sum_i \theta_{i(s_t)}^{(t)}}, \quad \text{and} \quad \hat{\rho}_{m_jkj|c_j}^{(t)} = \frac{\sum_i \theta_{i(c_j)}^{(t)} \times I\left(y_{im_jj}^{(t)} = k\right)}{\sum_i \theta_{i(c_j)}^{(t)}}. \tag{3.3}
$$

For the invariant η or ρ over time for practical interpretation due to the consistent meaning of latent classes across time, the parameter estimators become

$$
\hat{\eta}_{c_j|s_t}^{(j)} = \frac{\sum_{t=1}^{T} \sum_i \theta_{i(s_t,c_j)}^{(t)}}{\sum_{t=1}^{T} \sum_i \theta_{i(s_t)}^{(t)}} \quad \text{and} \quad \hat{\rho}_{m_jkj|c_j} = \frac{\sum_{t=1}^{T} \left\{ \sum_i \theta_{i(c_j)}^{(t)} \times I\left(y_{im_jj}^{(t)} = k\right) \right\}}{\sum_{t=1}^{T} \sum_i \theta_{i(c_j)}^{(t)}}. \tag{3.4}
$$

3.2. Estimation of JLTA

E-step computes the conditional probability that the ith individual belongs to stage sequence $\mathbf{s} = (s_1, \ldots, s_T)$ and the class $\mathbf{c} = (\mathbf{c_1}, \ldots, \mathbf{c_T})$ given $\mathbf{y}_i$. The posterior probabilities can be obtained by $\theta_i(\mathbf{s}, \mathbf{c}) = P(\mathbf{U} = \mathbf{s}, \mathbf{C} = \mathbf{c} \mid \mathbf{Y}_i = \mathbf{y}_i) = L^*_{(\text{JLTA})i}/L_{(\text{JLTA})i}$. Using the ordinary indicator functions, the complete-data likelihood derived in (2.3) can be expressed as follows.

$$
L^*_{(\text{JLTA})i} = \delta_{s_1}^{I(u_{i1}=s_1)} \prod_{t=2}^{T} \tau_{s_t|s_{t-1}}^{(t)\ I(U_{it}=s_t,U_{i,t-1}=s_{t-1})} \prod_{t=1}^{T} \prod_{j=1}^{J} \left[\eta_{c_j|s_t}^{(j,t)\ I(C_{ij}=c_j,U_{it}=s_t)} \prod_{m_j=1}^{M_j} \prod_{k=1}^{r_{m_j}} \rho_{m_jkj|c_j}^{(t)\ I(y_{im_jjt}=k,C_{ij}^{(t)}=c_j)} \right]. \tag{3.5}
$$

Using the likelihood given in (3.5), the expectation of the log-complete data likelihood can be obtained in terms of θ,

$$
E\left(\sum_{i=1}^{n} \log L^*_{(\text{JLTA})i}\right) = \sum_{i=1}^{n} \sum_{s_1=1}^{S} \theta_{i(s_1)} \log(\delta_{s_1})
$$

$$
+ \sum_{i=1}^{n} \sum_{t=2}^{T} \sum_{s_t=1}^{S} \sum_{s_{t-1}=1}^{S} \theta_{i(s_t,s_{t-1})}^{(t)} \log \tau_{s_t|s_{t-1}}^{(t)}
$$

$$+ \sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{c_j=1}^{C_j} \sum_{s_t=1}^{S} \theta_{i(s_t,c_j)}^{(t)} \log \eta_{c_j|s_t}^{(j,t)}$$

$$+ \sum_{i=1}^{n} \sum_{t=1}^{T} \sum_{j=1}^{J} \left\{ \sum_{c_j=1}^{C_j} \theta_{i(c_j)}^{(t)} \left[\sum_{m_j=1}^{M_j} \sum_{k=1}^{r_{m_j}} I\left(y_{im_j j}^{(t)} = k\right) \log \rho_{m_j k j|c_j}^{(t)} \right] \right\}.$$

The M-step focuses on the maximization of the expected complete-data likelihood of JLCA model with latent transition with respect to the model parameters. The resulting parameter estimators are

$$\hat{\delta}_{s_1} = \frac{\sum_i \theta_{i(s_1)}}{n}, \quad \hat{\tau}_{s_t|s_{t-1}}^{(t)} = \frac{\sum_i \theta_{i(s_t,s_{t-1})}^{(t)}}{\sum_i \theta_{s_{t-1}}^{(t)}}, \quad \hat{\eta}_{c_j|s_t}^{(j,t)} = \frac{\sum_i \theta_{i(s_t,c_j)}^{(t)}}{\sum_i \theta_{i(s_t)}^{(t)}}, \quad \text{and} \quad \hat{\rho}_{m_j k j|c_j}^{(t)} = \frac{\sum_i \theta_{i(c_j)}^{(t)} \times I\left(y_{im_j j}^{(t)} = k\right)}{\sum_i \theta_{i(c_j)}^{(t)}}. \quad (3.6)$$

The parameters of η and ρ become (3.4) under the invariant assumption over time, in the same way in JLCPA. For τ, it may be of interest to consider the invariance assumption of the transition probability matrix across time to simplify the model when $T \geq 3$. The null hypothesis of equal transition probabilities over time can be tested by comparing the model fit statistics of two models. The transition probability estimator under invariance assumption becomes

$$\hat{\tau}_{s_t|s_{t-1}} = \frac{\sum_{t=2}^{T} \sum_i \theta_{i(s_t,s_{t-1})}^{(t)}}{\sum_{t=2}^{T} \sum_i \theta_{s_{t-1}}^{(t)}}. \quad (3.7)$$

3.3. Model diagnosis

It is important to assess the model with objective fit measures to understand whether the fitted model is appropriate to the data. LC model which properly reflects the underlying latent structure of the data enables us draw valid statistical inferences. Pearson χ^2 and LRT statistics generally do not hold for latent class model, and bootstrap p-value obtained from the empirical distribution can be considered as criterion for the absolute performance of the model (Chung *et al.*, 2011). For the calculation of bootstrap p-value, we perform parametric bootstrap procedure by generating 100 bootstrap samples of G^2 statistic, and calculate the proportion of the bootstrapped G^2s that are larger than observed G^2. The model whose bootstrap p-value is larger than the significance level is considered as a proper model for the data. Next, we can compare the performance of several plausible models and select the best one. The relative model fit statistics such as Akaike information criterion (AIC) and Bayesian information criterion (BIC) can be considered to determine which LC model is better for explaining a data set. AIC tends to prefer the complex model with a lot of parameters while BIC usually indicates that a comparatively simple model is the optimal model. The model with p-value to be larger than 0.05, and the model with smaller AIC (or BIC) is relatively preferred, but the interpretability of the latent classes also should be considered in order to reflect the underlying latent structure of the data properly.

4. Application to adolescents emotional well-being

4.1. Data description

The study of Angel *et al.* (2009) analyzed a longitudinal survey data to assess the well-being of low-income children and families in Boston, Chicago, and San Antonio. Each household had a family income below 200% of the federal poverty line, a female caregiver, and a child (aged 0 to 4 years) or

Table 1: Selection of the number of latent class for each latent component at wave 1

Latent variable	Number of classes	AIC	BIC	Bootstrap p-value
Depression	2	4822.1	4883.6	0.00
	3	4807.0	4901.6	0.05
	4	4799.6	4927.4	0.39
	5	4801.6	4962.5	0.54
Anxiety	2	4585.0	4646.5	0.00
	3	4524.9	4619.5	0.71
	4	4527.2	4654.9	0.65
	5	4536.3	4697.2	0.48

AIC = Akaike information criterion; BIC = Bayesian information criterion.

adolescent (aged 10 to 14 years) at the time of the first interview. This study focuses on the $N = 839$ sample households with adolescent, and their responses on the manifest items related to the adolescent emotional well-being and parent-adolescent relationship over three waves in 1999, 2001, and 2005.

For the study, we used 6 manifest items on the adolescent depression, 6 items measuring adolescent anxiety. Questionnaires on depression behaviors are "do you...?" (a) often feel no interest in things; (b) often feel lonely; (c) often feel blue; (d) often feel worthless; (e) often feel hopeless about future; (f) sometimes think about suicide. Items on anxiety behaviors are whether do you (a) often feel nervous; (b) often feel tense; (c) often feel scared suddenly; (d) often feel panic; (e) often feel restless; (f) often feel fearful. For the interpretational simplicity, all the polychotomous items were changed into dichotomous indicating true or not.

4.2. Model selection

JLCPA and JLTA have several latent variables and complicated structure over time which can cause identifiability problem. Prior to the construction of JLCPA (or JLTA) model, it is required to specify the appropriate latent structure of each latent variable. The joint latent structure of the multiple latent variables can be determined, once the latent structure of the each latent variable is specified (Jeon *et al.*, 2017). The stability of model was examined by using 100 different random starting values to prevent a local optimum problem. We adopt the information criteria (AIC, BIC), and bootstrap p-value with significance level $\alpha = 0.05$ to assess the goodness-of-fit, and we also take into account the interpretability of each latent class model.

Table 1 presents AIC, BIC and bootstrap p-value estimated using the data at wave 1 with varying number of latent classes. The best class structure for *Depression* is determined as 3-class model by considering AIC, BIC, bootstrap p-value and the interpretability of latent class. Similarly, the best class structure of the *Anxiety* variable is determined as 3-class model. The optimal number of classes for each latent variable hold the same for the data at wave 2 and 3.

Figure 2 represents the ρ-probabilities of manifest items for the 3-class depression groups (left) and 3-class anxiety groups (right) at each wave. For both depression and anxiety groups, normal groups are illustrated as dotted line, mild groups as dashed line, and severe groups as solid line. Although the ρ-probabilities show slight fluctuations, their ranks and meaning of latent classes are stable over time. Based on these item response probabilities, the primary measurement parameters were constrained as invariant over time.

Based on the pre-specified latent structures, the optimal number of joint latent class was selected. We called the identified joint latent class variable as *Emotional well-being*. Goodness-of-fit measures were calculated and presented in Table 2 for the various joint latent classes. Similar to the previous procedure, the best structure for the joint latent variable is the model with three joint classes, showing
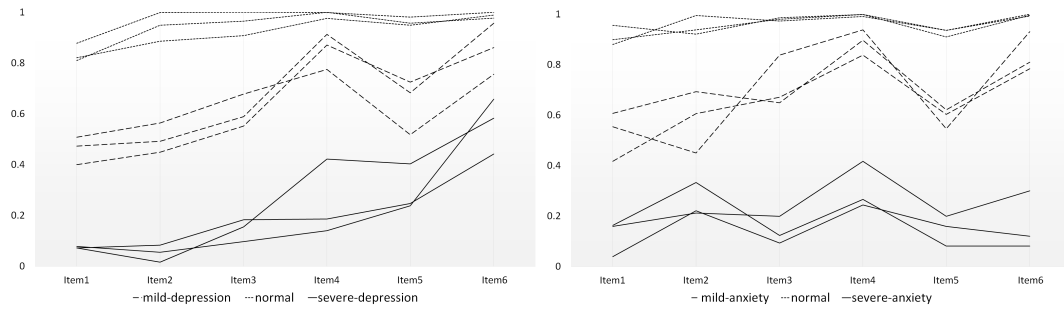
Figure 2: *The estimated ρ-parameters of depression (left) and anxiety (right) at each wave.*

Table 2: Selection of the number of joint latent class with different number of classes at wave 1

Joint latent variable	Number of classes	AIC	BIC	*p*-value
	2	25963.75	26602.59	0.00
Emotional well-being	3	25640.17	26350.00	1.00
	4	25678.25	26459.07	1.00

AIC = Akaike information criterion; BIC = Bayesian information criterion.

the smallest AIC, BIC and acceptable bootstrap *p*-value. At wave 2 and wave 3, the optimal number of joint latent classes also show the same.

Figure 3 represents the η-parameters of the 3-class depression groups (upper) and 3-class anxiety groups (lower) given three joint latent classes at each wave; each wave is illustrated as each dotted/dashed/solid line for each joint latent class. The η-parameters rarely fluctuate over time, and their meaning of latent classes given joint class is very stable over time, except for the mild-anxiety class given severe-symptoms, which can be due to its rareness. Therefore, the secondary measurement parameters η are constrained to be invariant over time.

JLCPA has an extra number of latent component (latent profile) to be determined based on the pre-specified latent class and joint class structures. Table 3 presents the goodness-of-fit for JLTA and JLCPA with different number of profile, and the JLCPA model with 2-profile showed the smallest BIC and the model with 4-profile showed the smallest AIC. Considering model interpretability and law of parsimony, we decided to choose 2-profile latent model with 3-joint class, 3-class for *Depression* and 3-class for *Anxiety* as the best model (Figure 4). JLTA also appears to be a competitive model in terms of model fit; therefore, we infer from both JLCPA and JLTA.

4.3. Parameter estimates

We estimated the parameters of our interest under the selected model structure. Table 4 shows the estimated ρ-parameters of the latent variable for *Depression* and *Anxiety*. JLCPA and JLTA provide similar estimates for item-response probabilities. With regard to the *Depression* variable, individuals in the *Normal* class have small probabilities for all items, those in the *Mild-depression* class have higher probabilities than the *Normal* class, and those in the *Severe-depression* class have high probabilities of responding "Yes" to mild depressive symptoms and moderate probabilities for severe depressive symptoms. Similarly, with regard to the *Anxiety* variable, the item-response probabilities are small in the *Normal* class, medium in the *Mild-anxiety* class, and and high in the *Severe-anxiety* class showing the tendency of individuals belonging to the class.

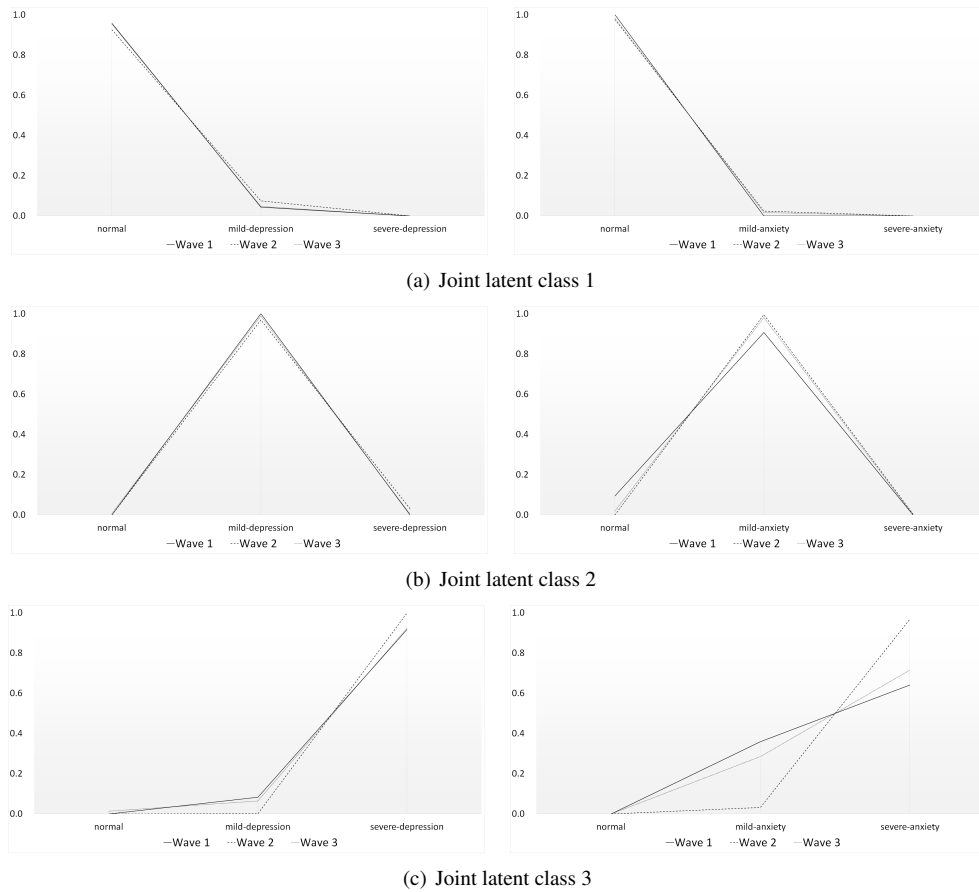Table 5 represents the probability of belonging to the class. From JLCPA results, the first joint

(a) Joint latent class 1

(b) Joint latent class 2

(c) Joint latent class 3

Figure 3: *The estimated η-parameters of depression (left) and anxiety (right) at each wave.*

Table 3: Diagnostic statistics for JLTA model and JLCPA models with the different number of profiles

Model	Number of profiles	AIC	BIC
	2	31898.03	32409.08
JLCPA	3	31893.10	32494.99
	4	31805.02	32495.93
JLTA	N/A	31879.14	32413.87

JLTA = joint latent transition analysis; JLCPA = joint latent class profile analysis; AIC = Akaike information criterion; BIC = Bayesian information criterion.

class can be labeled as the *Normal emotions* because the probabilities of belonging to the class of *Normal* are 0.937 and 0.993 for the *Depression* and *Anxiety*. The second joint class can be named as *Mild symptoms* as the probability of belonging to the class of *Mild-depression* for the *Depression* and *Mild-anxiety* for the *Anxiety* are both over 0.9. The last joint class shows the high probability of belonging to the class of *Severe-depression* and *Severe-anxiety*, so it can be labeled as *Severe symptoms*. JLTA also shows similar results.

Based on these latent structure, the conditional probabilities of joint class membership for a particular class profile (the tertiary measurement parameters of JLCPA) identified the representative se-
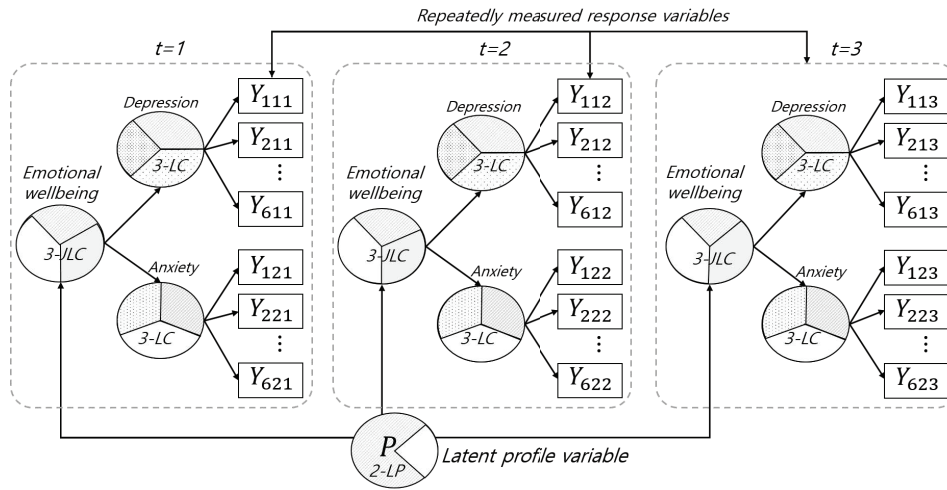
Figure 4: *Model structure of the selected joint latent class profile analysis.*

Table 4: The estimated ρ-parameters of item-response probabilities for each latent variable

| Manifest item | Latent class for Depression | | | | | |
| | JLCPA | | | JLTA | | |
	Normal	Mild-depression	Severe-depression	Normal	Mild-depression	Severe-depression
Feeling no interest in things	0.134	0.590	0.922	0.132	0.585	0.922
Feeling lonely	0.061	0.508	0.953	0.059	0.501	0.947
Feeling blue	0.045	0.412	0.861	0.043	0.410	0.859
Feeling worthless	0.006	0.154	0.791	0.006	0.150	0.791
Feeling hopeless about future	0.052	0.315	0.763	0.050	0.313	0.762
Sometimes think about suicide	0.011	0.119	0.482	0.012	0.117	0.480

| Manifest item | Latent class for Anxiety | | | | | |
| | JLCPA | | | JLTA | | |
	Normal	Mild-anxiety	Severe-anxiety	Normal	Mild-anxiety	Severe-anxiety
Feeling nervous	0.079	0.455	0.893	0.077	0.448	0.890
Feeling tense	0.038	0.401	0.775	0.035	0.396	0.771
Feeling scared suddenly	0.032	0.256	0.858	0.031	0.252	0.846
Feeling panic	0.004	0.106	0.662	0.004	0.103	0.649
Feeling restless	0.063	0.411	0.827	0.060	0.405	0.826
Feeling fearful	0.010	0.147	0.812	0.010	0.140	0.807

JLCPA = joint latent class profile analysis; JLTA = joint latent transition analysis.

quential patterns of *Adolescent emotional well-being*. In Table 6, the most common profile can be labeled as *Deterioration* and its prevalence is 71.6%. Among individuals in '*Deterioration*,' the probabilities of belonging to the joint class of *Normal emotions* are consistently decreased over time and those of belonging to other class of *Mild symptoms* are slightly increased over time (see the first two rows in Table 6). The model also identified other profiles of juveniles, some of whom tended to intensify their depressive and anxious symptoms over time and others who got better, moving towards *Normal emotions* or *Severe symptoms* from *Mild symptoms*, labeled as '*Polarization*.'

Table 7 presents the estimates of marginal probabilities at wave 1 and transition probabilities from JLTA. At wave 1, the class membership of *Normal emotions* is 0.503, that of *Mild symptoms* is 0.390, and that of *Severe symptoms* is 0.107. At wave 2, individuals among *Normal emotions* class at wave

Table 5: The estimated η-parameters of belonging to a class for each given joint class

Latent variable	Class	Joint class for Emotional well-being					
		JLCPA			JLTA		
		Normal emotions	Mild symptoms	Severe symptoms	Normal emotions	Mild symptoms	Severe symptoms
Depression	Normal	0.937	0.014	0.000	0.923	0.001	0.000
	Mild-depression	0.063	0.931	0.023	0.076	0.931	0.005
	Severe-depression	0.000	0.055	0.977	0.001	0.068	0.995
Anxiety	Normal	0.993	0.017	0.000	0.977	0.000	0.000
	Mild-anxiety	0.006	0.967	0.139	0.023	0.975	0.093
	Severe-anxiety	0.001	0.015	0.861	0.001	0.025	0.907

JLCPA = joint latent class profile analysis; JLTA = joint latent transition analysis.

Table 6: The tertiary measurements estimates of γ- and the profile prevalence δ-parameters from JLCPA

Profile	Joint class	Wave 1	Wave 2	Wave 3
Deterioration (71.6%)	Normal emotions	0.621	0.541	0.477
	Mild symptoms	0.259	0.370	0.388
	Severe symptoms	0.120	0.089	0.135
Polarization (28.4%)	Normal emotions	0.238	0.267	0.405
	Mild symptoms	0.518	0.502	0.291
	Severe symptoms	0.234	0.231	0.304

JLCPA = joint latent class profile analysis.

Table 7: The initial joint class prevalence estimates of δ- at wave 1 and transition probability τ-parameter from JLTA

Joint Class		Joint class for Emotional well-being						
		Wave 2				Wave3		
		Normal emotions	Mild symptoms	Severe symptoms		Normal emotions	Mild symptoms	Severe symptoms
Normal emotions (0.503)		0.736	0.228	0.036		0.698	0.271	0.032
Mild symptoms (0.390)	Wave 1	0.334	0.559	0.107	Wave 2	0.332	0.502	0.166
Severe symptoms (0.107)		0.176	0.275	0.549		0.227	0.394	0.380

JLTA = joint latent transition analysis.

1 tend to maintain at the same class *Normal emotions* with the probability of 0.736, while some move to *Mild symptoms* with probability of 0.228, and few move to *Severe symptoms* with probability of 0.036. That is, JLTA provides transition parameter estimates for two adjacent time points, such as the transition probability from wave 1 to wave 2 and the transition probability from wave 2 to wave 3. Transition probabilities are useful to explain the tendency to shift or maintain among a latent class for a given time point, in a pairwise way. However, these pairwise transition estimates are difficult to interpret for whole time period, because the number of possible combinations of joint latent classes significantly increases (J^T) as time points T increases. In contrast, JLCPA is useful for identifying how joint latent class prevalence change over time, and it does not need the combination of joint latent class. Therefore, JLCPA is easier to understand or interpret the flow over the entire period. Note that the more time points one considers, the more useful JLCPA would be versus JLTA.

4.4. Estimates by adolescent-parent relationship group

We now expand the discussion by considering the relationship with parents that affect the emotional well-being of adolescents, and perform our JLCPA according to the parent-adolescent-relationship.

Table 8: The contribution of manifest items for subgrouping parent-adolescent relationship

Manifest item	Class for Parent-adolescent relationship		
	Trust & no anger	Trust & squabbling	Apathy
Seek parent's attention	0.937	0.958	0.425
Communication	0.937	0.863	0.139
Trust their parents	0.985	0.975	0.624
Not discussing problems w/ them	0.782	0.348	0.530
Rarely getting upset	0.754	0.118	0.391
Rarely avoiding parents	0.697	0.362	0.439
Rarely feeling angry	0.812	0.462	0.337

Table 9: The estimated tertiary measurements according to the relationship with parent

Relationship group	Profile	Joint class	Wave 1	Wave 2	Wave 3
Trust & no anger	Deterioration	Normal emotions	0.870	0.879	0.759
		Mild symptoms	0.108	0.121	0.241
		Severe symptoms	0.022	0.000	0.000
	Polarization	Normal emotions	0.382	0.073	0.123
		Mild symptoms	0.617	0.812	0.708
		Severe symptoms	0.001	0.115	0.169
Trust & squabbling	Deterioration	Normal emotions	0.450	0.571	0.584
		Mild symptoms	0.526	0.385	0.325
		Severe symptoms	0.024	0.044	0.091
	Polarization	Normal emotions	0.000	0.017	0.117
		Mild symptoms	0.510	0.458	0.483
		Severe symptoms	0.490	0.525	0.400
Apathy	Deterioration	Normal emotions	0.440	0.467	0.382
		Mild symptoms	0.510	0.533	0.498
		Severe symptoms	0.050	0.000	0.120
	Polarization	Normal emotions	0.000	0.139	0.466
		Mild symptoms	0.092	0.115	0.000
		Severe symptoms	0.908	0.746	0.534

For the parent-adolescent relationship, we divided sample youths into subgroups using 7 items at wave 1: whether do you (a) seek your caregiver's point of view; (b) tell your caregiver about problem; (c) trust your caregiver; (d) feel satisfied discussing problems with your caregiver; (e) rarely get upset about your caregiver; (f) never hesitate to bother your caregiver with problem; (g) rarely feel angry with your caregiver. Here, all the polychotomous items were also changed into dichotomous.

The optimal segmentation was determined as three groups by LCA in terms of model interpretation and simplicity (AIC = 5848.8, BIC = 5957.6). The subgroups were labeled as '*Trust & no anger*', '*Trust & squabbling*', and '*Apathy*', based on the estimated item-response probabilities as presented in Table 8. Table 9 shows that the sequential patterns of *Adolescent emotional well-being* differs among subgroups depending on the relationship with parent. Adolescents in '*Trust & no anger*' group having good relationship with their parents, have consistently high probabilities of belonging to the *Normal emotion* higher than 0.75, implying that they are likely to remain normal over time, while their probabilities of *Severe symptoms* are almost zero over time. However, adolescents in the '*Trust & squabbling*' group show less tendency of belonging to *Normal emotion* than those in '*Trust & no anger*' group, and higher tendencies of having symptoms either *Mild symptoms* or *Severe symptoms*. For the '*Apathy*' group, the joint class prevalence of *Severe symptoms* in *Polarization* profile shows the highest tendencies among the three relationship groups. These results imply that the negative relationship with parent may have an adverse effect on adolescent's emotional wellbeing.

5. Conclusion

This paper proposes joint latent class analysis for longitudinal data in two different approaches of the as joint latent class profile analysis (JLCPA) and joint latent transition analysis (JLTA). By reflecting the dependencies among latent variables in cross-sectional and longitudinal view, our models allows an examination of the associations among latent variables of different attributes as well as an investigation of the joint stage-sequential progression for the mixture of multiple latent class variables. These models can be applied to other research subjects such as disease progression considering gene-gene interactions, or poly-substance use behavior patterns consisted of alcohol and cigarettes. We applied our models to investigate the sequential patterns of adolescents' emotional well-being, depression and anxiety, which are repeatedly measured as several survey items. Empirical results revealed three joint latent classes of normal emotions, mild symptoms, and severe symptoms, under both JLCPA and JLTA. JLCPA seems more useful to interpret than JLTA as time points to consider increase. Our JLCPA results presented two representative profiles for the adolescent emotional well-being over time, and their tendencies showed different according to the parent-adolescent-relationship subgroups. The proposed models can be generalized to incorporate covariates of individual characteristics, such as gender, race, and age. However, such inclusion of covariates may cause computational complexity requiring alternative estimation methods, which are left to future studies.

Acknowledgements

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