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Magnetic dependence of cyclotron resonance in the electronpiezoelectric phonon interacting materials

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Abstract Based on quantum transport theory, we investigated theoretically the magnetic field dependence of the quantum optical transition of quasi 2-dimensional Landau splitting system, in CdS and ZnO. Through the analysis of the current work, we found the increasing properties of the cyclotron resonance line-profiles (CRLPs) which show the absorption power and the cyclotron resonance line-widths (CRLWs) with the magnetic field in CdS and ZnO. We also found that that CRLWs, $\gamma_{total}(B)$ of CdS $< \gamma_{total}(B)$ of ZnO in the magnetic field region B < 15 Tesla.

Keywords Cyclotron resonance, quantum transport theory, CdS and ZnO, equilibrium average projection scheme, scattering factor, absorption power

Introduction

The study of cyclotron resonance (CR) has been known to be a good tool for investigating the transport behavior of electrons in low-dimensional resonant system.¹ We use the projected Liouville equation method with the equilibrium average projection scheme (EAPS). The merit of using EAPS is that the absorption power formula (line-profiles) and the scattering factor function (line-widths) can be obtained in a onestep process by expanding the quantum transport theory.²⁻⁷ In the previous work,⁸⁻¹⁰ we applied the EAPS theory in Ge and Si, since there are abundant experimental dates in non-confining potential systems. We compared our results of numerical calculations of the EAPS theory with existing experimental data and showed a good agreement between them.¹¹ However, it was restricted for non-confining potential systems with the extremely weak coupling (EWC) approximation. Hence, it is important to obtain an explicit expression of the CRLPs for a given confining potential system on the basis of a theoretical formulation. We suggested a more precise procedure of expansion and application of EAPS in low-dimensional electron systems with the moderately weak coupling (MWC) approximation. In the MWC scheme, the distribution components can provide an adequate explanation of the quantum transition processes.¹² Through the numerical calculation of the theoretical result, we investigate the optical transition which show the absorption power and the scattering factor, which show the scattering effect in the electron-piezoelectric phonon interaction system. Finally, we shall be devoted to some discussion and draw conclusions.

Theory

When a static magnetic field $\vec{B} = B\hat{z}$ is applied to an

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electron system, the single electron energy state is quantized to the Landau levels. We select a system of electrons confined in an infinite square well potential between z = 0 and $z = L_z$ in the z -direction. We use the eigenvalue and eigenstate of Ref [8] of the square well potential system. We suppose that an oscillatory electric field $E(t) = E_0 \exp(i\omega t)$ is applied along the z -axis, which gives the absorption power delivered to the system as

$$P(\omega) = \frac{E_0^2}{2} \operatorname{Re}\{\sigma(\omega)\}$$
(1)

where "Re" denotes the real component and $\sigma(\omega)$ is the optical conductivity tensor that is the coefficient of the current formula. Here the absorption power represents the optical cyclotron resonance line-profiles (CRLPs), and the scattering factor function represents the optical cyclotron resonance line-widths (CRLWs). We consider the electron-piezoelectric phonon interacting system and then we have the Hamiltonian of the system as below,¹³

$$H_{s} = H_{e} + H_{p} + V$$

$$H_{e} = \sum_{\alpha} \langle \alpha | h_{e} | \alpha \rangle a_{\alpha}^{\dagger} a_{\alpha}$$

$$H_{p} = \sum_{q} h \omega_{q} b_{q}^{\dagger} b_{q}$$

$$V = \sum_{q} \sum_{k,\lambda} C_{k,\lambda}(q) a_{k}^{\dagger} a_{\lambda}(b_{q} + b_{-q}^{\dagger}).$$
(2)

Where, H_e is the electron Hamiltonian, H_p is the phonon Hamiltonian and V is the electron-piezoelectric phonon (or impurity) interaction Hamiltonian. The $b_q^{\dagger}(b_q)$ is the creation operator (annihilation operator) of boson particle, and \vec{q} is phonon (or impurity) wave vector. The interaction Hamiltonian of electron-piezoelectric phonon (or impurity) interacting system is V, where the coupling matrix element of electron-phonon interaction $C_{k,\lambda}(q)$ is $C_{k,\lambda}(q) \equiv |V_q|^2 \langle k | \exp(i\vec{q} \cdot \vec{r}) | \lambda \rangle$, \vec{r} is the position vector of electron. The electron-piezoelectric phonon interaction parameter V_q is given by

$$\left|V_{q}\right|^{2} = \frac{\bar{K}^{2}\hbar\nu_{s}e^{2}}{2\chi\varepsilon_{0}\Omega}\frac{1}{q}.$$
(3)

Here the \bar{K} is the electrochemical constant, the v_s is the sound velocity in solid, the Ω is the volume of the system, the ε_0 is the permittivity of free space and the χ is the dielectric constant. For the optical quantum transition system under a circularly polarized external field, the current operator J^{\pm} are defined

$$J^{+} = \sum_{\alpha} j_{\alpha}^{+} a_{\alpha+1}^{\dagger} a_{\alpha} \text{ and } J^{-} = \sum_{\beta} (j_{\beta}^{+})^{*} a_{\beta} a_{\beta+1}$$

where $J^{\pm} \equiv J_x \pm i J_y$ are two components of the single electron current operator *J*. Using Landau gauge, the magnetic field *B* is perpendicular to the barriers of the well, and the distance between the barriers, which are assumed to be infinitely high. Since the wave function vanish at z = 0 and $z = L_z$, the eigenfunctions $\Psi_{\alpha,k_{y\alpha},k_{z,\alpha}}$ and the corresponding $E_{\alpha,k_{z\alpha}}$ as

$$\begin{split} \Psi_{\alpha,k_{y\alpha},k_{z,\alpha}}(x,y,z) &= \left\langle \vec{x} \middle| N_{\alpha},k_{y\alpha},k_{z,\alpha} \right\rangle \\ &= \frac{1}{\sqrt{L_y L_z}} \exp[i(k_{y\alpha}y + k_{z\alpha}z)] \\ &\times \Phi_{N_\alpha}(x - x_\alpha) \sin(k_z z) \\ \Phi_{N_\alpha}(x) &= \frac{1}{\sqrt{2^{N_\alpha} N_\alpha! r_0 \sqrt{\pi}}} \exp\left[-\frac{(x - x_\alpha)^2}{2r_0^2}\right] \\ &\times H_\alpha\left(\frac{x - x_\alpha}{r_0}\right) \\ E_{\alpha,k_{z\alpha}} &= \left(N_\alpha + \frac{1}{2}\right) \hbar \omega_c + n_\alpha^2 \varepsilon_0, \end{split}$$

where

$$\varepsilon_{0} = \frac{\hbar^{2} \pi^{2}}{2m^{*} L_{z\alpha}^{2}} \qquad N_{\alpha} = 0, 1, 2, 3, ...$$
$$n_{\alpha} = 1, 2, 3, ... = N_{\alpha} + 1, \qquad (4)$$

here $r_0 = (\hbar/m\omega_c)^{1/2}$ is the radius of cyclotron motion, $H_{\alpha}(x)$ is the Hermite polynomials. From the eigenstate, we can obtain the matrix element of the current as

$$j_{\alpha}^{+} \equiv \langle \alpha + 1 | j^{+} | \alpha \rangle$$
$$= -ie \left[\frac{2(N_{\alpha} + 1)\hbar\omega}{m^{*}} \right]^{\frac{1}{2}},$$
(5)

here m^* is the effective mass of electron. In the previous work,⁸ we derived the current in the ω space as

$$J^{-}(\omega) = \operatorname{Re}\{\sigma_{\pm}(\omega)\}E_{+}(\omega), \qquad (6)$$

where the conductivity tensor is

$$\sigma_{\pm}(\omega) = \frac{\left(-\frac{i}{\hbar}\right)\Lambda_{\mp}}{i\omega - A_{\mp} + Q_{\mp}(\omega)}$$
(7)

The scattering factor function is

$$Q_{\mp}(\omega) \equiv \frac{i}{\hbar\Lambda_{\mp}} Tr\{J^{-}L_{s}G_{\mp}^{'}(\omega)P_{\mp}^{'}L_{s}L^{+}\rho_{s}\},\qquad(8)$$

and the propagator is

$$G'_{\mp}(\omega) \equiv \frac{1}{\hbar \bar{\omega} - P'_{\mp} L_s},\tag{9}$$

here ρ_s is the equilibrium density matrix, $\bar{\omega} = \omega + i\eta$, and the η is an infinitesimal value. The equilibrium part of Liouville operators implies $L_s X \equiv [H_s, X]$ and the non-equilibrium part of response current Liouville operator implies $L^+X = (-i/\omega)[J^+, X]$ for an arbitrary operator X. We expand the propagator with the conventional series expansion representation.

$$G_{\mp}^{'}(\omega) \equiv \frac{1}{\hbar\bar{\omega} - P_{\mp}^{'}L_s} = G_d^{'}\sum_{n=0}^{\infty} \left[P_{\mp}^{'}L_{\nu}G_d^{'}\right]^n$$
$$G_d^{'}(\omega) \equiv \frac{1}{\hbar\bar{\omega} - P_{\mp}^{'}L_d} = G_d\sum_{n=0}^{\infty} \left[P_{\mp}L_dG_d\right]^n$$

$$G_d(\omega) \equiv \frac{1}{h\bar{\omega} - L_d} \tag{10}$$

Using the properties of projection operator we obtain the relations,

$$\left(\frac{i}{\hbar\Lambda_{\mp}^{(\alpha)}}\right) \langle Tr\{J^{-}XP_{\mp}^{'}L_{d}L^{+}\rho_{s}\}\rangle_{B}
= \left(\frac{i}{\hbar\Lambda_{\mp}^{(\alpha)}}\right) \langle Tr\{J^{-}XL_{d}L^{+}\rho_{s}\}\rangle_{B}
- \left(\frac{i}{\hbar\Lambda_{\mp}^{(\alpha)2}}\right) \langle Tr\{J^{-}XL^{+}\rho_{s}\}\rangle_{B} \langle Tr\{J^{-}L_{d}L^{+}\rho_{s}\}\rangle_{B} = 0
\left(\frac{i}{\hbar\Lambda_{\mp}^{(\alpha)}}\right) \langle Tr\{J^{-}L_{d}P_{\mp}^{'}XL^{+}\rho_{s}\}\rangle_{B}
= \left(\frac{i}{\hbar\Lambda_{\mp}^{(\alpha)}}\right) \langle Tr\{J^{-}L_{d}XL^{+}\rho_{s}\}\rangle_{B}
- \left(\frac{i}{\hbar\Lambda_{\mp}^{(\alpha)2}}\right) \langle Tr\{J^{-}L_{d}L^{+}\rho_{s}\}\rangle_{B} \langle Tr\{J^{-}XL^{+}\rho_{s}\}\rangle_{B}
= 0 \qquad (11)$$

Since the average of odd background terms are zero, we use the useful relation as below

$$\langle Tr\{YP'L_{\nu}L^{+}\rho_{s}\}\rangle_{B} = \langle Tr\{YL_{\nu}L^{+}\rho_{s}\}\rangle_{B}$$

$$Tr\{R_{\mu}L_{1}L_{2}L_{3}....L_{n}L^{+}\rho_{s}\}$$

$$= (-1)^{n+1}\langle L^{+}L_{n}...L_{3}L_{2}L_{1}R_{\mu}\rangle_{B},$$

$$(12)$$

here $\langle \dots \rangle_B$ is the ensemble average of background particle states (for example, phonon state). $\langle \dots \rangle$ is the ensemble average of electron states and background particle states. Then we obtain the simple scattering factor function as

$$Q_{\mp}(\omega) \equiv \frac{i}{\hbar \Lambda_{\mp}} \langle L^+ L_{\nu} G_d L_{\nu} J^- \rangle \tag{13}$$

We obtain the matrix elements of dynamic variable

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and through the continuous approximation of the quantum integral, the absorption power formula is obtained finally as below

$$P(\omega) \propto \left(\frac{e^2 \omega_c^2}{\pi^2 \hbar \omega}\right)$$
$$\times \frac{\gamma_{total}(\omega) \sum_{N_{\alpha}} \int_{-\infty}^{\infty} dk_{z_{\alpha}} (N_{\alpha} + 1)(f_{\alpha} - f_{\alpha+1})}{(\omega - \omega_0)^2 + \gamma_{total}^2(\omega)},$$
(14)

and the CRLWs $\gamma^{\mp}(\omega)$ in integration form as below

$$\gamma_{total}(\omega) \equiv \operatorname{Re}\{Q_{\mp}(\omega)\} \equiv \sum_{\mp} \gamma_{\alpha,\beta}^{\mp} = \left(\frac{-\Omega}{4\pi\hbar^2 v_s}\right)$$
$$\times \frac{\sum_{\mp} \sum_{N_{\alpha}=0} \sum_{N_{\beta}=0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Y_{\alpha,\beta}^{\mp} dk_{z_{\alpha}} dq_{z}}{\sum_{N_{\alpha}=0} \int_{-\infty}^{\infty} dk_{z_{\alpha}} (N_{\alpha}+1)(f_{\alpha+1}-f_{\alpha})},$$
(15)

where the integrand-factor is

$$Y_{\alpha,\beta}^{\mp} \equiv Y_{\alpha,\beta}^{A\mp} + Y_{\alpha,\beta}^{B\mp} + Y_{\alpha,\beta}^{C\mp} + Y_{\alpha,\beta}^{D\mp} + Y_{\alpha,\beta}^{E\mp} + Y_{\alpha,\beta}^{F\mp},$$
(16)

and

$$\begin{split} Y_{\alpha,\beta}^{A\mp} &\equiv S_{\beta,\alpha+1}^{\mp} \left(\begin{array}{c} \alpha,\beta\\ \beta,\alpha\end{array}\right) (N_{\alpha}+1)(f_{\alpha+1}-f_{\alpha}) \\ &\times \left[N_{q}^{\pm}+\left(1-F_{\beta}^{\pm}\right)\right] \\ Y_{\alpha,\beta}^{B\mp} &\equiv -S_{\beta,\alpha+1}^{\mp} \left(\begin{array}{c} \alpha,\beta\\ \beta+1,\alpha+1\end{array}\right) \sqrt{(N_{\alpha}+1)(N_{\beta}+1)} \\ &\times \left(f_{\beta+1}^{\pm}-f_{\beta}^{\pm}\right) \left[N_{q}^{\pm}\mp f_{\alpha+1}\right] \\ Y_{\alpha,\beta}^{C\mp} &\equiv -U_{\alpha,\beta+1}^{\mp} \left(\begin{array}{c} \beta,\alpha\\ \alpha+1,\beta+1\end{array}\right) \sqrt{(N_{\alpha}+1)(N_{\beta}+1)} \\ &\times \left(f_{\beta+1}^{\pm}-f_{\beta}^{\pm}\right) \left[N_{q}^{\pm}\pm\left(1-f_{\beta+1}^{\mp}\right)\right] \\ Y_{\alpha,\beta}^{D\mp} &\equiv U_{\alpha,\beta}^{\mp} \left(\begin{array}{c} \beta,\alpha\\ \alpha+1,\beta\end{array}\right) (N_{\alpha}+1)(f_{\alpha+1}-f_{\alpha}) \left[N_{q}^{\pm}f_{\beta}^{\mp}\right] \\ Y_{\alpha,\beta}^{E\mp} &\equiv -\left[W_{1}^{\mp} \left(\begin{array}{c} \beta,\alpha\\ \alpha,\beta\end{array}\right) + W_{1}^{-} \left(\begin{array}{c} \beta,\alpha\\ \alpha+1,\beta+1\end{array}\right) \\ &- W_{1}^{+} \left(\begin{array}{c} \beta,\alpha\\ \alpha,\beta\end{array}\right) - W_{1}^{+} \left(\begin{array}{c} \beta,\alpha\\ \alpha+1,\beta+1\end{array}\right) \end{split}$$

$$\times (N_{\alpha} + 1)(f_{\alpha+1} - f_{\alpha})$$

$$Y_{\alpha,\beta}^{F\mp} \equiv -\left[Z^{-} \begin{pmatrix} \alpha,\beta\\ \beta+1,\alpha+1 \end{pmatrix} (f_{\beta+1}^{-} + f_{\beta}^{-}) + Z^{+} \begin{pmatrix} \alpha,\beta\\ \beta+1,\alpha+1 \end{pmatrix} (f_{\beta+1}^{-} + f_{\beta}^{-})\right]$$

$$\times \sqrt{(N_{\alpha} + 1)(N_{\beta} + 1)}(f_{\alpha+1} + f_{\alpha}), \quad (17)$$

here terms of electron-phonon interacting parts are

$$\begin{split} S_{\beta,\alpha}^{\mp} \left(\begin{array}{c} k,\eta\\ \mu,\nu \end{array} \right) \\ &\equiv \left[V \left(q_{z}, q_{\perp 1}^{\mp\beta\alpha} \right)^{2} \sqrt{\left(q_{\perp 1}^{\mp\beta\alpha} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 1}^{\mp\beta\alpha} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 2}^{\mp\beta\alpha} \right)^{2} \sqrt{\left(q_{\perp 2}^{\mp\beta\alpha} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 2}^{\mp\beta\alpha} \right)^{2} \right) \right] \\ &= \left[V \left(q_{z}, q_{\perp 3}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 3}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 3}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 4}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 4}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 4}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &= \left[V \left(q_{z}, q_{\perp 4}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 6}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 6}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &= \left[V \left(q_{z}, q_{\perp 6}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 6}^{\mp\alpha\beta} \right)^{2} \sqrt{\left(q_{\perp 6}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 5}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 8}^{\mp\alpha} \right)^{2} \sqrt{\left(q_{\perp 6}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 6}^{\mp\alpha\beta} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 8}^{\mp\alpha} \right)^{2} \sqrt{\left(q_{\perp 6}^{\mp\alpha\beta} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 6}^{\mp\alpha} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 8}^{\mp\alpha} \right)^{2} \sqrt{\left(q_{\perp 7}^{\mp\alpha} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 7}^{\mp\alpha} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 8}^{\mp\alpha} \right)^{2} \sqrt{\left(q_{\perp 8}^{\mp\alpha} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 7}^{\mp\alpha} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 8}^{\mp\alpha} \right)^{2} \sqrt{\left(q_{\perp 8}^{\mp\alpha} \right)^{2} + q_{z}^{2}} K_{\mu,\nu}^{k,\eta} \left(\frac{r^{2}}{2} \left(q_{\perp 8}^{\mp\alpha} \right)^{2} \right) \right] \\ &+ \left[V \left(q_{z}, q_{\perp 8}^{\mp\alpha} \right)^{2} \sqrt{\left(q_{\pm 8}^{\mp\alpha} \right)^{2} + q_{z}^{$$

The Fermi-Dirac distribution functions are

$$f_{\alpha} = \frac{1}{e^{\varepsilon_{\alpha}/k_{B}T} + 1} \qquad f_{\beta}^{\pm} = \frac{1}{e^{\varepsilon_{\beta}^{\pm}/k_{B}T} + 1},$$
 (19)

where eigenvalues are

$$\begin{aligned} \varepsilon_{\alpha} &= \left[\left(N_{\alpha} + \frac{1}{2} \right) \hbar \omega_{c} + \frac{\hbar^{2} k_{z\alpha}^{2}}{2m^{*}} + (\varepsilon_{c} - \varepsilon_{F}) \right] \\ \varepsilon_{\beta}^{\pm} &= \left[\left(N_{\beta} + \frac{1}{2} \right) \hbar \omega_{c} + \frac{\hbar^{2} k_{z\beta}^{2}}{2m^{*}} \\ &+ (\varepsilon_{c} - \varepsilon_{F}) \right] \delta_{k_{\beta},k_{\beta} \pm q_{yz}} \\ &= \left[\left(N_{\beta} + \frac{1}{2} \right) \hbar \omega_{c} + \frac{\hbar^{2} (k_{z\alpha} \pm q_{z})^{2}}{2m^{*}} + (\varepsilon_{c} - \varepsilon_{F}) \right], \end{aligned}$$

$$(20)$$

where ω_c is the cyclotron resonance frequency, $k_{z\alpha}$ is the z component of electron wave vector and the energy are as below

$$\varepsilon_c - \varepsilon_F(T) = 0.5 \left[\varepsilon_g(T) - \frac{kT}{T - \xi} - \frac{3}{4} k_B T \ln\left(\frac{\bar{m}}{m^*}\right) \right],$$
(21)

where ε_c is the conduction band minimum energy, $\varepsilon_F(T)$ is the Fermi energy, $\varepsilon_g(T)$ is the band gap energy at T, k and ξ are characteristic constant of the material, \bar{m} is the density of states effective mass of hole. Here the phonon energy is

$$\varepsilon(q,T) = \frac{\hbar\omega_q}{k_B T} = \frac{\hbar\nu_s}{k_B T} \sqrt{\left(q_{\perp n}^{\mp k\lambda}\right)^2 + q_z^2}.$$
 (22)

If $N_{\alpha} < N_{\beta}$ and $N_k < N_{\lambda}$, the K-matrix is

$$K_{k,\lambda}^{\alpha,\beta} \equiv \sqrt{\frac{N_{\alpha}!}{N_{\beta}!}} \sqrt{\frac{N_{k}!}{N_{\lambda}!}} (\sqrt{t})^{N_{\beta}-N_{\alpha}} (\sqrt{t})^{N_{\lambda}-N_{k}} \times \exp(-t) L_{N_{\alpha}}^{N_{\beta}-N_{\alpha}}(t) L_{N_{k}}^{N_{\lambda}-N_{k}}(t), \qquad (23)$$

where the Legendre function is

$$L_{n}^{m}(t) = \frac{1}{(n!)} \exp(t) \frac{1}{t^{m}} \left(\frac{d^{n}}{dt^{n}}\right) [t^{n+m} \exp(-t)]$$



Figure 1. Magnetic field dependence of the CRLWs, $\gamma_{total}(B)$ of CdS at T = 50, 120 and 210 K.

$$t \equiv \frac{r_0^2 (q_x^2 + q_y^2)}{2}.$$
 (24)

The result can be applied directly to numerical analysis through wave vector integration.

Result and discussion

Through the numerical calculation of Eq. (14) and Eq. (15), we analyze absorption power and line-widths in CdS and ZnO. It is well known that the piezoelectricpotential scattering is dominant for pure CdS and ZnO. We use $m^* = 0.19m_0$ and $\bar{m} = 0.7m_0$ which are the effective masses of constants of CdS are $\rho =$ 4.82×10^3 kg/m³, $v_{sl} = 4.28 \times 10^3$ m/s is the longitudinal sound velocity, $v_{st} = 1.81 \times 10^3$ m/s is the transverse sound velocity, $k = 8.58 \times$ 10^{-4} eV/K, $\xi = 235$ K and $|K|^2 = 2.98 \times 10^{-2}$. The speed of sound v_s in Eq. (22) shall be replaced by the average value v_s of v_{sl} and v_{st} , as $\bar{v}_s =$ $(v_{sl} + v_{st})/2$ and the energy gap $\varepsilon_g(T)$ replaced by $\bar{\varepsilon}_{g} = 2.56$ eV in approximation by noting that the variation against the temperature is very small. More accurate value of $\varepsilon_q(T)$ can be obtained by Eq. (21) if the characteristic constants k and ξ are available. We choose $\varepsilon_0 = 8.854 \times 10^{-12} \text{ c}^2/\text{Nm}^2$. For ZnO,



Figure 2. Magnetic field dependence of the CRLWs, $\gamma_{total}(B)$ of ZnO at T = 50, 120 and 210 K.

we have $m^* = 0.27m_0$ and $\bar{m} = 1.8m_0$, $\rho = 4.09 \times 10^3$ kg/m³, $v_{sl} = 5.61 \times 10^3$ m/s, $v_{st} = 2.98 \times 10^3$ m/s, $\bar{e}_g = 3.42$ eV, $k = 17.88 \times 10^{-4}$ eV/K and $\xi = 204$ K. In Fig. 1, it is shown that the comparison of the magnetic field dependence of CRLWs, $\gamma_{total}(B)$ of CdS, and $\gamma_{total}(B)$ of ZnO in Fig. 2, at T = 50, 120, 210 K. Our results reveal that $\gamma_{total}(B)$ of CdS $<\gamma_{total}(B)$ of ZnO in the magnetic field dependence of CRLWs, $\gamma_{total}(B)$ of CdS $<\gamma_{total}(B)$ of ZnO in the magnetic field region B < 15 Tesla. The magnetic field dependence of CRLWs, $\gamma_{total}(B)$ of CdS and $\gamma_{total}(B)$ of ZnO at T = 50, 120, 210 K is plotted in log scale. As shown, $\gamma_{total}(B)$ of CdS increases as the magnetic field increases, and $\gamma_{total}(B)$



Figure 3. The relative frequency dependence of the absorption power, P(B) of CdS with $\lambda = 119^{\Box}\mu m$ at T = 50, 120 and 210 K.

of ZnO increases as the magnetic field increases. However, $\gamma_{total}(B)$ of ZnO decreases when the magnetic field in the high magnetic field is larger than B = 17 Tesla at T = 50, 120, 210 K. We can read the relative frequency dependence of the absorption power, P(B) of CdS (in Fig. 3) and ZnO (in Fig. 4) with $\lambda = 119 \ \mu m$ at T = 50, 120, 210 K. The analysis of the relative frequency dependence of the absorption power represents the magnetic field dependent property of the absorption power given for an external field wavelength and the conditions of the system. The broadening effects near the resonance peaks



Figure 4. The relative frequency dependence of the absorption power, P(B) of ZnO with $\lambda = 119^{\Box}\mu m$ at T = 50, 120 and 210 K.

for various external fields appeared. The results indicate that the EAPS is a useful method to explain the resonant phenomena based on the quantum transition and scattering effect in a microscopic view.

Conclusions

As a conclusion, the relatively easy approach to the analysis of the magnetic field dependence of CRLPs and CRLWs is one of the merits of our EAPS theory. We analyzed the absorption power P(B) of CdS and we saw broadening effect of absorption power P(B) increases as magnetic field increase. This result implies that the scattering effect of the phonon increase as magnetic field increase. We also analyzed

similarly the thermal properties of ZnO. The results of this work will help to analyze experimental the scattering mechanism in the electron-piezoelectric phonon interacting materials.

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