

A Study on the Desired Target Signal Estimation using MUSIC and LCMV Beamforming Algorithm in Wireless Coherent Channel

Kwan Hyeong Lee

Associate Professor, Division of Human IT Convergence, Daejin University, Korea
khlee@daejin.ac.kr

Abstract

In this paper, we studied to direction of arrival (DoA) estimation to use DoA and optimum weight algorithms in coherent interference channels. The DoA algorithm have been considerable attention in signal processing with coherent signals and a limited number of snapshots in a noise and an interference environment. This paper is a proposed method for the desired signal estimation using MUSIC algorithm and adaptive beamforming to compare classical subspace techniques. Also, the proposed method is combined the updated weight value with LCMV beamforming algorithm in adaptive antenna array system for direction of arrival estimation of desired signal. The proposed algorithm can be used with combination to MUSIC algorithm, linearly constrained minimum variance beamforming (LCMV) and the weight value method to accurately desired signal estimation. Through simulation, we compare the proposed method with classical direction of in order to desired signals estimation. We show that the propose method has achieved good resolution performance better that classical direction arrival estimation algorithm. The simulation results show the effectiveness of the proposed method.

Keywords: LCMV, DoA, Adaptive Array, Coherent, MUSIC

1. Introduction

Adaptive array signal processing has been an important technology in radar¹, sonar, and wireless communication. Adaptive array signal processing appears as a potential technology to improve the spectrum efficiency [1-2]. Adaptive array signal processing has focused on methods for high resolution direction of arrival(DoA) estimation and optimum beamforming. DoA estimation is important on array signal processing in wireless interference channel. A many the DoA algorithms have been studied for the desired signal estimation of the incident signals on the receiver [3]. Among of DoA algorithms, Bartlett and Capon algorithms based on Fourier method are traditional DoA signal processing algorithms, but this two methods have poor resolution [4-5]. MUSIC and ESPRIT using subspace method have a high resolution because divide with the signal and noise subspace from eigen-decomposition of the covariance matrix of the received signals. However,

the subspace method is required much the computational burden. The subspace method usually need to the decorrelation techniques because don't be directly applied in coherent channel [6]. MUSIC algorithm is generally used for DoA estimation, and optimum beamforming is used as the Linearly constrained minimum variance beamforming(LCMV) algorithm. Drawback of these algorithms takes the severe degradation of the estimation accuracy in the coherent signals. An adaptive beamforming algorithm provide a distortionless response to the desired signal while removing noise and interference [7]. However, adaptive beamformer can suffer significant performance degradation in the presence of the mismatch between the actual direction of arrival of the signal and the look direction of the beamformers. In this paper, we propose a novel approach to robust adaptive array antenna using MUSIC algorithm and LCMV beamforming, which improves robustness to uncertainty in the desired signal direction. When signal to noise ratio is high, the proposed algorithm places more emphasis on the observations, estimates direction of arrival of the actual signal reliably and has nearly optimal performance. When signal to noise ratio is low, it relies on the priori knowledge about the source direction of arrival and has wider main beam which is robust the excellent performance of the proposed robust adaptive beamforming. Linear prediction method was studied to improve the DoA estimation of the single direction [8]. The linear prediction is usually used in time series problems, and can also be used in array signal processing of the coherent channel. The linear prediction method uses of multiple overlapping observation sequences for the prediction of unknown signals. This method don't require eigen-decomposition of the covariance matrix of the received signals. Also, the method has a higher resolution than the conventional Fourier-method. However, the linear prediction fails to estimation when the number of snapshots is small. That is, the linear prediction method is searching the optimal weight value which is minimum the prediction error. The rest of this paper is organized as follows. Section2 represent the signal model. Section 3 describes the Optimal weight vector. The proposed algorithm is provided in section 4. Section 5 shows simulation results of the proposed algorithm, and section 5 draws the conclusion.

2. Signal model

2.1 Linear Prediction signal method

The linear prediction method is output by linear combination of past data. If the parameter doesn't change for any time, the system is time invariant. The linear time invariant output is as [9].

$$y(n) = \sum_{j=0}^q b_j x(n-j) - \sum_{k=1}^p a_k y(n-k) \quad (1)$$

The equation(1) is differential equation consisting of signal(x), output signal(y), and scalar(b_j, a_k) and $j = 0, 1, \dots, q$, $k = 1, 2, \dots, p$. Transfer function of the in equation (1) is as follows

$$y(n) + \sum_{k=1}^p a_k y(n-k) = \sum_{j=0}^q b_j x(n-j) \quad (2)$$

if $a_0 = 1$,

$$\sum_{k=1}^p a_k y(n-k) = \sum_{j=0}^q b_j x(n-j) \quad (3)$$

In equation (3) can be transformed into z-domain.

$$\sum_{k=1}^p a_k z^{-k} Y(z) = \sum_{j=0}^q b_j z^{-j} X(z) \tag{4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{j=0}^q b_j z^{-j}}{\sum_{k=0}^p a_k z^{-k}} \tag{5}$$

2.2 All Pole Linear Prediction Signal Model

In equation (1), the linear prediction normal equation decided to the parameter of all pole linear system from linear system equation. We assume that the output signal is wide sense stationary random process, and estimate the output from latest past sample linear combination. The linear prediction expected value is as [10-11].

$$\hat{y}(n) = - \sum_{k=1}^p a_k y(n - k) \tag{6}$$

In equation (6) is sample number of output by p^{th} order. The estimation error between output signal and N sample prediction output is as follows

$$e(n) = y(n) - \hat{y}(n) \tag{7}$$

In equation (7) can be rewritten as follows

$$e(n) = y(n) + \sum_{k=1}^p a_k y(n - k) \tag{8}$$

The total mean square error of in equation (8) is as follows

$$E = \sum_n [e(n)]^2 \tag{9}$$

$$= \sum_n [y(n)]^2 - 2y(n)\hat{y}(n) + [\hat{y}(n)]^2 \tag{10}$$

In equation (9), we know that the optimal predictor is to minimize the mean square error. The minimize of the optimal can decide the differential equation so that prediction mean square error is minimized. By applying the differential equation to in equation (9).

$$\frac{\partial E}{\partial a_k} = 0 \tag{11}$$

$$= \frac{\partial}{\partial a_k} \left(\sum_n ([y(n)]^2 - 2y(n)\hat{y}(n) + [\hat{y}(n)]^2) \right) = 0 \tag{12}$$

$$\frac{\partial}{\partial a_k} \hat{y}(n) = -y(n - k) \tag{13}$$

The correlation matrix is as follows

$$\phi(i, k) = \sum_n y(n-i) y(n-k) \quad (14)$$

Substituting in equation (14) into equation (12)

$$-\phi(0, k) = \sum_{i=1}^P a_i \phi(i, k) \quad (15)$$

3. Optimal weight vector

The most direction of arrival methods depends on the array correlation matrix. They are received by an array of N-elements. Each received signal includes additive zero mean Gaussian noise. The array output y can be given in the following from [12-13].

$$y(n) = w^T x(n) \quad (16)$$

where

$$x(n) = [a(\theta_1) \quad a(\theta_2) \quad a(\theta_D) \quad \begin{bmatrix} s_1(n) \\ s_2(n) \\ \vdots \\ s_D(n) \end{bmatrix}] + N(n) \quad (17)$$

$$= A s(n) + N(n) \quad (18)$$

Where, $s(n)$ is source signal correlation matrix, $N(n)$ is noise signal. $a(\theta)$ is an element array steering vector for the direction of arrival, $A(M \times D)$ is steering vector matrix. It is understood that the arriving signals are time varying and the calculations are based upon time snapshots of the incident signal. Thus, each of the complex signals arrives at angles and is intercepted by the M antenna elements. It is initially assumed that the arriving signals are monochromatic and the number of arriving signals ($D < M$). It is understood that the arriving signals are time varying and thus our calculations are based upon time snapshots of the incoming signal. Obviously if the transmitters are moving, the matrix of steering vectors is changing with time and the corresponding arrival angles are changing. Unless otherwise stated, the time dependence will be suppressed in equation (16) and equation (17). In order to simplify the notation let us define the $M \times M$ array correlation matrix R_{xx} as

$$R_{xx} = E[x x^H] = E[(As + N)(s^H A^H + N^H)] \quad (19)$$

$$= AE[s s^H]A^H + E[N N^H] \quad (20)$$

$$= A R_{ss} A^H + R_{NN} \quad (21)$$

Where, R_{ss} is a source correlation matrix and R_{NN} is a noise correlation matrix. $E[\]$ denotes the expectation value. The array correlation matrix and the source correlation matrix are found by the expected value of the respective absolute values squared. If we do not know the exact statistics for the noise and signals, but we can assume that the process is ergodic, then we can approximate the correlation by use of a time averaged correlation. In that the correlation matrices are defined by

$$R_{xx} = \frac{1}{k} \sum_{k=1}^M x(k)x^H(k) \quad , \quad R_{ss} = \frac{1}{k} \sum_{k=1}^M s(k)s^H(k) \quad , \quad R_{NN} = \frac{1}{k} \sum_{k=1}^M n(k)n^H(k) \quad (22)$$

When the signals are uncorrelated, source correlation matrix obviously has to be a diagonal matrix because

off diagonal elements have no correlation. When the signals are partly correlated, source correlation matrix is nonsingular. When the signals are coherent, source correlation matrix becomes singular because the rows are linear combinations of each other. $X = [x_1(k), x_2(k), \dots, x_M(k)]$ is the signal vector on array antenna $s(t)$ is signal steering, $n(t)$ is noise signal. In equation (16) can be rewritten as follow

$$X(k) = \sum_{k=1}^M A e^{j2\pi f(t-(k-1)\tau)+\varphi} + N(k) \tag{23}$$

Where φ is the phase of the k -th source, A is the amplitude of the k -th source, f is the frequency of the k -th source, τ is the time delay between adjacent hydrophones corresponding to the k -th sources signal. The output of a narrowband beamformer is given by

$$Y(k) = W^H X(k) \tag{24}$$

Here $W = [w_1, w_2, \dots, w_M]^T$ is the complex vector of beamformer weights, and $()^T$ and $()^H$ are the transpose and hermitian transpose, respectively. The signal to noise ratio has the following

$$SNR = \frac{W^H R_{SS} W}{W^H R_{NN} W} \tag{25}$$

Where $R_{SS} = E[s(t)s^H(t)]$, $R_{NN} = E[N(t)N^H(t)]$, R_s is signal covariance matrices and R_{NN} is noise covariance matrices. $E[\]$ is expected value. To finding the maximum of in equation (27) is equivalent to the following optimization problem as follow

$$\min_W W^H R_{NN} W \quad \text{subject to} \quad W^H a = 1 \tag{26}$$

LCMV can be written as follow

$$W_{LCMV} = \frac{R_{NN}^{-1} a}{a^H R_{NN}^{-1} a} \tag{27}$$

The sample covariance matrix can be written

$$R = \frac{1}{k} \sum_{k=1}^M X(k)X^H(k) \tag{28}$$

4. Proposed DoA algorithm

LCMV beamformer provides a distortionless response in the direction of the desired signal, while suppressing noise and interference. However, if there is uncertainty in DOA of the desired signal, the performance of LCMV beamformer is known to degrade severely. For DOA uncertainty, additional linear constraints can be imposed to reduce sensitivity to pointing error. The constraints is on the beamformer output at the values of phase near presumed DOA. The weight vector of LCMV beamformer can be found from the following constrained minimization problem.

$$\min_W W^H R_{xx} W \quad \text{subject to} \quad Z^H W = F \tag{29}$$

Where R_{xx} is the training data covariance matrix. Z is $M \times J$ matrix of steering vectors for the constrained length

$$Z = [a(\theta_1), a(\theta_2), \dots, a(\theta_j)] \tag{30}$$

The constrained weight vector is given by

$$W_{LC} = R_{xx}^{-1} Z (Z^H R_{xx}^{-1} Z)^{-1} F \quad (31)$$

Here, F is the $J \times 1$ vector of constraints. We applied MUSIC algorithm to look direction. MUSIC algorithm is divided signal subspace (E_S) and noise subspace (E_N) using eigen decomposition and eigen value. MUSIC spectrum is as follow

$$P_{LC_MU} = \frac{a(\theta)^H W_{LC} W_{LC}^H a(\theta)}{a(\theta)^H W_{LC} E_N W_{LC}^H E_N^H a(\theta)} \quad (32)$$

5. Simulation

We assumed that there are uncorrelated signal sources all direction and equal power interferers all directional. We use uniform linear array antenna, and array element distance is half wavelength. Array elements are 10 number, and target position are 3 objects $[-10^\circ, 0^\circ, 10^\circ]$ and 4 numbers $[-10^\circ, 0^\circ, 5^\circ, 10^\circ]$. We want to estimate of 3 signals $[-10^\circ, 0^\circ, 10^\circ]$ by MUSIC algorithm using optimal weight vector in figure1, but figure 3 is only estimated the 2 object signals which is $[-10^\circ, 0^\circ, X^\circ]$. Notation $\langle X^\circ \rangle$ don't estimate for the desired object. Figure 2 is shown to estimate the desired object by the proposed algorithm using optimal weight vector in object signals $[-10^\circ, 0^\circ, 10^\circ]$. Direction of arrival of the proposed algorithm estimated correctly for the desired object signals $[-10^\circ, 0^\circ, 10^\circ]$ among the impinging signals on the receiver. In figure3, we show the response of the classical MUSIC for the estimation desired signals in coherent channels. We want to estimate of 4 signals in figure3, but the figure is only estimated the 3 signals which is $[-10^\circ, 0^\circ, X^\circ, 10^\circ]$. Notation X don't estimate for the desired signal. Figure 4 shows by the proposed algorithm. Direction of arrival of the proposed algorithm estimated correctly for all the targets of the desired signals among the impinging signals on the receiver.

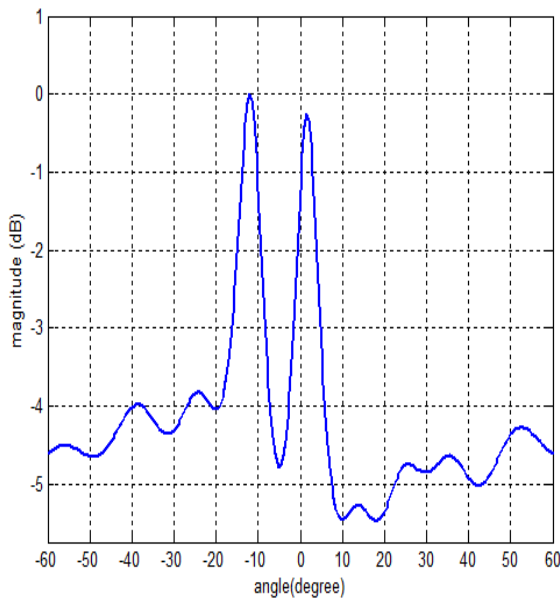


Figure 1. DoA of the MUSIC algorithm

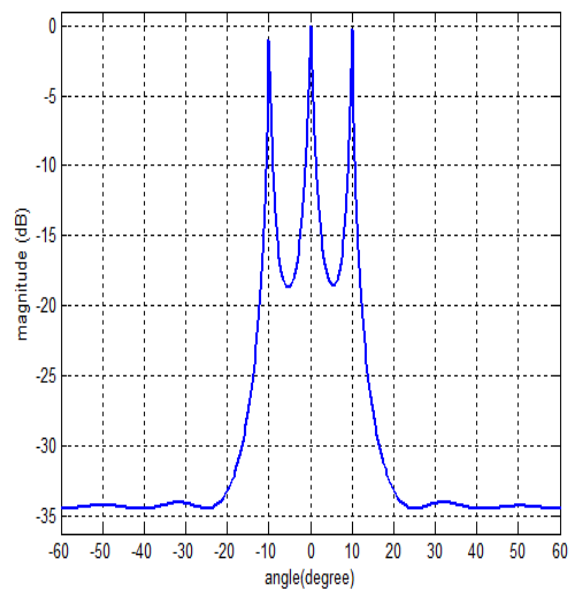


Figure 2. DoA of the Proposed algorithm

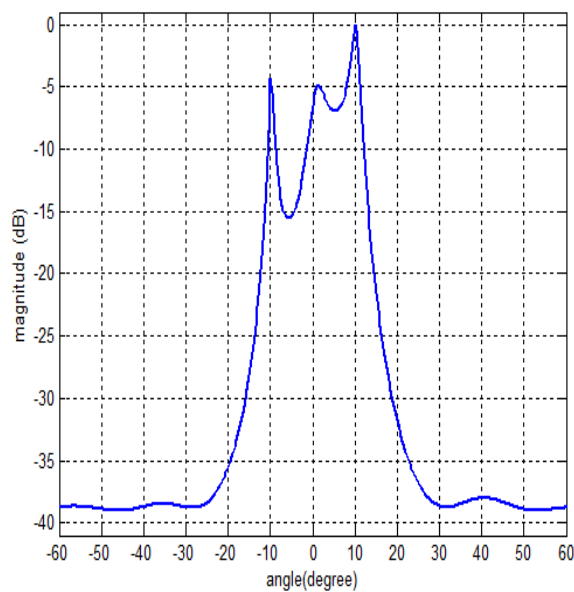


Figure 3. DoA of the MUSIC algorithm

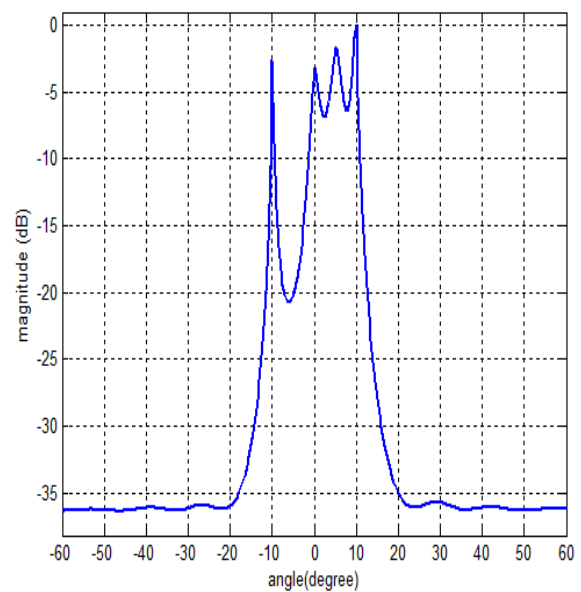


Figure 4. DoA of the Proposed algorithm

6. Conclusion

This paper proposed direction of arrival algorithm in order to correctly the estimation desired object. In this paper, we proposed to combine LCMV algorithm with MUSIC algorithm in order to the desired signal estimation in the noise and the interference channel. The proposed algorithm of this paper removes the interference and the noise signal in coherent channel. This paper proposed the algorithm which is removal the interference and the noise signal in coherent channel. The proposed algorithm is acquisition updated weigh matrix before removal noise and interference signal to get the desired signal. It is necessary to detect the number of sources before the direction of arrival estimation. LCMV- MUSIC algorithm in this paper described here yields considerably superior performance as compared with the classical MUSIC algorithm. As a result of computer simulation, the proposed algorithm is showed good performance better than general MUSIC algorithm in order to estimation desired signal in coherent channels.

References

- [1] F. Colone, D. Cristallini, D.Cerutti Maori, and P. Lombardo, "Direction of arrival estimation performance comparison of dual cancelled channels space time adaptive processing techniques," *IET journals &Magazines*, vol. 8, no. 1, pp. 17-26, Jan 2014. DOI: [10.1049/iet-rsn.2012.0368](https://doi.org/10.1049/iet-rsn.2012.0368).
- [2] N.Le Bihan, S.Miron, and J.I.Mars, "MUSIC Algorithm for vector sensors array using biquaternions," *IEEE Trans. On Signal processing*, vol. 55, no. 9, pp. 4523-4533, Aug 2007. DOI: [10.1109/TSP.2007.896067](https://doi.org/10.1109/TSP.2007.896067).
- [3] B.Porat and Benjamin Friedlander, "Analysis of the asymptotic relative efficiency of the MUSIC algorithm," *IEEE Trans on Acoustics, speech, and signal processing* vol. 34, no. 4, pp.532-544, Apr 1988. DOI: [10.1109/29.1557](https://doi.org/10.1109/29.1557).
- [4] Wen.Fei, Wan.Qun, Fan Rong, and Wei Hewen, "Improved MUSIC Algorithm for Multiple noncoherent Subarrays," *IEEE Trans on signal processing Letters*, vol. 21, no. 5, pp.527-530, Feb 2014. DOI: [10.1109/LSP.2014.2308271](https://doi.org/10.1109/LSP.2014.2308271).
- [5] Wang.Xiang, Huang.Zhital, and Zhou.Yiyu, "Underdetermined DOA estimation and blind separation of non disjoint sources in time frequency domain based on sparse representation method," *IEEE Trans. on system engineering and electronics journal*, vol. 25, no.1, pp.247-258, Mar 2014. DOI: [10.1109/JSEE.2014.00003](https://doi.org/10.1109/JSEE.2014.00003).
- [6] G.Bienveny and L.kopp, "Source power estimation method associated with high resolution bearing estimator, Acoustics, speech, and signal processing," in *Proc. 1981 IEEE international conference on Acoustics, Speech, and*

Signal Processing(ICASSP), Atlanta, Georgia, pp.153-159, March.31 - April 1, 1981.

DOI: [10.1109/ICASSP.1981.1171301](https://doi.org/10.1109/ICASSP.1981.1171301).

- [7] J.Fang, J.Shen, H.Li, and S.Li, "Super-resolution compressed sensing:An iterative reweighted algorithm for joint parameter learning and sparse signal recovery," *IEEE Trans. on signal processing Letters*, , vol. 21, no.6, pp.761-765, Apr 2014. DOI: [10.1109/LSP.2014.2316004](https://doi.org/10.1109/LSP.2014.2316004).
- [8] Edmond Nicolau. and Dragos Azharia. "Adaptive Arrays", Elsevier, 1983.
- [9] Alesander Bertrand and Marc Moonen, "Distributed LCMV beamforming in a wireless sensor network with signal channel per-node signal Transmission," *IEEE Trans Signal Precessing*, vol. 61, no.13, pp.3447-3459, July 2013. DOI: [10.1109/TSP.2013.2259486](https://doi.org/10.1109/TSP.2013.2259486).
- [10] Chen.H.Haihua, Chan, Sling Chow, G.Zhang Zhiguo, and Ho.Ka Leung, "Adaptive beamforming and recursive DOA estimation using frequency invariant uniform concentric spherical arrays," *IEEE Trans. on circuits and systems*, vol. 55, no. 10 , pp.3077-3089, Nov.2008. DOI: [10.1109/TCSI.2008.924126](https://doi.org/10.1109/TCSI.2008.924126)
- [11] B. Allen and M.Ghavami. "Adaptive Array Systems," John Willey&Sons, 2005.
- [12] A.J.Fenn, "Evaluation of adaptive phased array antenna far field nulling performance in the near field region," *IEEE Trans on Antennas and Propagation*, vol. 38, no. 2 , pp.173-185, Feb.1990. DOI: [10.1109/LSP.2013.2290948](https://doi.org/10.1109/LSP.2013.2290948)
- [13] Ruan and R.C. de Lamare, "Robust adaptive beamforming using a low complexity shrinkage based mismatch estimation algorithm," *IEEE Trans on signal processing Letters*, vol. 21, no. 1, pp.60-64, Jan 2014. DOI: [10.1109/ LSP .2013.2290948](https://doi.org/10.1109/LSP.2013.2290948).