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Distributor's Lot-sizing and Pricing Policy with Ordering Cost inclusive of a Freight Cost under Trade Credit in a Two-stage Supply Chain

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Abstract

As an effective means of price discrimination, some suppliers offer trade credit to the distributors in order to stimulate the demand for the product they produce. The availability of the delay in payments from the supplier enables discount of the distributor's selling price from a wider range of the price option in anticipation of increased customer's demand. Since the distributor's lot-size is affected by the demand for the customer, the distributor's lot-size and the selling price determination problem is interdependent and must be solved at the same time. Also, in many common business transactions, the distributor pays the shipping cost for the order and hence, the distributor's ordering cost consists of a fixed ordering cost and the shipping cost that depend on the order quantity. In this regard, we deal with the joint lot-size and price determination problem when the supplier allows delay in payments for an order of a product. The positive effects of credit transactions can be integrated into the EOQ (economic order quantity) model through the consideration of retailing situations, where the customer's demand is a function of the distributor's selling price. It is also assumed that the distributor's order cost consists of a fixed ordering cost and the variable shipping cost. We formulate the distributor's mathematical model from which the solution algorithm is derived based on properties of an optimal solution. A numerical example is presented to illustrate the algorithm developed.

Keywords: *Credit period, Lot-Size, Pricing, Price elasticity demand function, Supply Chain*

1. INTRODUCTION

While deriving the economic order quantity (EOQ) expression, it is implicitly assumed that the distributors pay for the products as soon as they receive them from a supplier. However, in practice, a supplier will allow a certain fixed period (credit period) for settling the amount the buyer owes to him for the products supplied.

In this regard, many research works appeared that deal with the problem of determining the EOQ under a fixed period. Goyal [1] and Teng et al. [2] studied the mathematical model for obtaining an EOQ when the supplier permits a fixed credit period in payments of the product. However, the availability of opportunity to delay in payment suggested by the supplier becomes the effective means reducing the inventory holding costs for distributor. It is known that one of the major reasons for the supplier to offer a trade credit to the distributors is to stimulate demand for the product he produces. The benefit of trade credit from the supplier is that it

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positively influences the purchasing and marketing decisions for the distributor. The supplier usually expects to be compensated for the capital losses incurred during the delay in payments through increased sales. The positive effect of credit transactions can be integrated into the EOQ model through the retailing situations where the customer's demand is a function of the selling price set by the distributor. The availability of the credit period from the supplier enables discount of the distributor's selling price from a wider range of the price option expect to increase the customer's demand. Since the lot size for the distributor is affected by the customer's demand rate of product, the distributor's lot-size and the selling price determination problem is interdependent and must be solved at the same time (we will call this the DLSP problem).

Based upon the above observations, some research works dealt with the DLSP problem under trade credit assuming that the demand rate for the customer is a decreasing function of distributor's selling price. Chang et al. [3], Dye and Ouyang [4], and Ouyang et al. [5] dealt with the DLSP problem when the demand is a constant price elasticity function of distributor's selling price under trade credit. Avinadav et al. [6] and Shi et al. [7] dealt with the same problem without considering trade credit assuming that the demand rate is represented by a linearly decreasing function of selling price.

All the above mentioned research papers tacitly assumed that the ordering cost consists of a fixed order cost without taking into account the shipping cost for the transportation of order quantity. However, in many practical transactions, the distributor pays the shipping cost for the order quantity. Aucamp[8], Lee[9] and Lippman[10, 11] evaluated the inventory model with the ordering cost consisting of a fixed cost and shipping cost charged by the size of the order quantity.

However, in common business transactions, the purchased quantity may be transported in unit loads, i.e., containers, pallets, boxes, and others. Therefore, there is a base rate for the first unit load and there is an incremental rates for each more freight unit loads considered as special model by Lee[9]. In this regard, Shinn[12] introduced the EOQ model with ordering cost inclusive of a shipping cost under the condition of permissible delay in payments. He also assumed that the distributor's ordering cost consists of a fixed order cost and a shipping cost to be charged depending on each additional unit load required. In this paper, we extend the model evaluated by Shinn[12] to the case of the DLSP problem when the supplier permits a certain credit period for the distributor 's order quantity. It is also assumed that the distributor's ordering cost consists of a fixed ordering cost and a shipping cost to be charged depending on each additional unit load required. In Section 2, we formulate the mathematical model to determine the distributor's optimal lot sizing and the pricing policy. Based on the characteristics of an optimal solution, a solution algorithm is given in Section 3. A numerical example is presented in Section 4, which is followed by conclusions.

2. MATHEMATICAL MODEL DEVELOPMENT

2.1 Assumptions and Notations

In this study, the assumptions and notations are essentially the same to the model mentioned by Shinn[13] except for the condition of the customer's demand which is represented by a function of distributor's selling price.

Following assumptions and notations are used.

(1) The customer's demand is represented by a constant price elasticity function of selling price set by the distributor.

(2) Shortages are not allowed.

(3) The supplier proposes a fixed credit period and sales revenue generated during the credit period is deposited in an interest with rate I. At the end of the credit period, the price of the product is settled and the distributor begins paying the capital opportunity cost for the products in inventory with rate R ($R \geq I$).

(4) The distributor pays the shipping cost for the transport of the order quantity.

D = the customer's annual demand, as a function of distributor's selling price (P); $D = KP^{-\beta}$.
K = scaling factor (> 0)
β = index of price elasticity (> 0)
P = the distributor's unit selling price(< P_u)
C = unit purchase cost.
T = replenishment cycle time.
Q = order size.
H = inventory holding cost, excluding the capital opportunity cost.
I = earned interest rate (as a percentage).
R = capital opportunity cost (as a percentage).
tc = credit period set by the supplier.
$S(Q)$ = ordering cost for Q, $(j-1)U < Q \le jU$, $j = 1, 2, \cdots, n$; $A + F_j$.
A = fixed order cost.
F_j = shipping cost for Q, $(j-1)U < Q \le jU, j = 1, 2, \cdots, n$; $S_0 + (j-1)S$, $S_0 \ge S$.

Namely, as stated by Lee[9], there is a base rate S_0 for the first U shipping unit and there is an incremental rates S for each U more shipping units. In the case of $S_0 = S$, it can be interpreted U as the capacity of each cargo and S as the unit cargo cost. Therefore, in this model, the shipping cost of each order equals jS when the order size satisfies $(j-1)U < Q \le jU$. Also, note that the inequality $S_0 > S$ implies that there is some quantity discount in the shipping cost for changing the order size from $(j - 1)U$ to jU. This special case applies in many real situations, for instance, in postal service charges.

In this model, the distributor's objective is to maximize the annual net profit, $\pi(Q,P)$ and $\pi(Q,P)$ consists of the following five elements;

1) Annual sales revenue = DP ;
2) Annual purchasing cost = DC ;
3) Annual inventory holding cost = $\frac{QH}{2}$;
4) Annual ordering cost = $\frac{D(A+S_0+(j-1)S)}{Q}$ for $(j-1)U < Q \le jU$;
5) Annual capital opportunity cost(refer to Shinn[12])
   (1) Case 1: $Dtc \le Q$
   $= \frac{C(R-I)D^2 tc^2}{2Q} + \frac{CRQ}{2} - CRDtc$;
   (2) Case 2: $Dtc > Q$)
   $= \frac{CIQ}{2} - CIDtc$.

Then, the distributor's annual net profit $\pi(Q,P)$ can be formulated as

$\pi(Q,P)$ = Sales revenue – Purchasing cost – Ordering cost – Inventory holding cost – Capital opportunity cost.

Depending on the relative size Dtc to Q, $\pi(Q,P)$ has two different expressions, as follows;

Case 1: $Dtc \le Q$

$$\pi_{1,j}(Q,P) = DP - DC - \frac{D(A+S_0+(j-1)S)}{Q} - \frac{QH}{2} - \left(\frac{C(R-I)D^2 tc^2}{2Q} + \frac{CRQ}{2} - CRDtc\right),\ (j-1)U < Q \le jU, \quad (1)$$
$j = 1, 2, \cdots, n$.

Case 2: $Dtc > Q$

$$\pi_{2,j}(Q,P) = DP - DC - \frac{D(A+S_0+(j-1)S)}{Q} - \frac{QH}{2} - \left(\frac{CIQ}{2} - CIDtc\right),\ (j-1)U < Q \le jU,\ j = 1, 2, \cdots, n. \quad (2)$$

3. DETERMINATION OF OPTIMAL POLICY

The problem is to find an optimal lot-size Q^* and an optimal selling price P^*, which maximizes $\pi(Q,P)$. For a fixed P, $\pi(Q,P)$ is a concave function of Q for every i and j, and there exists a unique value $Q_{i,j}$, which maximizes $\pi_{i,j}(Q,P)$, $i=1,2, j=1,2,\cdots,n$ as follows;

$$Q_{1,j}=\sqrt{\frac{2(A_1+S_0+(j-1)S)D}{H_1}} \text{ where } A_1=A+\frac{C(R-I)Dtc^2}{2}, \ D=KP^{-e} \text{ and } H_1=H+CR \quad (3)$$

$$Q_{2,j}=\sqrt{\frac{2(A+S_0+(j-1)S)D}{H_2}} \text{ where } D=KP^{-e} \text{ and } H_2=H+CI. \quad (4)$$

Note that for $P=P^0$ fixed, $\pi(Q,P^0)$ is the same model as stated by Shinn[12]. So, we are able to adopt the following two theorems of Shinn[12] in finding the optimal lot-size Q^* for a fixed *P*.

Theorem 1(for Case 1). Suppose $(m-1)U<Dtc \le mU$ for some *m*. Let *a* be the index such that $(a-1)U<Q_{1,0}\le aU$ where $Q_{1,0}=\sqrt{2(A_1+S_0-S)D/H_1}$.

If $a>m$ and $Q_{1,a}\le aU$, then $Q_0=(a-1)U, Q_{1,a}$,

If $a>m$ and $Q_{1,a}>aU$, then $Q_0=(a-1)U, aU$,

If $a\le m$ and $Q_{1,m}\le Dtc$, then Q^* must less than *Dtc*,

If $a\le m$ and $Q_{1,m}\le mU$, then $Q_0=Q_{1,m}$,

If $a\le m$ and $Q_{1,m}>mU$, then $Q_0=mU$.

Theorem 2(for Case 2). Suppose $(m-1)U<Dtc \le mU$ for some *m*. Let *b* be the index such that $(b-1)U<Q_{2,0}\le bU$ where $Q_{2,0}=\sqrt{2(A+S_0-S)D/H_2}$.

If $b<m$ and $Q_{2,a}\le bU$, then $Q_0=(b-1)U, Q_{2,b}$,

If $b<m$ and $Q_{2,a}>bU$, then $Q_0=(b-1)U, bU$,

If $b=m$ and $Q_{2,b}\le Dtc$, then $Q_0=(b-1)U, Q_{2,b}$,

If $b=m$ and $Q_{2,b}\le Dtc$, then $Q_0=(b-1)U$,

If $b>m$, then $Q_0=(m-1)U$.

Theorem 1 and 2 state that for $P=P^0$ fixed, only the elements in set $\Omega=\{jU, Q_{1,j}(P^0), Q_{2,j}(P^0), j=1,2,\cdots,n)$ become candidates for an optimal lot size $Q^*(P^0)$ where $Q_{i,j}(P^0)$ is obtained by substituting *P* with P^0 in equations (3) and (4). Noting that some elements in Ω do not need to be considered in search of $Q^*(P)$, we formulate the following conditions $Q_{i,j}(P)$ and *jU* must satisfy to become a candidate of $Q^*(P)$.

(Cond. 1) The conditions for $Q_{i,j}(P)$ to be a candidate of $Q^*(P)$ are:

$$Q_{1,j}(P)\ge Dtc \text{ and } (j-1)U<Q_{1,j}(P)\le jU \text{ for Case 1,} \quad (5)$$

$$Q_{2,j}(P)<Dtc \text{ and } (j-1)U<Q_{2,j}(P)\le jU \text{ for Case 2.} \quad (6)$$

(Cond. 2) The conditions for jU to be a candidate of $Q^*(P)$ are:

$jU \geq Dt_c$ and $jU < Q_{1,j}(P)$ for Case 1, (7)

$jU < Dt_c$ and $jU < Q_{2,j}(P)$ for Case 2. (8)

For $Q_{1,j}(P)$ to be a candidate of $Q^*(P)$ in Case 1, $Q_{1,j}(P)$ must lie on $((j\text{-}1)U, jU]$ and also $Q_{1,j}(P) \geq Dt_c$ must hold. For jU to be a candidate of $Q^*(P)$ in Case 1, $\pi_{1,j}(P,Q)$ must be increasing at $Q = jU$. Namely, the conditions $jU < Q_{1,j}(P)$ and $jU \geq Dt_c$ must be satisfied. The conditions for Case 2, equations (6) and (8), are justified in a similar way.

Now, let us consider $Q_{1,j}(P) \geq Dt_c$ in equation (5). Since the customer's demand D is also a function of P, the inequality can be written as

$$Q_{1,j}(P) = \sqrt{\frac{2D(A+S_0+(j-1)S)+D^2C(R-I)t_c^{\,2}}{H+CR}} \geq Dt_c = KP^{-\beta}. \quad (9)$$

Rearranging equation (9),

$$P \geq \left(K(H+CI)t_c^{\,2}/2(A+S_0+(j-1)S)\right)^{1/\beta}. \quad (10)$$

Let

$$P1_j = \left(K(H+CI)t_c^{\,2}/2(A+S_0+(j-1)S)\right)^{1/\beta}. \quad (11)$$

It is self-evident that for any $P \geq P1_j$, the inequality $Q_{1,j}(P) \geq Dt_c$ holds. Similarly, $(j-1)U < Q_{1,j}(P)$ in equation (5) can be rewritten as

$$P < P2_j, \text{ where } P2_j = \left(\frac{KC(R-I)t_c^{\,2}}{\sqrt{(A+S_0+(j-1)S)^2+C(R-I)H_1 t_c^{\,2}((j-1)U)^2}-(A+S_0+(j-1)S)}\right)^{1/\beta}. \quad (12)$$

Also, from $Q_{1,j}(P) \leq jU$ in equation (5), we have

$$P \geq P3_j, \text{ where } P3_j = \left(\frac{KC(R-I)t_c^{\,2}}{\sqrt{(A+S_0+(j-1)S)^2+C(R-I)H_1 t_c^{\,2}(jU)^2}-(A+S_0+(j-1)S)}\right)^{1/\beta}. \quad (13)$$

With a similar procedure, other price ranges are obtained from inequalities in equations (6), (7) and (8). They are:

$$P < P1_j, \text{ where } P1_j = \left(K(H+CI)t_c^{\,2}/2(A+S_0+(j-1)S)\right)^{1/\beta} \text{ from } Q_{2,j}(P) < Dt_c, \quad (14)$$

$$P \geq P4_j, \text{ where } P4_j = \left(\frac{2K(A+S_0+(j-1)S)}{H_2(jU)^2}\right)^{1/\beta} \text{ from } Q_{2,j}(P) \leq jU, \quad (15)$$

$$P < P5_j, \text{ where } P5_j = \left(\frac{2K(A+S_0+(j-1)S)}{H_2((j-1)U)^2}\right)^{1/\beta} \text{ from } Q_{2,j}(P) > (j-1)U, \quad (16)$$

$$P \geq P6_j, \text{ where } P6_j = \left(\frac{Kt_c}{jU}\right)^{1/\beta} \text{ from } Dt_c \leq jU. \quad (17)$$

We consider that $Q_{1,j}(P)$ determined with P value which satisfies all the three inequalities (10), (12) and (13) can be a candidate for $Q^*(P)$. In other words, $Q_{1,j}(P)$ can be a candidate for $Q^*(P)$ only if P is in the price interval, $PIQ_j = \{P | P3_j \leq P < P2_j \text{ and } P \geq P1_j\}$. Utilizing the price ranges in equations (10) to (17), we can find the following price intervals which correspond to (**Cond. 1**) and (**Cond. 2**).

(PR-1) Price interval on which $Q_{i,j}(P)$, i =1, 2 and j = 1, 2 , ⋯, n becomes a candidate for $Q^*(P)$:

$$PQ_j = \{P | P3_j \le P < P2_j \text{ and } P \ge P1_j\} \text{ for Case 1,} \tag{18}$$

$$PQ_j = \{P | P4_j \le P < P5_j \text{ and } P < P1_j\} \text{ for Case 2.} \tag{19}$$

(PR-2) Price interval on which U_j, j = 1, 2 , ⋯, n becomes a candidate for $Q^*(P)$:

$$PN_j = \{P | P < P3_j \text{ and } P \ge P6_j\} \text{ for Case 1,} \tag{20}$$

$$PN_j = \{P | P < P4_j \text{ and } P < P6_j\} \text{ for Case 2.} \tag{21}$$

The price intervals we present have a significant role in solving the model. For example, we consider (**PR-1**). If $P \in PQ_j$, $Q_{i,j}(P)$ satisfies (**Cond. 1**) and becomes a candidate of $Q^*(P)$. Substituting Q with $Q_{i,j}(P)$ in $\pi_{i,j}(Q,P)$, we have a problem of maximizing $\pi_{i,j}(Q_{i,j}(P),P)$ which is a single variable function of P. Let $\pi_{i,j}{}^0(P) = \pi_{i,j}(Q_{i,j}(P),P)$, $i = 1,2, j = 1, 2, \cdots n$. Note that $\pi_{i,j}{}^0(P)$ is valid only on the interval $P \in PQ_j$. Likewise, if $P \in PN_j$, then U_j satisfies (**Cond. 2**). Substituting Q with U_j in $\pi_{i,j}(Q,P)$, we have $\pi_{i,j}(U_j,P)$, $i = 1,2, j = 1, 2, \cdots n$, which is also a single variable function of P because U_j is a constant. Then, we can find the distributor's optimal solution (Q^*, P^*) which maximizes $\pi(Q,P)$ by searching over $\pi_{i,j}{}^0(P)$ and $\pi_{i,j}(U_j,P)$, and

$$\max_{Q,P} \pi(Q,P) = max\left[\max_{P \in PQ_j} \pi_{1,j}{}^0(P), \max_{P \in PN_j} \pi_{1,j}(U_j,P), \max_{P \in PQ_j} \pi_{2,j}{}^0(P), \max_{P \in PN_j} \pi_{2,j}(U_j,P)\right]. \tag{22}$$

Now, we are going to investigate the properties of $\pi_{i,j}{}^0(P)$ and $\pi_{i,j}(U_j,P)$. With $Q = Q_{i,j}(P)$ as a function of P, the following single variable functions are obtained:

$$\pi_{1,j}{}^0(P) = D\{P - C(1 - Rt_c)\} - \sqrt{2H_1 D(A_1 + S_0 + (j-1)S)}, \tag{23}$$

$$\pi_{2,j}{}^0(P) = D\{P - C(1 - It_c)\} - \sqrt{2H_2 D(A + S_0 + (j-1)S)}, \tag{24}$$

where $j = 1, 2, \cdots n$ and $D = KP^{-\beta}$. Utilizing *Mathematica* by Wolfram[13] to obtain derivatives of $\pi_{i,j}{}^0(P)$, we have

$$\pi_{1,j}{}^0(P)' = D + D'\{P - C(1 - Rt_c)\} - D'(2A_1 - A + S_0 + (j-1)S)\sqrt{H_1/2D(A_1 + S_0 + (j-1)S)} \tag{25}$$

$$\pi_{2,j}{}^0(P)' = D + D'\{P - C(1 - It_c)\} - D'\sqrt{H_2(A + S_0 + (j-1)S)/2D}. \tag{26}$$

Also, the second order condition for concavity is

$$\pi_{1,j}{}^0(P)'' = \beta D P^{-2}\{P(\beta - 1) - C(\beta + 1)(1 - Rt_c) - f_1(P)\} < 0, \tag{27}$$

$$\pi_{2,j}{}^0(P)'' = \beta D P^{-2}\{P(\beta - 1) - C(\beta + 1)(1 - It_c) - f_2(P)\} < 0, \tag{28}$$

where $f_1(P) = \frac{\sqrt{H_1}[(\beta+2)(A+S_0+(j-1)S)^2 + (\beta+1)DCt_c{}^2(R-I)\{3(A+S_0+(j-1)S)+DCt_c{}^2(R-I)\}]}{\sqrt{D\{2(A_1+S_0+(j-1)S\}^3}}$ and $f_2(P) = \sqrt{\frac{H_2(\beta+2)^2(A+S_0+(j-1)S)}{8D}}$.

For $\beta \le 1$, $\pi_{i,j}{}^0(P)' > 0$ and $\pi_{i,j}{}^0(P)$ is an increasing function of P. Thus an optimal value $P_{i,j}$ of $\pi_{i,j}{}^0(P)$ occurs at the maximum point of the price interval (PQ_j) corresponding to $Q_{i,j}(P)$. For $\beta > 1$, it can be shown that both $f_1(P)$ and $f_2(P)$ become positive and the second order condition is satisfied if $P < C(1 - Rt_c)(\beta + 1)/(\beta - 1)$. Note that, given the second order assumption, $P_{i,j}$ is the one which has the minimum absolute value of $\pi_{i,j}{}^0(P)'$ on the price interval (PQ_j).

Table 1. Results of Step 1

j	$Q = Q_{1,j}(P)$			$Q = N_j$		
	$P \in PQ_j$ and $P \leq P_u$	$P_{1,j}$	$Q_{1,j}(P_{1,j})$	$P \in PN_j$ and $P \leq P_u$	$P_{1,j}$	N_j
1	∅	-	-	∅	-	-
2	[5.79,6.69]	5.79	1,001	[5.62,5.79]	5.62	1,000
3	[5.12,5.96]	**5.12**[a]	**1,265**[a]	∅	-	-
4	∅	-	-	∅	-	-
5	∅	-	-	∅	-	-
6	∅	-	-	∅	-	-
7	∅	-	-	∅	-	-
8	∅	-	-	∅	-	-
9	∅	-	-	∅	-	-
10	∅	-	-	∅	-	-

[a] Optimal solution for Case 1(Annual net profit = $8,809).

Now, with $Q = N_j$, the following results are obtained for $\pi_{i,j}(Q, P)$:

$$\pi_{1,j}(N_j, P)' = \frac{D}{P}\left\{(1-\beta)P + \beta C(1 - Rt_c) + \frac{\beta(2A_1 - A + S_0 + (j-1)S)}{jN_j}\right\}, \tag{29}$$

$$\pi_{1,j}(N_j, P)'' = \frac{\beta D}{P^2}\left[(\beta-1)P - (\beta+1)\left\{C(1 - Rt_c) + \frac{A + S_0 + (j-1)S}{jN_j}\right\} - f_3(P)\right], \tag{30}$$

where $f_3(P) = (2\beta+1)Ct_c^{\,2}(R - I)D/N_j > 0$,

$$\pi_{2,j}(N_j, P)' = \frac{D}{P}\left\{(1-\beta)P + \beta C(1 - It_c) + \frac{\beta(A + S_0 + (j-1)S)}{jN_j}\right\}, \tag{31}$$

$$\pi_{2,j}(N_j, P)'' = \frac{\beta D}{P^2}\left[(\beta-1)P - (\beta+1)\left\{C(1 - It_c) + \frac{A + S_0 + (j-1)S}{jN_j}\right\}\right]. \tag{32}$$

For $\beta \leq 1$, $\pi_{i,j}(N_j, P)$ is increasing in P and $\beta > 1$, $f_3(P) > 0$. It can be shown that if $P < C(1 - Rt_c)(\beta+1)/(\beta-1)$holds, $\pi_{i,j}(N_j, P)$ is concave. Based on these characteristics of $\pi_{i,j}(N_j, P)$, an optimal value $P_{i,j}$ of $\pi_{i,j}(N_j, P)$ can be easily determined on the corresponding price interval PN_j, $j = 1, 2, \cdots, n$. Note that for problem with $e > 1$, the algorithm is valid only when $P < C(1 - Rt_c)(\beta+1)/(\beta-1)$.

Now, we present the solution algorithm to determine the distributor's optimal lot-size and selling price.

Solution Algorithm

Step 1. **(for Case 1)** This step identifies all candidate values Q_0 for Q satisfying $Q_0 \geq Dt_c$. For each Q_0 its optimal price $P_{1,j}$ is determined from the corresponding price interval.

1.1. Compute $P_{1,j}$ which maximizes $\pi_{1,j}{}^0(P)$ in the following price intervals: $P \in PQ_j$ and $P \leq P_u$ with $Q_0 = Q_{1,j}(P)$, $j = 1, 2, \cdots, n$, where P_u is a given upper limit of distributor's selling price.

1.2. Compute $P_{1,j}$ which maximizes $\pi_{1,j}(P, N_j)$ in the following price intervals: $P \in PN_j$ and $P \leq P_u$ with $Q_0 = N_j$, $j = 1, 2, \cdots, n$, where P_u is a given upper limit of distributor's selling price.

Step 2. **(for Case 2)** This step identifies all candidate values Q_0 for Q satisfying $Q_0 < Dt_c$. For each Q_0 its optimal price $P_{2,j}$ is determined from the corresponding price interval.

2.1 Compute $P_{2,j}$ which maximizes $\pi_{2,j}{}^0(P)$ in the following price intervals: $P \in PQ_j$ and $P \leq P_u$ with $Q_0 = Q_{1,j}(P)$, $j = 1, 2, \cdots, n$, where P_u is a given upper limit of distributor's selling price.

2.2 Compute $P_{1,j}$ which maximizes $\pi_{2,j}(P, N_j)$ in the following price intervals: $P \in PN_j$ and $P \leq P_u$ with $Q_0 = N_j$, $j = 1, 2, \cdots, n$, where P_u is a given upper limit of distributor's selling price.

Step 3. Select the optimal lot size (Q^*) and selling price (P^*) which gives the maximum annual net profit among those obtained in the previous steps.

Table 2. Results of Step 2

j	$Q = Q_{2,j}(P)$			$Q = N_j$		
	$P \in PQ_j$ and $P \leq P_u$	$P_{2,j}$	$Q_{1,j}(P_{2,j})$	$P \in PN_j$ and $P \leq P_u$	$P_{2,j}$	N_j
1	∅	-	-	[0,6.69]	5.05	500
2	∅	-	-	[0,5.62]	$\mathbf{4.96}^a$	$\mathbf{1,000}^a$
3	[4.47,5.12]	4.95	1320	[0,4.47]	4.47	1,500
4	[3.70,4.65]	4.65	1501	[0,3.70]	3.70	2,000
5	[3.21,3.83]	3.83	2002	[0,3.21]	3.21	2,500
6	[2.86,3.31]	3.31	2504	[0,2.86]	2.86	3,000
7	[2.61,2.95]	2.95	3005	[0,2.61]	2.61	3,500
8	[2.41,2.69]	2.69	3495	[0,2.41]	2.41	4,000
9	[2.26,2.48]	2.48	4000	[0,2.26]	2.26	4,500
10	[2.13,2.31]	2.31	4511	[0,2.13]	2.13	5,000

[a] Optimal solution for Case 2. This solution is also the global optimum with its annual net profit $8,843.

4. NUMERICAL EXAMPLE

The following problem is considered, to illustrate the solution algorithm.

Let $K = 2.5 \times 10^5$, $\beta = 2.5$, A = \$50, C = \$3, tc = 0.3, H = \$0.1, I = 0.1(= 10%), R = 0.15(= 15%), U = 500, S_0 = 10, S = 8, $j = 1,\ 2,\ \cdots, 10$, $P_u = 6.68$. A computer program written in C programming language was developed to solve the numerical example as follows;

Step 1. (for Case 1)
Step 1.1. Solving equation (25) numerically in the price interval corresponding to $Q_0 = Q_{1,j}(P)$, $j = 1,\ 2,\ \cdots, 10$, we obtaine $P_{1,j}$ and these result are presented Table 1.
Step 1.2. Solving equation (29) numerically in the price interval corresponding to $Q_0 = jU$, $j = 1,\ 2,\ \cdots, 10$, we obtaine $P_{1,j}$ and these result are presented Table 1.
Step 2. (for Case 2)
Step 2.1. Solving equation (26) numerically in the price interval corresponding to $Q_0 = Q_{2,j}(P)$, $j = 1,\ 2,\ \cdots, 10$, we obtaine $P_{2,j}$ and these result are presented Table 2.
Step 2.2. Solving equation (31) numerically in the price interval corresponding to $Q_0 = jU$, $j = 1,\ 2,\ \cdots, 10$, we obtaine $P_{2,j}$ and these result are presented Table 2.
Step 3. From the results in steps 1 and 2, an optimal solution (Q^*, P^*) becomes (1,000, 4.96) with its maximum annual net profit \$8,843.

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5. CONCLUSION

In this paper, we have extended the model evaluated by Shinn[12] to the case of the distributor's lot-sizing and pricing determination problem in an environment in which the customer's demand of the product is a constant price elasticity function of the distributor's selling price when the supplier provides a fixed credit period for settling the amount the distributor owes to him. It is also assumed that the distributor's ordering cost consists of a fixed order cost and a variable shipping cost to be charged depending on each additional unit load required. In many real situations, the purchased quantity may be transported in unit loads, i.e., containers, pallets, boxes, and others. And hence, there is a base rate for the first unit load and there is an incremental rate for each more freight unit loads. Therefore, the model presented in this paper seems more realistic.

For a distributor who benefits from the delay in payments set by supplier, it is common that he lowers his selling price to some extent expecting that he can make more profitable by stimulating the customer demand. Also, in many real situations, the purchased quantity may be transported in unit loads, i.e., containers, pallets, boxes, and others. And hence, there is a base rate for the first unit load and there is an incremental rate for each more freight unit loads. In view of this, we think that the model presented in this paper may be more realistic for some real world problems. For the system presented, a mathematical model was developed. After formulating the mathematical model, we found the properties of the annual net profit and developed the solution algorithm.

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