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The Nature of a Method Course for Prospective Secondary Mathematics Teachers[1]

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Through this study, we aimed to capture the nature of a mathematics method course, called "the Curriculum Development and Teaching Methods in Mathematics Education" which is a pedagogy course for teaching for secondary school mathematics taught at a university located in a south eastern part of South Korea. The research participants include three junior students who took the methods course and a local high school math teacher with two professors. The research has three parts. First, we designed a method course to prepare the junior or senior students for a teaching practicum. The individual students gave a mini lecture about a secondary mathematical topic as a course requirement. Second, the three students watched a classroom video-clip of the high school teacher and analyzed his instruction before the actual classroom visits. Furthermore, by "Let's Learn" program for students, the course was associated with a local community through the students and so that they could visit the teacher's classroom three times to observe his math classroom teaching. The students discussed the difference between their own mini lectures and the actual math classroom teaching to develop an understanding of what it entails to teach an actual math class. Third, the first author supervised the students' activities in the program including their report for it to bring out their findings to the class of the method course. We found out this method course provided the students with the experience of various aspects of actual math lesson as well as learning theories about the pedagogy for teaching for secondary school mathematics. We conclude that this course gives a model for the method course in mathematics education for secondary school mathematics.

Keywords: teaching practicum, components of lesson, pedagogical content knowledge
MESC Classification: B50, C70, D40
MSC2010 Classification: 97C70, 97D40, 97D80, 97B50

I. INTRODUCTION

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Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) results show high mathematical achievement in countries, such as Singapore, South Korea, China, Hong Kong, and Japan. Alerted by U.S. students' mediocre mathematical achievement in international comparisons examinations, U.S. researchers such as Stigler and Hiebert (1999) conducted a video analysis of mathematics classroom instructions in three different countries (i.e., Germany, Japan, and the U.S.). Studies reported that while U.S. teachers limited themselves to telling procedures, Japanese teachers developed concepts; While U.S. students spent most of their class time practicing and rote memorization, Japanese students were involved in non-practice activities to make conjectures and to reason (Firestone, Mayrowetz, & Fairman, 1998; Firestone, Winter, & Fitz, 2000).

As Black, Harrison, Lee, Marshall and Wiliam (2004) point out that what is significant is what is happening "inside the black box" (i.e., inside the classroom), as opposed to the input such as the provision of building resources (Cohen, Raudenbush, & Ball, 2003) or the output such as student achievement scores, scholars started to pay close attention to what is happening inside the classroom, during a classroom instruction. Formative assessment research (Black & Wiliam, 1998, 2009; Pryor & Crossouard, 2008; Torrance & Pryor, 1998) can be interpreted as one of such endeavors. Researchers also paid attention to classroom discourse practices in mathematics education (e.g., Cazden, 2001; Nathan & Knuth, 2003; Walshaw & Anthony, 2008; Wood, Williams, & McNeal, 2006), and closely examined the relationship between interaction patterns and the levels of students' mathematical thinking. Another line of research relates to Japan's lesson study approach.

When teachers have a robust understanding in mathematics and teaching, it will influence increased students' mathematical achievement. It seems more likely to have impact on changing pre-service teachers' mathematical knowledge for teaching than in changing in-service teachers' mathematical knowledge for teaching. In the current study, the researchers aim to develop an instructional model for teaching Korean undergraduate mathematics education course, "the Curriculum Development and Teaching Methods in Mathematics Education." The course teaches important theories and practices of mathematics education. Moreover, the course has junior students conduct their own demonstrative lessons for 15-20 minutes, so that students could prepare for the teaching practicum in the following year.

It is not common for the department of mathematics education to have an association with the local secondary schools for improving mathematics education courses. In this study, we are privileged to work closely with a local high school and introduce three selected undergraduate students to an onsite expert high school mathematics teacher. The local high school mathematics teacher provided comments and feedback for the three students' demonstrative lessons and kindly invited them to his actual secondary mathematics classroom. We propose that this kind of interaction be encouraged among the students who are pre-service teachers and the onsite teachers, so that students can have a better grasp of what an actual mathematics teaching would look like.

II. THEORETICAL BACKGROUND

1. CCSSI'S MATHEMATICAL PRACTICE (MP)

The standards for Mathematical Practice (MP) in the Common Core State Standards Initiative (CCSSI, 2010) describe eight Mathematical Practice that "mathematics educators should seek to develop in their students" (p. 6). The Mathematical Practice is derived based on (a) the National Council of Teachers of Mathematics (NCTM)'s process standards (NCTM, 2000) and (b) the mathematical proficiency specified in the National Research Council (NRC)'s report, *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001). The NCTM process standards include mathematics processes, such as problem solving, reasoning and proof, communication, representation, and connections. The NRC's report *Adding It Up* includes proficiencies, such as adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition. Based on these two standards documents, the CCSSI-M's Mathematical Practice includes the following mathematical practices:

(MP1) make sense of problems and persevere in solving them;
(MP2) reason abstractly and quantitatively;
(MP3) construct viable arguments and critique of the reasoning of others;
(MP4) model with mathematics;
(MP5) use appropriate tools strategically;
(MP6) attend to precision;
(MP7) look for and make use of structure;
(MP8) look for and express regularity in repeated reasoning. (CCSSI, 2010, pp. 6-8)

Focusing on mathematics processes such as *reasoning and proof, communication,* and *connections* and the third mathematical practice (MP3), *construct viable arguments and critique the reasoning of others.* Stein, Engle, Smith, and Hughes (2008) proposed five practices for mathematics teachers to orchestrate productive mathematical discussion: "(a) anticipating students' mathematical responses, (b) monitoring student responses, (c) purposefully selecting student responses for public display, (d) purposefully sequencing student responses, and (e) connecting student responses" (pp. 322-331). Smith and Stein

(2011) outlined vivid examples of each of the five practices in their recent book for orchestrating classroom discourse. Similarly focusing on the classroom discourse strategies, Cho, Park, Lee, & Lee (2016) discussed three characteristics of teachers' questioning strategies that help formulate productive mathematical discourse such as openness, sharedness, and productivity.

2. MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

Shulman (1986) proposed a special domain of teacher knowledge, which he called pedagogical content knowledge (PCK). PCK bridges content knowledge and the practice of teaching. After developing an empirical approach to understand the content knowledge for teaching, Ball, Thames, and Phelps (2008) made refinements to the map of teacher content knowledge. They hypothesized that Shulman's content knowledge could be subdivided into common content knowledge (CCK) and specialized content knowledge (SCK); and that Shulman's PCK could be divided into knowledge of content and students (KCS) and knowledge of content and teaching (KCT).

Common content knowledge (CCK) is looking in student work for "simply calculating an answer or correctly solving mathematics problems" (Ball et al., 2008, p. 399). It is the mathematical knowledge and skills used in settings other than teaching. Examples of CCK are as follows: "knowing that a square is a rectangle; that 0/7 is 0; and that the diagonals of a parallelogram are not necessarily perpendicular" (p. 399).

Specialized content knowledge (SCK) is mathematical knowledge and skill unique to teaching that is not needed for purposes other than teaching. SCK involves "looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general" (Ball et al., 2008, p. 400). It requires appreciating the "difference between 'take-away' and 'comparison' models of subtraction, and between 'measurement' and 'partitive' models of division," (p.400) as demonstrated in detailed way in Ma (1999). An example of SCK is being able to create a story problem that represents division by fractions, which was the most difficult task for elementary school teachers in Ma (1999).

Knowledge of content and student (KCS) is knowledge that combines knowing about students and knowing about mathematics. When assigning a task, teachers need to "(a) anticipate what students are likely to do with it and whether they will find it easy or hard, and (b) be able to hear and interpret students' emerging and incomplete thinking" (Ball et al., 2008, p. 401) as expressed in their own language. The task requires "an interaction between specific mathematical understanding and familiarity with students and their mathematical thinking" (p. 401). Central to these tasks is knowledge of common student conceptions and misconceptions about particular mathematical content. Examples of KCS are familiarity with common errors and deciding which of the several errors students are most likely to make.

Knowledge of content and teaching (KCT) is combining knowing about teaching and knowing about mathematics. It involves mathematical knowledge of the *design of instruction,* such as "(a) sequencing particular content for instruction; (b) choosing which examples to start with and which examples to use to take students deeper into the content; (c) evaluating the instructional advantages and disadvantages of representations used to teach a specific idea; and (d) identifying what different methods and procedures afford instructionally" (Ball et al., 2008, p. 401). Examples of KCT are "(a) knowing different instructionally viable models for place value (e.g., money model, coffee stirrers bundled with rubber bands, and base-ten blocks and unit cubes); (b) knowing what each can reveal about the subtraction algorithm; and (c) knowing how to deploy them effectively" (p. 402).

Ball et al.'s (2008) major argument was that the notion of specialized content knowledge (SCK) is in need of further work, to understand the most important dimensions of teachers' professional knowledge. For example, in teaching two-digit subtraction, being able to carry out the subtraction algorithm is necessary but not sufficient for teaching it. To have SCK means that the teacher is being able to (a) interpret and analyze student error, (b) evaluate alternative algorithms, and (c) explain procedures. When students produce nonstandard/alternative approaches, teachers need to be able to describe the method student is using (e.g., is it legitimate to do this? Why? Would it work in general?) and justify it mathematically. In explaining procedures, teachers must know rationale for procedures, meanings for terms, and explanations for concepts. To represent the meaning of the subtraction algorithm, Ball et al. (2008) describe and make contrasts among three different ways to explain the procedures of two-digit subtraction (e.g., a set of procedural directions, using money model, and using straws rubber banded into groups of ten).

For the current study, we noticed that the domains of SCK, KCS, and KCT were particularly important and so we used the notion of mathematical knowledge for teaching (MKT) used by Ball et al. (2008) as our conceptual framework to develop and analyze the components of a secondary school mathematics lessons, and to capture how they were represented in the pre-service teachers' development of lesson planning materials and their demonstrative lessons.

3. THE CURRENT STUDY

This paper describes a research program, called "Let's Learn" of which the Principal Investigator (PI) was the first author in a university located in a Southeastern part of South Korea. The program was a small group research program for students which was sponsored by "the Center for Teaching and Learning" of the university. In the program, participants were required to extend their activities in an on-campus course to a local community. An undergraduate course, "the Curriculum Development and Teaching Methods (CDTM) in Mathematics Education" is the one at the level of junior undergraduates who are pre-service mathematics teachers. This course is a prerequisite for a teaching practicum that all the students should carry out in their last academic year. The course not only prepared students for the teaching practicum, but also prepared them for the "national teacher employment test" (Kim, Ham, & Paine, 2011, p. 51). One of the components of which is conducting a demonstrative lesson for around 15-20 minutes. In this course, nearly 40 students, as pre-service mathematics teachers, took turns to conduct their own demonstrative lesson for 15-20 minutes. The individual students selected a secondary mathematics textbook to analyze in-depth, designed a math lesson by developing lesson planning material, and conducted a demonstrative lesson, as a part of the undergraduate coursework. As a part of the research, three undergraduate who took the CDTM coursework were invited to participate in the Let's Learn program, and they were introduced to the onsite expert mathematics teacher, who taught at a nearby, local high school.

The purpose of this research was to increase students' understanding of secondary school mathematics instruction, and to promote and cultivate classroom observation and analysis skills. The research questions the researchers pose are as the following: What did the three students who participated in the Let's Learn research program learn about the components or processes of secondary school mathematics lesson, based on (a) observing their colleagues' demonstrative lessons and the listening to the PI's feedback as part of the undergraduate coursework, (b) observing onsite expert mathematics teacher's actual secondary school mathematics classroom lesson and interacting with the onsite teacher and his classroom students, an experience provided by the Let's Learn research program, and (c) conducting their own 15-minute demonstrative lessons during the CDTM coursework? The researchers intermediated the three students' meetings with the onsite expert teacher, but did not pose constraints on the nature of their interactions that transpired during the meetings. Moreover, when the three students observed the actual classroom lessons, what components did they particularly focus on for their classroom observation and analysis? To answer the research questions, interviews were proceeded with the three students and with the onsite expert secondary mathematics teacher.

An overarching goal of this research was to develop an instructional model for teaching an undergraduate course, "the Curriculum Development and Teaching Methods (CDTM) in Mathematics Education." The main activities of the CDTM coursework were composed of four major ones. First, all the students who took the CDTM coursework were encouraged to develop lesson planning materials in advance for the chapter and units they were assigned. This task requires them to conduct an extensive study of teaching materials, the textbooks, and more broadly, the overall secondary school mathematics curriculum. Second, all the students in the coursework were provided with exemplary videotaped lessons of students from previous years during the first few weeks of the semester, and were given instructions for weaving the sequence of a lesson, and for conducting the study of textbook and other teaching materials, so that they could seek a possible direction for how he/she would conduct his/her own demonstrative lesson. Moreover, they were informed of how they would be evaluated in a checklist format, which will be explicated further in the findings section. Third, all the students in the coursework had Q-and-A meetings with the professor (the first author) twice, before his/her own demonstrative lesson, as a course requirement. Fourth, as a part of the "Let's Learn" research program, the three selected, among the students enrolled in the CDTM coursework, were introduced to a high school, onsite expert mathematics teacher, so that they could get a better grasp of an actual secondary school mathematics classroom setting. This fourth one was one of the most important activities of "Let's Learn" research program. See Figure 1 for the brief summary of the major activities.

Our Goal for an Instructional Model for CDTM		
The Undergraduate Coursework ➢ All students enrolled in the CDTM course (a) conducted an extensive study of teaching materials, textbook, and secondary school mathematics curriculum, and (b) developed lesson planning material, which will be used as a guideline for conducting demonstrative lessons. ➢ All students enrolled in the CDTM course were provided with exemplary videotaped lessons of students from previous years. ➢ Two Q-and-A meetings were held with the professor, before each student's demonstrative lesson. ➢ The three students, among the students in the CDTM course, were recommended to participate in the Let's Learn research study and were introduced to the high school, onsite expert mathematics teacher.	⇔	**Let's Learn Research Program** ➢ The three students, who participated in the Let's Learn research program, analyzed the components of mathematics classroom instruction, which was provided in the undergraduate coursework. ➢ The three students made visits to the nearby high school and made observation of the onsite, high school teacher's mathematics instruction. ➢ The three students compared and contrasted the similarities and differences of the actual secondary mathematics instruction with the demonstrative lessons they conducted in the undergraduate coursework. ➢ The three students sought evaluation feedback and comments from the onsite expert mathematics teacher about their own demonstrative lessons.
The overarching goals of this research: o developing an instructional model for a method course for prospective secondary mathematics teachers; o integrating the components of undergraduate coursework and "Let's Learn" research program for the prospective teachers to work closely with local schools;		

Figure 1. A summary of the major activities of the undergraduate coursework and "Let's Learn" research program

The first and third activities of the CDTM coursework will be explicated more in-depth in the following two paragraphs. As a consequence of the extensive study of the textbooks of grades 6-8 (middle school) and grades 9-12 (high school), all the students in the CDTM coursework are exposed to acquire extensive curricular knowledge of secondary school mathematics. They review several textbooks with different publishers and select one that they could learn the most and that they could do their best for their demonstrative lesson. They develop lesson planning material that has balanced parts for introductory, body, and concluding elements of instructional flow.

The Q-and-A meetings were held, before each student in the CDTM coursework had conducted his/her demonstrative lesson. The professor provided comments and feedback on the lesson planning material that students had turned in. Generally, the students had difficulties in the following areas – (a) making connections between exploratory activity and the lesson's principal mathematical concept; (b) finding real-world problems that match well with the lesson content; and (c) expecting secondary school students' misconceptions through thought experiments. Since the lesson planning material that the students developed will be used as a guideline for structuring their demonstrative lesson, the professor provided direction for the students how they could revise some part of the lesson planning material to strengthen it, and guidance how they can overcome the difficulties they presented. After conducting the demonstrative lesson, all students received extensive feedback on how their demonstrative lesson went, by reviewing what and how the mathematics contents were taught and reviewing what was written down on the chalkboard.

The three selected students, among the students in the CDTM coursework, participated in the Let's Learn research program and were engaged in additional work. They analyzed the components of mathematics classroom instruction, which was provided in the undergraduate coursework. They visited nearby high school and made observation of the onsite, high-school teacher's mathematics instruction. They took notes of how the secondary school teacher conducted his lesson plan, how the class was designed and organized, what classroom environment actually looked like, and what were the levels of the high school students' mathematical understandings. They compared and contrasted the similarities and differences of the actual secondary mathematics instruction with the demonstrative lessons they conducted in the undergraduate coursework. They sought evaluation feedback and comments from the onsite expert mathematics teacher about their own demonstrative lessons.

From these activities in "the Curriculum Development and Teaching Methods (CDTM) in Mathematics Education," the students develop an image of what secondary school mathematics classroom teaching looks like. Moreover, the three students who participated in the Let's Learn research study worked closely with a nearby local high school. They had an opportunity to meet with and work with the onsite expert mathematics teacher, the work of which helped them substantiate the meaning of what mathematics teaching is like in a secondary school setting.

III. METHODS

1. PARTICIPANTS AND SETTINGS

The study was started at a University located in Southeastern part of South Korea, and a local high school in 2015. The undergraduate coursework was provided in the fall semester of the year, and the "Let's Learn" program lasted for two months in the semester. Our early work has been presented at an International Conference of Joint Societies for Mathematics Education in 2017 (Kim, S. & Lee, S. H., 2017). The researchers developed more focused direction for our research and conducted an ongoing analysis, based on comments and feedback we received from the conference presentation.

The study participants include three students who enrolled in the first author's method course and a high-school, onsite expert teacher. The three students, one female and two males, took the undergraduate coursework "the Curriculum Development and Teaching Methods (CDTM) in Mathematics Education" at the University as juniors, which prepared them for their teaching practicum the following year. The high-school, onsite expert teacher holds a bachelor's and master's degrees in mathematics education. He has experience giving lectures for pre-service mathematics teachers at the University, and has experience giving a presentation of his own master's thesis at a nationwide mathematics education conference. The teacher made his classroom available for the three students to observe, provided videotaped classroom teaching materials available for them to watch, and reviewed and evaluated the three students' lesson planning documents, demonstrative lessons, and their work products.

2. DATA COLLECTION & ANALYSIS

The researchers adopted an exploratory case study research method for the following two reasons. First, a case study is "an empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2009, p. 18). In the current study, the three undergraduate students participated in the "Let's Learn" research program, while also taking the undergraduate coursework, "the Curriculum Development and Teaching Methods (CDTM) in Mathematics Education." Their participation in the "Let's

Learn" program had simultaneously influenced how they participated in the undergraduate coursework CDTM and how they formulated their understanding of the processes of secondary school mathematics teaching.

Second, the case study inquiry

- copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result
- relies on multiple sources of evidence, with data needing to converge in a triangulating fashion, and as another result
- benefits from the prior development of theoretical propositions to guide data collection and analysis. (Yin, 2009, p. 18)

The researchers collected the following data: videotapes of the three students' demonstrative lessons for CDTM coursework; lesson planning materials of the three; videotapes of the onsite high school mathematics teacher's actual mathematics instruction; four sets of "learning community" activity reports; field notes and research notes taken during the site visits and during ongoing analysis. In addition, aside from the "Let's Learn" research purpose, the videotapes of all students registered in the CDTM coursework and their lesson planning documents were collected.

Data analysis included first- and second- cycle coding, jottings and analytic memo, and the development of assertions and propositions, as suggested by Miles, Huberman, and Saldana (2014).

3. PROCEDURES

Eight research activities were conducted for the Let's Learn research study. The activities were reviewed by the first author and also feedback was received from the onsite expert teacher. In the first and second research activities, the three students, who participated in the Let's Learn research study, prepared for the classroom visits, by conducting library search, discussing key points to look for during their onsite visits, and establishing research questions to pursue. In the third and fourth research activities, the three students watched videotapes of the onsite expert teacher's mathematics instruction, which was recorded for his own professional development purpose. They also had an opportunity to observe the onsite expert teacher's actual mathematics lesson. In the fifth research activity, the three students reviewed and analyzed their classroom observation of the onsite expert teacher, and prepared several things to discuss with the onsite expert teacher. In the sixth, seventh, and eighth research activities, the three students met with the onsite expert teacher at the research site, and discussed the differences between the actual lessons of mathematics they observed at the local high school, with their own demonstrative lessons conducted for the University undergraduate course requirement. The onsite expert teacher also provided feedback for the components of the three students' demonstrative lessons. See Table 1.

Table 1. Summary of the research activities

	Dates	**Summary of the Research Activities**	**Nature of the Research Activities**
Research Activity 1	Oct 27	Conducted a library search to establish research questions	
Research Activity 2	Oct 29	Conducted a library search to establish research questions	
Research Activity 3	Nov 5	Watched a videotape of an actual mathematics instruction	
Research Activity 4	Nov 6	Observed the onsite expert teacher's actual mathematics instruction	First site visit
Research Activity 5	Nov 12	Conducted review and analysis of the classroom observation	On-going analysis
Research Activity 6	Nov 13	Consulted with the onsite expert teacher about the components of her lesson	Second site visit
Research Activity 7	Nov 26	Discussed the differences between the onsite expert teacher's actual mathematics lesson with the three students' demonstrative lessons	On-going analysis
Research Activity 8	Dec 11	Received feedback from the onsite expert teacher about the three students' demonstrative lessons	Third site visit

IV. RESULTS AND DISCUSSIONS

1. A MODEL OF THE COURSE "CDTM"

1) The Design for CDTM

The Q-and-A meetings were held before and after students' demonstrative lessons. The students who took the CDTM undergraduate coursework were asked to select their preferences for the grade-levels they were interested in pursuing deeper, from the range of grades 6-8 (middle school) and grades 9-12 (high school). Almost all students were assigned a grade-level from which that they showed their preferences. The group of students who were assigned the same grade-level met together with the professor, and selected a chapter they wished to work on. Then, it was the professor's discretion to select small units for the students to focus on for developing the lesson planning materials. From a range of a number of small units on a chapter, the professor selected a small unit that contained major issues that mathematics education community deems important (e.g., how to introduce the concept of negative integers to secondary school students).

After the students were assigned the chapters and small units, they conducted an extensive study of teaching materials, the textbook, and the secondary school mathematics curriculum documents individually, and developed lesson planning materials for their demonstrative lessons. The group of students met again with the professor with their lesson planning materials. As a group, they shared several (about three) mathematical concepts or representations for teaching, that they thought were particularly difficult when working on the units. The students in the same group discussed how they would represent the mathematical concept for the imaginary secondary school students to understand.

Right after individual students' demonstrative lesson performed for 15-20 minutes, they had opportunities to share their impression on their own lessons (e.g., some things that did not go as intended). The professor provided comments and feedback extemporaneously, mainly about mathematical concepts that went unaddressed, or about teaching style that could have taught in a better way. The professor also addressed comments from the audience (i.e., other students in the class). In other words, the feedback given extemporaneously was (a) to address important mathematical concepts that went unaddressed by the presenter, and (b) to address questions from the audience. The additional feedback was provided at the beginning of the next class meeting in a PowerPoint slide format, where a screenshot of the chalkboard was captured to point to the students' important mathematical concepts that have implications for connecting with the theories of mathematics education.

2) The Components of Demonstrative Lesson

The three students, who participated in the Let's Learn research study, analyzed the process of secondary school mathematics classroom instruction, based on their knowledge gained from the undergraduate coursework. The components of secondary school mathematics instruction suggested in the undergraduate coursework are as follows. We extracted these components from the composition of a lesson unit for one class in the secondary school mathematics textbooks written according to the National Curriculum and modified it referring to the research report for the Teaching Evaluation Standards of Mathematics published by the Korea Institute for Curriculum and Evaluation (2006). We also reflected the first author's experience of teaching of the course CDTM since 2006 for more than 10 years in the following components.

(a) Setting up learning objectives;
(b) Making connections to prior knowledge;
(c) Explorative approach for the core concepts, involving pair, or group work;
(d) Teaching for the core conceptual understanding;
(e) Reviewing example problems;
(f) Posing problems for students to solve, involving individual, pair, or group work;
(g) Develop a shared understanding with the whole group;
(h) Assessing student understanding with the whole group;
(i) Assigning homework problems; and
(j) Announcing the topic for the next class.

These instructional components can be categorized into three broader processes of instruction: introduction to the lesson (a, b, c), the body of the lesson (d, e, f, g, h), and the closing remarks (i, j). Figure 2 shows the design of the CDTM undergraduate coursework for students who are pre-service mathematics teachers, and shows how the components of the demonstrative lessons in the CDTM undergraduate coursework matches with the components of secondary school mathematics lessons. Figure 3 delineates the design of the CDTM undergraduate coursework in a slightly different format, and shows the interflow of the the CDTM course design and the Let's Learn research program (i.e., how they are tightly interwoven together to enhance the three students' understandings of the instructional processes of the secondary school mathematics lessons).

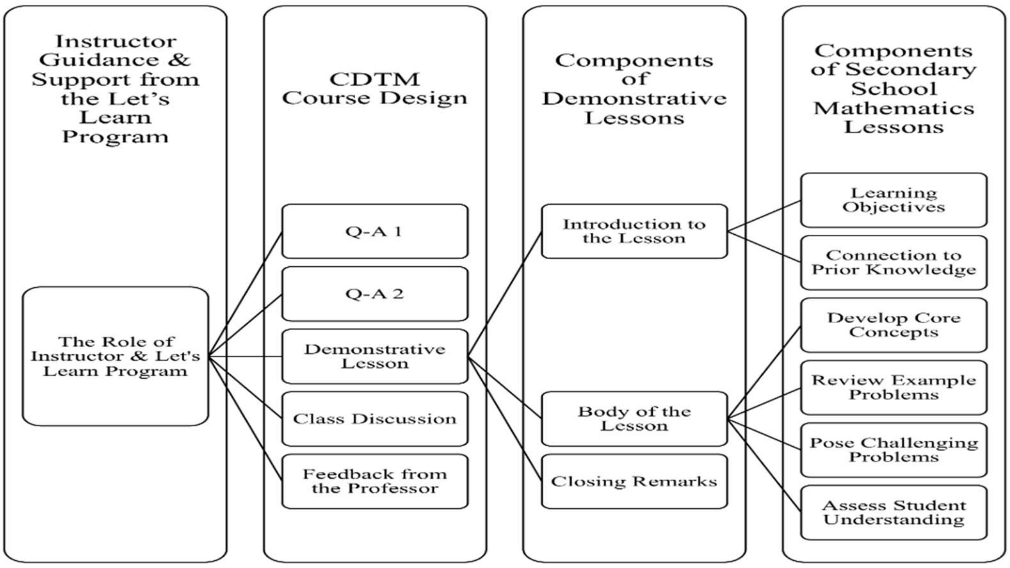

Figure 2. CDTM undergraduate course design and the instructional processes of the secondary school mathematics lessons

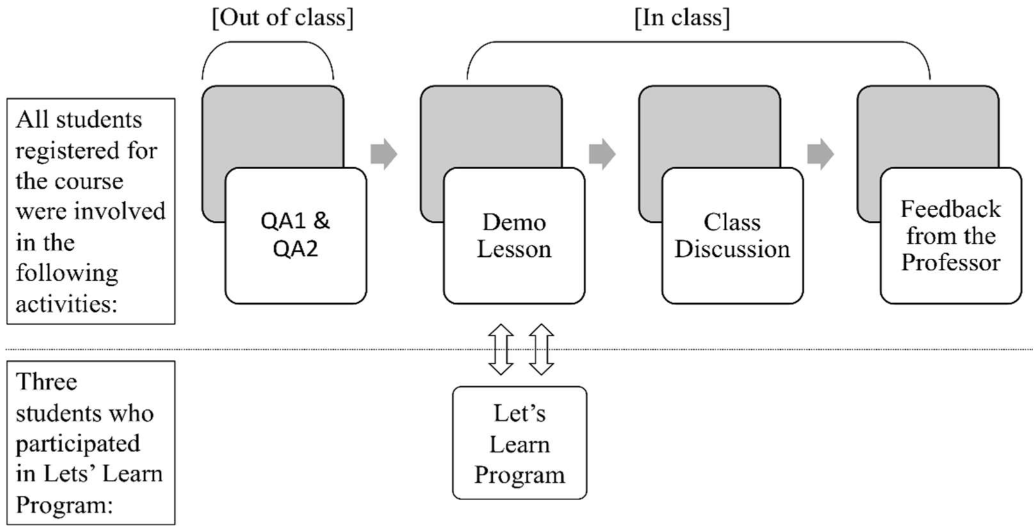

Figure 3. The interflow between the CDTM course design and the "Let's Learn" program

3) Assessment Criteria for Students' Demonstrative Lessons

The worksheet describing "the components of secondary school mathematics teaching" was distributed to the three students. They were given instruction to use this checklist worksheet for making observation of the high-school, onsite mathematics teacher's lesson, to match how the components are represented in his teaching. They were also given instruction that this checklist worksheet will be used for grading their own demonstrative lessons in the undergraduate coursework. In the checklist, there were the five big domains: (a) planning of the lesson, (b) mathematics content knowledge, (c) methods of teaching, (d) understanding of students, and (e) teaching techniques.

In the first domain of the planning of the lesson, two assessment criteria were sought. The learning objectives were described in the lesson planning material, based on the textbook and the secondary school curriculum documents. The connections to the previous lesson were made; the learning contents are delineated in relation to the prior and the subsequent grade-level contents.

In the second domain of the mathematics content knowledge, eight assessment criteria were sought. The mathematical concepts and principles were explained meaningfully, balancing both the informal or intuitive levels of understanding and the rigorous aspects of mathematics. Appropriate example problems were selected so that the mathematical content can be well-represented. The relevancy and the level of difficulty were taken into account when selecting formative assessment problems. The mathematical symbols and mathematical terminologies were used accurately. For teaching problem solving strategies, guidelines about understanding the problem and describing the plan for solving the problem were provided; the problems that encourage multiple solution strategies were presented; after reaching an answer, it was encouraged to reflect on the solution process and seek other possible ways to solve the problem. Multiple representations were used to explain a mathematical concept, such as graphs, equations, tables, pictures, formulas, mathematical expressions, and stories. Secondary school students' social or environmental phenomena were used as a resource for introducing mathematical concepts, principles, and laws. Related history of mathematics was used. This domain can be connected to Ball et al.'s (2008) notions of common content knowledge (CCK) and specialized content knowledge (SCK).

In the third domain of the methods of teaching, five assessment criteria were sought. Teacher questioning was provided which prompted secondary school students' mathematical thinking. To promote students' communication skills, there were opportunities for small- and whole-group discussions, and opportunities for representing students' mathematical thinking in spoken words and in written format. The problem sets contained a wide range of problems, so that students at different levels of understanding can be engaged in the mathematical tasks that they can handle. The content of the lesson can be made connections to other domains of mathematics and to other disciplinary subject areas. Manipulatives or assistive software tools were utilized in the class. This domain can be related to Ball et al.'s (2008) notion of the knowledge of content and teaching (KCT).

In the fourth domain of understanding of students, three assessment criteria were sought. Students' misconceptions were well-taken care of. Self-regulated learning was promoted. An expectation that all students can achieve highly, mathematically was expressed clearly; that persistence in learning mathematics is important was communicated effectively. This domain can be related to Ball et al.'s (2008) notion of the knowledge of content and students (KCS).

In the fifth domain of teaching techniques, four assessment criteria were sought. The lesson progressed as was planned in the lesson planning material. The homework assignment for secondary school students were presented clearly. What was written on the chalkboard was organized in a systematic manner. The summary of the lesson content was provided, and the content for next lesson was announced.

2. THE ONSITE EXPERT MATH TEACHER'S TEACHING GUIDANCE

1) Comments for the Demonstrative Lesson of the Student A

For an understanding of the contents of the unit, the onsite expert mathematics teacher commented, "The student A did his demonstrative lesson on a unit called, 'letters and expressions' at the level of grade 8. Because middle school students encounter letters in mathematical expressions almost for the first time, teachers have to explain the necessity why we are adopting letters in expressing the problem situation. The student A addressed the necessity of the letters precisely and has a clear understanding of the unit."

For the analysis of the lesson objective and the development process of problem-solving phase, the onsite expert mathematics teacher commented, "The lesson objective was 'students can express diverse real-world situations with letters and mathematical expressions.' The student A provided a learning goal that prompted students to utilize letters and expressions to describe problem situations in real-world contexts. He did not introduce the letter expression abruptly; instead, he addressed why students needed to use the letters in describing the problem situation, which I think was a good approach."

For the utilization of diverse sources of knowledge, the onsite expert mathematics teacher commented, "Because the number of shapes (e.g., circle, triangle, rectangle, etc.) is limited, there can be some problematic situation where the shapes cannot distinguish various sorts of mathematical objects (variables). To identify mathematically distinguishable objects, we adopt a number of symbols. The student A explained well, that alphabetical letters can be used to distinguish different variables."

For matching this student's demonstrative lesson to an actual class, the onsite expert mathematics teacher commented, "If we apply the student A's lesson contents in an actual mathematics class, students will learn why mathematicians use alphabetical symbols in expressing or solving mathematical problems. They will eventually learn other advantages of using letters, such as the flexible nature of the use of alphabetical symbols and the general nature of the use of alphabetical symbols."

2) Comments for the Demonstrative Lesson of the Student B

For an understanding of the contents of the unit, the onsite expert mathematics teacher commented, "The student B taught a unit on 'functions' at the level of grade 8. School students have already learned proportional relationships (direct and inverse proportion), but they might face difficulty when functional concept is introduced for the first time in grade 8. In the national curriculum document tells that the introduction of function should be as follows: 'students understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.' The student B described the function, using the terms 'proportion' and 'correspondence'. He explained the proportional relationship that students have learned in elementary school, and made connection to the concept of corresponding relation using an example."

For the analysis of the lesson objective and the development process of problem-solving phase, the onsite expert mathematics teacher commented, "The lesson objective was 'students can solve various problems using functions.' The student B grasped the intent of the curriculum material that the perspective of dependence and that of correspondence should be intermingled. She reminded students of the proportional relations (y=ax, y=a/x, $a \neq 0$ constant), and introduced the meaning of corresponding relationship using real-world examples. I think this approach was exceptionally powerful in helping students learn the functional relationship."

For the utilization of diverse sources of knowledge, the onsite expert mathematics teacher commented, "the student B not only approached functions problems quantitatively (e.g., applying known formula, executing precise calculations), but also approached qualitatively by interpreting verbal expressions into a rough graphical representation."

For matching the three students' demonstrative lesson to an actual class, the onsite expert mathematics teacher commented, "8^{th} graders' understanding of a functional relationship might be fixated to the dependence relationship; yet the student B nicely transitions the functional relationship from the dependence into a corresponding relationship."

V. CONCLUSIONS

By conducting this study, the researchers aimed to develop an instructional model for teaching Korean undergraduate mathematics education course, "the Curriculum Development and Teaching Methods (CDTM) in Mathematics Education." Among the students who took the CDTM undergraduate coursework as prospective mathematics teachers, three students were selected to participate in the "Let's Learn" research program. The three students observed their colleagues' demonstrative lessons and listened to the PI's feedback as part of the undergraduate coursework. Moreover, they had an opportunity to observe an onsite expert mathematics teacher's actual high school mathematics instruction and to interact with the onsite teacher and the high school students. This was an experience solely provided by the Let's Learn research program. Finally, the three students conducted their own 15-minute demonstrative lessons, a product of both the CDTM coursework and the Let's Learn research program.

The research questions the researchers pursued were: (a) what did the three pre-service mathematics teachers who participated in the Let's Learn research study learn about the components or the processes of secondary school mathematics instruction? (b) When the three students observed the actual classroom lessons, what components did they particularly focus on? As the researchers conducted the study, the overarching goals of this research were two-fold: (a) developing an instructional model for a method course for prospective secondary mathematics teachers; and (b) integrating the components of an undergraduate coursework, CDTM, and the Let's Learn research program for the three prospective mathematics teachers to work closely with local schools. The researchers carefully collected and analyzed the qualitative data, including the videotapes of three students' demonstrative lessons for CDTM coursework, their lesson planning materials for preparing the demonstrative lessons, and the videotapes of the onsite high school mathematics teacher's actual mathematics instruction. The researchers analyzed four sets of "learning community" activity reports, field notes and research notes taken during the site visits and during the ongoing analysis stages.

We could capture the design of the CDTM undergraduate coursework for pre-service mathematics teachers, and show how the components of the demonstrative lessons in the CDTM undergraduate coursework matches with the components of secondary school mathematics lessons (See Figure 2). We could also delineate the design of the CDTM undergraduate coursework in a slightly different format, and show the interflow of the CDTM course design and the Let's Learn research program (See Figure 3). That is, the CDTM course design and the Let's Learn research program were tightly interwoven together, which enhanced the three students' understandings of the instructional processes of the secondary school mathematics lessons.

The Mathematical Knowledge for Teaching (MKT) was a construct developed by researchers (e.g., Ball et al., 2008). Ball et al. (2008) refined the map of teacher knowledge, based on Shulman's (1986) concept of pedagogical content knowledge. Among the four domains of knowledge – common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT) – which comprise the Mathematical Knowledge for Teaching (MKT), Ball et al. (2008) argue that the notion of SCK is in need of further work. For the current study, we used the construct of mathematical knowledge for teaching (MKT) as our conceptual framework in defining and analyzing the components or processes of a secondary school mathematics lessons. For example, the assessment criteria developed for the CDTM coursework were well-aligned with three of the domains of MKT – SCK, KCT, and KCS – which we consider important to the teaching of mathematics.

Our research shows that the three students demonstrated a robust understanding of mathematics and teaching, specifically in the topics of "letters and expressions" and "functional relationships" at the level of grade 8. The three students were able to make most out of their unique experience by participating both in the CDTM undergraduate mathematics method coursework and in the Let's Learn research program, the latter part of which enabled them to interact with the local high school institution, the staffs, and the actual secondary school educational environment.

Through this research, we found out this method course provided our students who are pre-service teachers with the experience of various aspects of actual mathematics lesson as well as learning theories about the pedagogy for teaching for secondary school mathematics.

We conclude that this course gives a model for the methods course in mathematics education for secondary mathematics. This research implies that the method course must involve not only students' teaching practicum experiences in the class but also a connection with onsite expert math teachers and visits of the educational institution during the class.

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