

BPSK 변조의 최대 전송률 분석: 상관 정보원의 비직교 다중 접속 관점에서

정규혁*

Analysis of Achievable Data Rate under BPSK Modulation: CIS NOMA Perspective

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요약

본 논문은 연속 가우시안 입력 변조를 사용하는 대부분의 기존 NOMA 설계와는 대조적으로, BPSK 변조하에서 상관 정보원의 NOMA에 대한 최대 전송률을 분석한다. 먼저, BPSK 변조와 상관 정보원의 NOMA에 대한 최대 전송률의 폐쇄형 표현식을 유도한다. 다음, 수치적 결과를 통해, 강 채널 사용자에게 대해서는, 독립 정보원의 최대 전송률과 비교하여, 상관 정보원의 최대 전송률이 감소하는 것을 보여준다. 또한, 약 채널 사용자에게 대해서는, 독립 정보원의 최대 전송률과 비교하여, 상관 정보원의 최대 전송률이 증가하는 것을 입증한다. 추가로, 수신 신호와 사용자 간 간섭의 확률 분포 함수의 심도 있는 분석을 통해 이론적 결과를 입증한다.

ABSTRACT

This paper investigates the achievable data rate for non-orthogonal multiple access(NOMA) with correlated information sources(CIS), under the binary phase shift keying(BPSK) modulation, in contrast to most of the existing NOMA designs using continuous Gaussian input modulations. First, the closed-form expression for the achievable data rate of NOMA with CIS and BPSK is derived, for both users. Then it is shown by numerical results that for the stronger channel user, the achievable data rate of CIS reduces, compared with that of independent information sources(IIS). We also demonstrate that for the weaker channel user, the achievable data rate of CIS increases, compared with that of IIS. In addition, the intensive analyses of the probability density function(PDF) of the observation and the inter-user interference(UI) are provided to verify our theoretical results.

키워드

NOMA, Superposition Coding, Successive Interference Cancellation, Power Allocation, Correlation Coefficient, BPSK
비직교 다중 접속, 중첩 코딩, 순차적 간섭 제거, 전력 할당, 상관 관계 계수, BPSK

1. Introduction

Recently, non-orthogonal multiple access(NOMA) has emerged as a key enabling multiple access(MA) technology to meet the unprecedented

requirements of forthcoming future fifth-generation(5G) wireless mobile networks, due to high spectral efficiency, massive connectivity, and low transmission latency[1, 2], compared to orthogonal multiple access(OMA)[3-5]. In this paper, we

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mainly consider the power-domain NOMA[6, 7]. The users with better channel conditions can eliminate other users' messages by applying successive interference cancellation(SIC) before decoding their own messages[8].

Lately, the bit-error rate(BER) performance of NOMA was analyzed[9]. The impacts of local oscillator imperfection for NOMA were studied[10]. In addition, the BER expression with randomly generated signals was derived[11]. The BER expression for the two and three-user cases was studied[12]. The average symbol error rate(SER) expressions for the two-user case were considered in[13].

As the state-of-the-art advances in NOMA, the optimal power control was investigated based on individual QoS constraints[14], whereas the energy harvesting in NOMA was studied for machine-to-machine(M2M) communications[15].

As in the above-mentioned literature, in NOMA, information sources are assumed to be independent, i.e., independent information sources(IIS). However, sometimes the common information needs to be transmitted. For example, many mobile game users are enjoying the mobile game together, in the same cell of 5G networks. In this case, game companies need to broadcast the screen of live players. Then such data can be modeled as correlated information sources(CIS)[16].

In this paper, we investigate the achievable data rate of NOMA for CIS, especially under the binary phase shift keying(BPSK) modulation, in contrast to most of the existing NOMA designs using continuous Gaussian input distributions. First, we derive the analytical expression for the achievable data rate of NOMA for CIS, for both users, respectively. Then it is shown that the achievable data rate of the stronger channel user in NOMA for CIS decreases, compared to that for IIS, whereas the achievable data rate of the weaker channel user in NOMA for CIS increases, compared

to that for IIS.

The remainder of this paper is organized as follows. In Section II, the system and channel model are described. The analytical expression for the achievable data rate of NOMA for CIS is derived in Section III. The results are presented and discussed in Section IV. Finally, the conclusions are presented in Section V.

II. System and Channel Model

In a cellular downlink NOMA transmission system, all the users are assumed to be experiencing block fading, in a narrow band of frequencies. For wideband systems, orthogonal frequency division multiplexing(OFDM) can transform a frequency-selective channel into slow fading ones. A base station and paired users are within the cell. The complex channel coefficient between the m th user and the base station is denoted by h_m , $m=1,2$. The channels are sorted as $|h_1| > |h_2|$. The base station sends the superimposed signal $x = \sqrt{\alpha P_A} s_1 + \sqrt{(1-\alpha)P_A} s_2$, where s_m is the message for the m th user with unit power, $E[|s_1|^2] = E[|s_2|^2] = 1$, where $E[u]$ represents the expectation of the random variable(RV) u , α is the power allocation factor, with $0 \leq \alpha \leq 1$, and P_A is the total allocated power. The correlation coefficient is $\rho_{1,2} = E[s_1 s_2^*]$. Owing to correlation, the power of the superimposed signal x is not equal to P_A . Therefore, for the constant total transmitted power P at the base station, P_A is effectively scaled by[17]

$$P_A = \frac{P}{1 + 2\text{Re}\{\rho_{1,2}\}\sqrt{\alpha}\sqrt{1-\alpha}} \quad (1)$$

where $\text{Re}\{z\}$ is the real part of a complex number z . It should be noted that for IIS, $P_A = P$. The observation at the m th user is given by

$$\begin{aligned} r_m &= |h_m| x + n_m \\ &= |h_m| \sqrt{\alpha P_A} s_1 + |h_m| \sqrt{(1-\alpha)P_A} s_2 + n_m, \end{aligned} \quad (2)$$

where $n_m \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian noise(AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ represents the distribution of a Gaussian RV with mean μ and variance Σ . Two uniform sources are given by

$$\begin{cases} P(b_1=0) = p_1 = 1/2 \\ P(b_1=1) = 1-p_1 = 1/2 \end{cases} \quad (3)$$

$$\begin{cases} P(b_2=0) = p_2 = 1/2 \\ P(b_2=1) = 1-p_2 = 1/2 \end{cases}.$$

For IIS, the joint probability mass function (PMF) is given by

$$\begin{cases} P(b_1=0, b_2=0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \\ P(b_1=0, b_2=1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \\ P(b_1=1, b_2=0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \\ P(b_1=1, b_2=1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \end{cases} \quad (4)$$

For CIS, the joint PMF is given by

$$\begin{cases} P(b_1=0, b_2=0) = q_{0,0} \\ P(b_1=0, b_2=1) = q_{0,1} = \frac{1}{2} - q_{0,0} \\ P(b_1=1, b_2=0) = q_{1,0} = \frac{1}{2} - q_{0,0} \\ P(b_1=1, b_2=1) = q_{1,1} = q_{0,0} \end{cases} \quad (5)$$

In this paper, we assume that CIS has the positive correlation, $\rho_{1,2} > 0$, which corresponds to $\frac{1}{4} < q_{0,0} < \frac{1}{2}$. Then the correlation coefficient is calculated as[16]

$$\rho_{1,2} = E[s_1 s_2^*] = 4q_{0,0} - 1. \quad (6)$$

In this paper, we consider the BPSK modulation,

$s_m \in \{+1, -1\}$, and to ensure the user fairness, we consider the power allocation range, $\alpha \leq 0.5$. It is also assumed that the conventional bit-to-symbol mapping:

$$\begin{cases} s_1(b_1=0) = +1 & s_2(b_2=0) = +1 \\ s_1(b_1=1) = -1 & s_2(b_2=1) = -1. \end{cases} \quad (7)$$

III. Derivation of Achievable Data Rate

Shannon originally defined the channel capacity as the mutual information $\mathcal{I}(y;x) = h(y) - h(y|x)$ where $h(y) = -E[\log_2 p_Y(y)]$ is the differential entropy, and $p_Y(y)$ is the probability density function (PDF). We start the derivation from the above-mentioned definition.

First, if the perfect SIC is assumed, the achievable data rate R_1 is given by

$$\begin{aligned} R_1 &= \mathcal{I}(r_1; b_1 | b_2) \\ &= h(r_1 | b_2) - h(r_1 | b_1, b_2). \end{aligned} \quad (8)$$

Then, the conditional differential entropy $h(r_1 | b_2)$ is calculated by

$$\begin{aligned} h(r_1 | b_2) &= h(|h_m| \sqrt{\alpha P_A} s_1(b_1) + |h_m| \sqrt{(1-\alpha)P_A} s_2(b_2) + n_1 | b_2) \\ &= h(|h_m| \sqrt{\alpha P_A} s_1(b_1) + n_m | b_2) \\ &= h(y_1 | b_2), \end{aligned} \quad (9)$$

where $y_1 = |h_m| \sqrt{\alpha P_A} s_1(b_1) + n_m$ is the observation after SIC is performed on r_1 . Then, the conditional PDF of $P_{Y_1 | B_2}(y_1 | b_2)$ is represented by

$$\begin{aligned} P_{Y_1 | B_2}(y_1 | b_2) &= \\ &= \frac{q_{0,b_2}}{P(b_2)} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_1 - |h_m| \sqrt{P_A \alpha})^2}{2N_0/2}} \\ &+ \frac{q_{1,b_2}}{P(b_2)} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_1 + |h_m| \sqrt{P_A \alpha})^2}{2N_0/2}}. \end{aligned} \quad (10)$$

Thus the differential entropy is calculated by

$$\begin{aligned} h(y_1 | b_2) &= -E[\log_2 P_{Y_1 | B_2}(y_1 | b_2)] \\ &= -\sum_{b_2=0}^1 P(b_2) \int_{-\infty}^{+\infty} P_{Y_1 | B_2}(y_1 | b_2) \log_2 P_{Y_1 | B_2}(y_1 | b_2) dy_2 \\ &= -\int_{-\infty}^{+\infty} P_{Y_1 | B_2}(y_1 | b_2 = 0) \log_2 P_{Y_1 | B_2}(y_1 | b_2 = 0) dy_2. \end{aligned} \quad (11)$$

And the conditional differential entropy $h(r_1 | b_1, b_2)$ is calculated by

$$\begin{aligned} h(r_1 | b_1, b_2) &= h(|h_m | \sqrt{\alpha P_A} s_1(b_1) + |h_m | \sqrt{(1-\alpha) P_A} s_2(b_2) + n_1 | b_1, b_2) \\ &= h(n_1 | b_1, b_2) \\ &= h(n_1) \\ &= \frac{1}{2} \log_2(2\pi e N_0/2). \end{aligned} \quad (12)$$

Second, the PDF of y_2 is represented by

$$\begin{aligned} P_{Y_2}(y_2) &= q_{0,0} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_d|(+\sqrt{P_d(1-\alpha)} + \sqrt{P_d\alpha}))^2}{2N_0/2}} \\ &+ q_{0,1} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_d|(-\sqrt{P_d(1-\alpha)} + \sqrt{P_d\alpha}))^2}{2N_0/2}} \\ &+ q_{1,0} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_d|(+\sqrt{P_d(1-\alpha)} - \sqrt{P_d\alpha}))^2}{2N_0/2}} \\ &+ q_{1,1} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_d|(-\sqrt{P_d(1-\alpha)} - \sqrt{P_d\alpha}))^2}{2N_0/2}} \end{aligned} \quad (13)$$

It should be noted that for the second user, since SIC cannot be performed, we have the received signal

$$y_2 = r_2. \quad (14)$$

Then, the differential entropy is calculated by

$$\begin{aligned} h(y_2) &= -E[\log_2 P_{Y_2}(y_2)] \\ &= -\int_{-\infty}^{+\infty} P_{Y_2}(y_2) \log_2 P_{Y_2}(y_2) dy_2. \end{aligned} \quad (15)$$

And the conditional differential entropy is calculated by

$$\begin{aligned} h(y_2 | b_2) &= -E[\log_2 P_{Y_2 | B_2}(y_2 | b_2)] \\ &= -\sum_{b_2=0}^1 P(b_2) \int_{-\infty}^{+\infty} P_{Y_2 | B_2}(y_2 | b_2) \log_2 P_{Y_2 | B_2}(y_2 | b_2) dy_2. \end{aligned} \quad (16)$$

where the conditional PDF is given by

$$\begin{aligned} P_{Y_2 | B_2}(y_2 | b_2) &= \frac{q_{0,b_2}}{P(b_2)} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_d|(+(-1)^{b_2} \sqrt{P_d(1-\alpha)} + \sqrt{P_d\alpha}))^2}{2N_0/2}} \\ &+ \frac{q_{1,b_2}}{P(b_2)} \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_2 - |h_d|(+(-1)^{b_2} \sqrt{P_d(1-\alpha)} - \sqrt{P_d\alpha}))^2}{2N_0/2}}. \end{aligned} \quad (17)$$

Then we have the achievable data rate R_2 for the second user

$$\begin{aligned} R_2 &= h(y_2) - h(y_2 | b_2) \\ &= h(y_2) + \sum_{b_2=0}^1 P(b_2) \int_{-\infty}^{+\infty} P_{Y_2 | B_2}(y_2 | b_2) \log_2 P_{Y_2 | B_2}(y_2 | b_2) dy_2 \\ &= h(y_2) + \int_{-\infty}^{+\infty} P_{Y_2 | B_2}(y_2 | b_2 = 0) \log_2 P_{Y_2 | B_2}(y_2 | b_2 = 0) dy_2 \\ &= -\int_{-\infty}^{+\infty} P_{Y_2}(y_2) \log_2 P_{Y_2}(y_2) dy_2 \\ &+ \int_{-\infty}^{+\infty} P_{Y_2 | B_2}(y_2 | b_2 = 0) \log_2 P_{Y_2 | B_2}(y_2 | b_2 = 0) dy_2 \end{aligned}$$

(18)

IV. Results and Discussions

It is assumed that $|h_1| = \sqrt{1}$ and $|h_2| = \sqrt{0.5}$. We consider the constant total transmitted signal

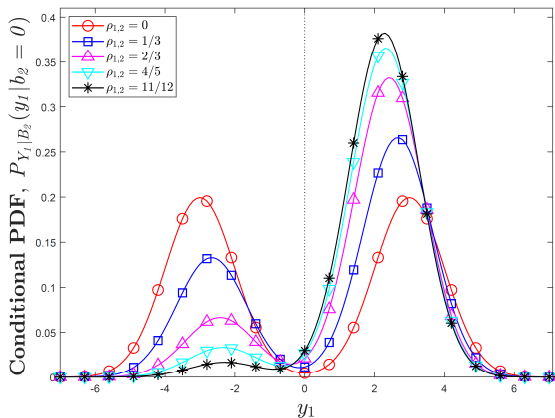


Fig. 1 Conditional PDF $P_{Y_1|B_2}(y_1|b_2=0)$ of observation for first user

power to noise power ratio (SNR) $P/N_0 = 15$.

First, for $\alpha=0.2$, we depict the PDF of $P_{Y_1|B_2}(y_1|b_2=0)$ for the first user, in order to investigate the impacts of CIS, in Fig. 1, for the various correlation coefficient, $\rho_{1,2}=0, \frac{1}{3}, \frac{2}{3}, \frac{4}{5}$ and $\frac{11}{12}$, which correspond to $q_{0,0} = \frac{1}{4}, \frac{2}{6}, \frac{5}{12}, \frac{11}{24}$ and $\frac{23}{48}$, respectively. As shown in Fig. 1, when the correlation coefficient $\rho_{1,2}$ increases, the double Gaussian bell-shape signal approaches a single Gaussian bell-shape signal, i.e., the entropy decreases. This is because the correlation coefficient $\rho_{1,2}$ increases. Thus, we expect that the achievable data rate R_1 will decrease.

In Fig. 2, we plot the achievable data rate R_1 , for the various correlation coefficient. As shown in Fig. 2, when the correlation coefficient $\rho_{1,2}$ increases, the achievable data rate R_1 decreases, due to the reduced entropy of the observation.

Second, in Fig. 3, the PDF of the observation $P_{Y_2}(y_2)$ for the second user is depicted for the various correlation coefficient $\rho_{1,2}$, for $\alpha=0.2$. As shown in Fig. 3, as the correlation coefficient $\rho_{1,2}$ increases, the the inter-user interference(IUI) looks

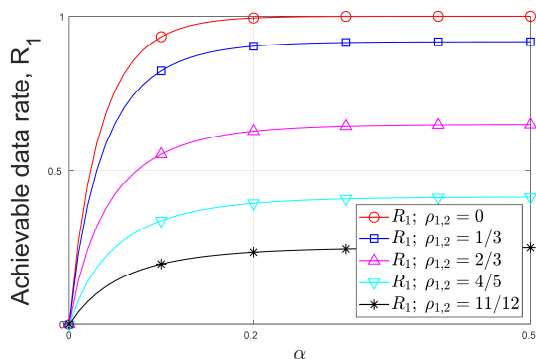


Fig. 2 Achievable data rate R_1 for first user

like disappearing. Also, we plot the PDF $P_{Y_2|B_2}(y_2|b_2=0)$ of the IUI+AWGN, in Fig. 4, for $\alpha=0.2$. As shown in Fig. 4, as the correlation coefficient $\rho_{1,2}$ increases, the IUI+AWGN approaches near AWGN, i.e., the IUI becomes less severe. Thus, we expect that the achievable data rate R_2 will increase.

In Fig. 5, we plot the achievable data rate R_2 , for the various correlation coefficient. As shown in Fig. 5, when the correlation coefficient $\rho_{1,2}$ increases, the achievable data rate R_1 also increases, due to the reduced IUI.

Lastly, we should mention that the results of both users are different. Specifically, this is because the achievable data rate of the stronger channel user is affected by the allocated power, whereas the achievable data rate of the weaker channel user is mainly influenced by the IUI. Physically, due to the correlation, the base station allocates less power, for the given constant total transmitted power. In addition, such correlation reduces the IUI, which determines the performance of the weaker channel user.

V. Conclusion

This paper investigated the achievable data rate

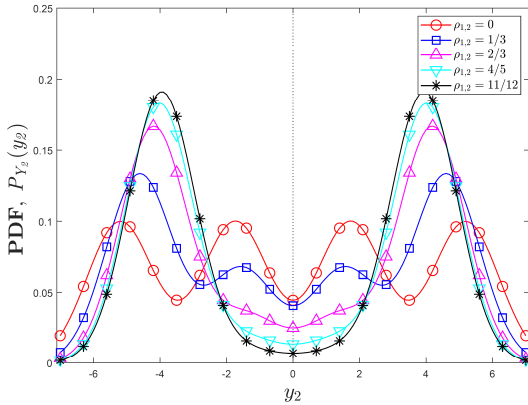


Fig. 3 PDF $P_{Y_2}(y_2)$ of observation for second user

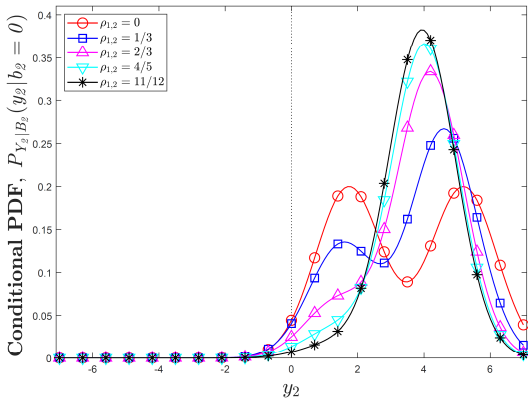


Fig. 4 Conditional PDF $P_{Y_2|B_2}(y_2 | b_2 = 0)$ of IUI+AWGN for second user

for NOMA with CIS, under the BPSK modulation. First, the analytical expression for the achievable data rate of NOMA with CIS and BPSK was derived, for both users. Then it was shown that for the stronger channel user, the achievable data rate of CIS reduces, whereas for the weaker channel user, the achievable data rate of CIS increases, compared with that of IIS, respectively. In addition, we also investigated the impacts of CIS on the PDF of the observation and the IUI, for both users.

As a direction of the future research, it would be interesting to design the modulation technique in

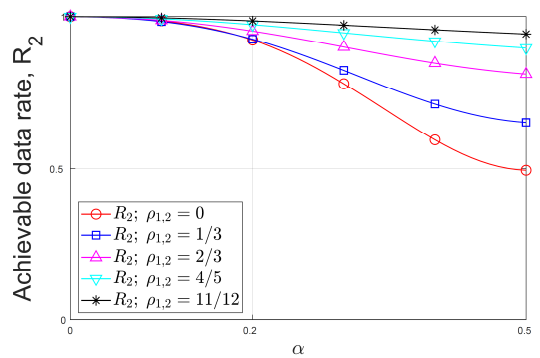


Fig. 5 Achievable data rate R_2 for second user

superposition coding for increasing the reduced achievable data rate of the stronger channel user, with a tolerable loss of the achievable data rate of the weaker channel user.

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