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ON INTERVAL VALUED INTUITIONISTIC FUZZY HYPERIDEALS OF ORDERED SEMIHYPERGROUPS

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Abstract. We introduce the notion of interval valued intuitionistic fuzzy hyperideals, bi-hyperideals and quasi-hyperideals of an ordered semihypergroup. We characterize an interval valued intuitionistic fuzzy hyperideal of an ordered semihypergroup in terms of its level subset. Moreover, we show that interval valued intuitionistic fuzzy bi-hyperideals and quasi-hyperideals coincide only in a particular class of ordered semihypergroups. Finally, we show that every interval valued intuitionistic fuzzy quasi-hyperideal is the intersection of an interval valued intuitionistic fuzzy left hyperideal and an interval valued intuitionistic fuzzy right hyperideal.

1. Introduction

The concept of hypergroups were first considered by Marty as an extension of groups, (see [10]). Later on, many generalizations of hypergroups are studied, such as semihypergroups and semihyperrings, (see [4, 14]). These concepts were extended upon and widely investigated. The concepts of ordered semihypergroups, ordered LA-semihypergroups and Γ-semihypergroups are generalized concepts of semihypergroups, (see [3, 6, 7, 12, 16, 17]). Moreover, semihyperrings can be generalized

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to the concept of ordered semihyperrings, (see [5]). Hence, many investigations into semigroups and semirings were to study and prove such generalizations.

One of the concepts which can be applied to an investigation of algebraic structures is the concept of fuzzy sets. The theory of fuzzy sets was first conceived by Zadeh, (see [18]). The concept of fuzzy sets is widely applied to logics, algebraic structures and computer sciences, (see [11, 13, 15]).

Therefore, there are many concepts that extend upon the notion of fuzzy sets. The one we consider in this paper is the concept of interval valued intuitionistic fuzzy sets, introduced by Atanassov and Gargov in 1989, (see [2]). Fuzzy sets may be applied not only to algebraic structures, but also to the concept of interval valued intuitionistic fuzzy sets. It can be applied to the study of Γ-semihypergroups and ternary semirings, (see [1, 9]).

In the present paper, we introduce various notions of interval valued intuitionistic fuzzy hyperideals of an ordered semihypergroup, as follows.

1. Interval valued intuitionistic fuzzy right (resp., left) hyperideals.
2. Interval valued intuitionistic fuzzy bi-hyperideals.
3. Interval valued intuitionistic fuzzy quasi-hyperideals.

Furthermore, we study their properties and show how they are interconnected.

2. Preliminaries

To make this paper a self-contained one, we introduce the basic concepts involved in this paper. This section is divided into two subsections. In the first subsection, the concept of ordered semihypergroups is provided. In the second subsection, we introduce the notion of interval valued intuitionistic fuzzy sets.

2.1. Ordered semihypergroups. In the study of algebras, algebraists usually try to extend well-known notions to more generalized concepts. In this paper we consider a generalized concept of semigroups, so-called ordered semihypergroups.

An *ordered hypergroupoid* is a structure $\langle S; \circ, \leq \rangle$ such that

1. $\langle S; \circ \rangle$ is a hypergroupoid, that is, $\circ\colon S \times S \to \mathrm{Sb}^*(S)$, where $\mathrm{Sb}^*(S)$ is the set of all nonempty subsets of S,

2. $\langle S; \leq \rangle$ is a partially ordered set, and
3. $x \leq y$ implies $u \circ x \leq u \circ y$ and $x \circ u \leq y \circ u$ for all $x, y, u \in S$.

We note that for every nonempty subsets A and B of S, $A \leq B$ means that for any $a \in A$, there exists $b \in B$ such that: $a \leq b$. Moreover, we denote $A \circ B := \bigcup_{(a,b) \in A \times B} a \circ b$. For any $c \in S$, instead of $\{c\} \circ A$ and $B \circ \{c\}$, we write $c \circ A$ and $B \circ c$, respectively.

Definition 2.1 ([7]). An ordered hypergroupoid $\langle S; \circ, \leq \rangle$ is said to be an *ordered semihypergroup* if

$$x \circ (y \circ z) = (x \circ y) \circ z$$

for all $x, y, z \in S$.

For simplicity, an ordered semihypergroup $\langle S; \circ, \leq \rangle$, will be written in its universe set as a bold letter $\mathbf{S}$.

An ordered semihypergroup $\mathbf{T} := \langle T; *, \preceq \rangle$ is an ordered subsemihypergroup of $\mathbf{S}$ if T is a nonempty subset of S, such that $* = \circ|_{T \times T}$ and $\preceq = \leq \cap (T \times T)$. For any set with $T \subseteq S$, we denote:

$$(T] := \{a \in S : a \leq t \text{ for some } t \in T\}.$$

Next, we introduce the core concepts which are widely and extensively investigated in the theory of ordered semihypergroups:

Let $\mathbf{S}$ be an ordered semihypergroup. A nonempty subset A of S satisfying

$$\text{for every } a \in S \text{ and } b \in B, a \leq b \text{ implies } a \in A$$

is said to be

1. a *right (resp., left) hyperideal* of $\mathbf{S}$ if $A \circ S \subseteq S$, (resp., $S \circ A \subseteq A$),
2. a *hyperideal* of $\mathbf{S}$ if it is both a right and a left hyperideal of $\mathbf{S}$,
3. a *bi-hyperideal* of $\mathbf{S}$ if $A \circ S \circ A \subseteq A$,
4. a *quasi-hyperideal* of $\mathbf{S}$ if $(A \circ S] \cap (S \circ A] \subseteq A$.

For more information about hyperideals of ordered semihypergroups, the readers are directed to the work of Heidari and Davvaz, (see [7]).

2.2. Interval valued intuitionistic fuzzy sets. The study of fuzzy sets was started by Zadeh in 1965. A *fuzzy set* on a nonempty set X is a mapping $\mu\colon X \to [0,1]$ from S to a unit closed interval, (see [18]). We denote the set of all fuzzy sets on X by $\mathsf{FS}(X)$. This concept can be applied not only for the study of algebraic systems, but also for computer science, logic, and automata. For two fuzzy sets $\mu_1, \mu_2 \in$

$\mathsf{FS}(X)$, a relation $\leq$ on $\mathsf{FS}(X)$ are defined by $\mu_1 \leq \mu_2$ if $\mu_1(x) \leq \mu_2(x)$ for all $x \in X$. We define binary operations min and max on $\mathsf{FS}(X)$ by $\min(\mu_1, \mu_2) := \min_{x\in X}(\mu_1(x), \mu_2(x))$

Let $[0,1]$ be a unit closed interval. An *interval number* γ *of* $[0,1]$ is a closed subinterval of $[0,1]$ of the form $[\gamma^-, \gamma^+]$, where $0 \leq \gamma^- \leq \gamma^+ \leq 1$. We denote the set of all interval numbers of $[0,1]$ by $D[0,1]$. A binary relation $\leq$ and binary operations min and max are defined on $D[0,1]$ as follows:

1. $\alpha \leq \beta$ if $\alpha^- \leq \beta^-$ and $\alpha^+ \leq \beta^+$.
2. $\min(\alpha, \beta) := [\min(\alpha^-, \beta^-), \min(\alpha^+, \beta^+)]$.
3. $\max(\alpha, \beta) := [\max(\alpha^-, \beta^-), \max(\alpha^+, \beta^+)]$.

An *interval valued fuzzy set* $\mathbf{f}$ on a nonempty set X is a mapping $\mathbf{f}\colon X \to D[0,1]$. This is denoted by $\mathsf{IVFS}(X)$, the set of all interval valued fuzzy sets on X.

Remark 2.2. For any interval valued fuzzy set $\mathbf{f}$ on X, it can be determined by two fuzzy sets $\mathbf{f}^-$ and $\mathbf{f}^+$ on X in such a way that $\mathbf{f}^- \leq \mathbf{f}^+$. That is, $\mathbf{f}(x) = [\mathbf{f}^-(x), \mathbf{f}^+(x)]$ for all $x \in X$, (see [19]).

Let $\mathbf{f}$ and $\mathbf{g}$ be interval valued fuzzy sets on X. A binary relation $\leq$ on $\mathsf{IVFS}(X)$ is defined by $\mathbf{f} \leq \mathbf{g}$ if $\mathbf{f}^-(x) \leq \mathbf{g}^-(x)$ and $\mathbf{f}^+(x) \leq \mathbf{g}^+(x)$ for all $x \in X$. In particular, since every $t \in [0,1]$ can be considered as an interval $[t,t]$, for any $\mathbf{f} \in \mathsf{IVFS}(X)$, $\mathbf{f} \leq t$ whenever $\mathbf{f}^-(x) \leq t$ and $\mathbf{f}^+(x) \leq t$ for all $x \in X$.

Therefore, we can see that the concept of interval valued fuzzy sets is a generalized concept of fuzzy sets in the sense that every fuzzy set is an interval valued fuzzy set. Indeed, a fuzzy set $\mathbf{f}$ on X can be considered as $\mathbf{f}(x) = [\mathbf{f}(x), \mathbf{f}(x)]$ for all $x \in X$. The special interval valued fuzzy sets $\mathbf{0}$ and $\mathbf{1}$ are defined by $\mathbf{0} \mapsto [0,0] = 0$ and $\mathbf{1} \mapsto [1,1] = 1$.

In 1989, Atanassov and Gargov extended the concept of interval valued fuzzy sets into the notion of interval valued intuitionistic fuzzy sets, (see [2]). Let X be a nonempty set and $\underline{\mathbf{f}}\colon X \to D[0,1] \times D[0,1]$. We can see that $\underline{\mathbf{f}}$ can be considered by a pair $(\mathbf{f}_1, \mathbf{f}_2)$ of two interval valued fuzzy sets on X.

Definition 2.3 ([2]). A mapping $\underline{\mathbf{f}}\colon X \to D[0,1] \times D[0,1]$ is an *interval valued intuitionistic fuzzy set* on X if $0 \leq \mathbf{f}_1^- + \mathbf{f}_2^- \leq 1$ and $0 \leq \mathbf{f}_1^+ + \mathbf{f}_2^+ \leq 1$. The *degree of membership* and the *degree of non-membership* to $\underline{\mathbf{f}}$ is used to denote $\mathbf{f}_1$ and $\mathbf{f}_2$, respectively. We denote the set of all interval valued intuitionistic fuzzy sets on X by $\mathsf{IVIFS}(X)$.

The special elements $\underline{\mathbf{0}}, \underline{\mathbf{1}} \in \mathsf{IVIFS}(X)$ are defined by $\underline{\mathbf{0}} \mapsto (\mathbf{0}, \mathbf{1})$ and $\underline{\mathbf{1}} \mapsto (\mathbf{1}, \mathbf{0})$. As such, it is not difficult to see that every interval valued fuzzy set on X is an interval valued intuitionistic fuzzy set on X. That is, if $\mathbf{f}$ is an interval valued fuzzy set on X, then $\underline{\mathbf{f}} = (\mathbf{f}, \mathbf{0})$ is also an interval valued intuitionistic fuzzy set on X as well.

3. Results

Let $\mathbf{S}$ be an ordered semihypergroup. We will call the element $\underline{\mathbf{f}} \in \mathsf{IVIFS}(S)$, where S is a universe set of an ordered semihypergroup $\mathbf{S}$, by an interval valued intuitionistic fuzzy set of $\mathbf{S}$. We also denote the set of all such mappings by $\mathsf{IVIFS}(\mathbf{S})$. We investigate some special kinds of interval valued intuitionistic fuzzy sets of $\mathbf{S}$ in this section.

Let $\mathbf{S}$ be an ordered semihypergroup. For any $a \in S$, we define the set:

$$A_a := \{(x, y) \in S \times S : a \leq x \circ y\}.$$

For any $\mathbf{s} = [\mathbf{s}_1, \mathbf{s}_2], \mathbf{t} = [\mathbf{t}_1, \mathbf{t}_2] \in D[0, 1]$ and $\underline{\mathbf{f}} \in \mathsf{IVIFS}(\mathbf{S})$, the set

$$\operatorname{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) := \{x \in S : \mathbf{f}_1(x) \geq \mathbf{s}(x) \text{ and } \mathbf{f}_2(x) \leq \mathbf{t}(x)\}$$

is called a *level subset* of $\underline{\mathbf{f}}$. On the set $\mathsf{IVIFS}(\mathbf{S})$, we define three binary operations $\cap$, $\cup$ and $*$ by

$$\underline{\mathbf{f}} \cap \underline{\mathbf{g}} := (\min\{\mathbf{f}_1, \mathbf{g}_1\}, \max\{\mathbf{f}_2, \mathbf{g}_2\}),$$

$$\underline{\mathbf{f}} \cup \underline{\mathbf{g}} := (\max\{\mathbf{f}_1, \mathbf{g}_1\}, \min\{\mathbf{f}_2, \mathbf{g}_2\})$$

and

$$\underline{\mathbf{f}} * \underline{\mathbf{g}} := (\mathbf{c}_1, \mathbf{c}_2),$$

where

$$\mathbf{c}_1(a) := \begin{cases} \sup_{(x,y) \in A_a}\{\min\{\mathbf{f}_1(x), \mathbf{g}_1(y)\}\} & \text{if } A_a \neq \emptyset \\ \mathbf{0} & \text{otherwise} \end{cases} \tag{1}$$

for all $a \in X$, and

$$\mathbf{c}_2(b) := \begin{cases} \inf_{(x,y) \in A_b}\{\max\{\mathbf{f}_2(x), \mathbf{g}_2(y)\}\} & \text{if } A_b \neq \emptyset \\ \mathbf{1} & \text{otherwise} \end{cases} \tag{2}$$

for all $b \in X$. Furthermore, for any $\underline{\mathbf{f}}, \underline{\mathbf{g}} \in \mathsf{IVIFS}(\mathbf{S})$, a binary relation $\leq$ on $\mathsf{IVIFS}(\mathbf{S})$ is defined by $\underline{\mathbf{f}} \leq \underline{\mathbf{g}}$ if $\mathbf{f}_1 \leq \mathbf{g}_1$ and $\mathbf{g}_2 \leq \mathbf{f}_2$. In particular, for any $t \in [0, 1]$, $([t, t], [t, t]) = (t, t) \geq \underline{\mathbf{f}}$ if $t \leq \mathbf{f}_1^-, \mathbf{f}_1^+$ and $\mathbf{f}_2^-, \mathbf{f}_2^+ \leq t$, (see [2]).

Definition 3.1. Let $\mathbf{S}$ be an ordered semihypergroup. An interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ is called an *interval valued intuitionistic fuzzy subsemihypergroup* of $\mathbf{S}$ if for any $x, y \in S$,

1. $\inf_{z\in x\circ y}\{\mathbf{f}_1(z)\} \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(y)\}$,
2. $\sup_{z\in x\circ y}\{\mathbf{f}_2(z)\} \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(y)\}$.

Example 3.2. Let $S = \{a, b, c\}$. Define the hyperoperation $\circ$ on S by the following table:

$\circ$	a	b	c
a	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a, c\}$	$\{a, c\}$	$\{a, c\}$

Define an order on S as follows:

$$\leq := \{(a, b), (a, c)\} \cup \Delta_S,$$

where Δ_S is an equality relation on S. That is, $\Delta_S := \{(x, x) \in S \times S : x \in S\}$. Then, $\mathbf{S} := \langle S; \circ, \leq\rangle$ is an ordered semihypergroup. We define an interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ by:

$$\underline{\mathbf{f}}(x) := \begin{cases} ([0.6, 0.8], [0.1, 0.2]) & \text{if } x = a, \\ ([0.6, 0.7], [0.1, 0.2]) & \text{if } x = b, \\ ([0.4, 0.7], [0, 0.15]) & \text{if } x = c, \end{cases}$$

for all $x \in S$. Therefore, $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy subsemihypergroup of $\mathbf{S}$.

Definition 3.3. Let $\mathbf{S}$ be an ordered semihypergroup. An interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ is called an *interval valued intuitionistic fuzzy right (resp., left) hyperideal* of $\mathbf{S}$ if for any $x, y \in S$,

1. $\inf_{z\in x\circ y}\{\mathbf{f}_1(z)\} \geq \mathbf{f}_1(x)$ (resp., $\inf_{z\in x\circ y}\{\mathbf{f}_1(z)\} \geq \mathbf{f}_1(y)$),
2. $\sup_{z\in x\circ y}\{\mathbf{f}_2(z)\} \leq \mathbf{f}_2(x)$ (resp., $\sup_{z\in x\circ y}\{\mathbf{f}_2(z)\} \leq \mathbf{f}_2(y)$),
3. $x \leq y$ implies $\underline{\mathbf{f}}(x) \geq \underline{\mathbf{f}}(y)$.

If an interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ is both an interval valued intuitionistic fuzzy left and an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$, then $\underline{\mathbf{f}}$ is said to be an *interval valued intuitionistic fuzzy hyperideal* of $\mathbf{S}$.

Example 3.4. Let $S = \{a, b, c\}$. Define the hyperoperation $\circ$ on S by the following table:

$\circ$	a	b	c
a	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a, b\}$	$\{c\}$

Define an order on S as follows:

$$\leq := \{(a, b)\} \cup \Delta_S.$$

Then, $\mathbf{S} := \langle S; \circ, \leq \rangle$ is an ordered semihypergroup. We define an interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ by:

$$\underline{\mathbf{f}}(x) := \begin{cases} ([0.3, 0.7], [0.1, 0.3]) & \text{if } x = a, \\ ([0.1, 0.5], [0.1, 0.3]) & \text{if } x = b, \\ ([0.3, 0.7], [0.1, 0.2]) & \text{if } x = c, \end{cases}$$

for all $x \in S$. Then, $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy left hyperideal of $\mathbf{S}$. But, $\underline{\mathbf{f}}$ is not an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$ since $\inf_{u \in c \circ b}\{\mathbf{f}_1(u)\} = \mathbf{f}_1(b) < \mathbf{f}_1(c)$.

Example 3.5. Let $\underline{\mathbf{f}}$ be as in Example 3.2. Then it is not difficult to show that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy hyperideal of $\mathbf{S}$.

Definition 3.6. Let $\mathbf{S}$ be an ordered semihypergroup. An interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ is called an *interval valued intuitionistic fuzzy bi-hyperideal* of $\mathbf{S}$ if it is an interval valued intuitionistic fuzzy subsemihypergroup of $\mathbf{S}$. Then, for any $x, y, z \in S$,

1. $\inf_{u \in x \circ y \circ z}\{\mathbf{f}_1(u)\} \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(z)\}$,
2. $\sup_{u \in x \circ y \circ z}\{\mathbf{f}_2(u)\} \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}$,
3. $x \leq y$ implies $\underline{\mathbf{f}}(x) \geq \underline{\mathbf{f}}(y)$.

Example 3.7. Let $S = \{a, b, c, d, e\}$. Define the hyperoperation $\circ$ on S by the following table:

$\circ$	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$
b	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$
c	$\{a\}$	$\{a\}$	$\{a, b, c\}$	$\{a\}$	$\{a, b, c\}$
d	$\{a, b, d\}$	$\{a, b, d\}$	S	$\{a, b, d\}$	S
e	$\{a, b, d\}$	$\{a, b, d\}$	S	$\{a, b, d\}$	S

Define an order on S as follows:

$$\leq := \{(a, b), (a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, e), (d, e)\} \cup \Delta_S$$

Then, $\mathbf{S} := \langle S; \circ, \leq \rangle$ is an ordered semihypergroup. We define an interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ by

$$\underline{\mathbf{f}}(x) := \begin{cases} ([0.7, 0.8], [0.1, 0.2]) & \text{if } x = a, \\ ([0.5, 0.6], [0.3, 0.4]) & \text{if } x = b, \\ ([0.1, 0.2], [0.7, 0.8]) & \text{if } x = c, d, e, \end{cases}$$

for all $x \in S$. Therefore, $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy bi-hyperideal of $\mathbf{S}$.

DEFINITION 3.8. Let $\mathbf{S}$ be an ordered semihypergroup. An interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ is called an *interval valued intuitionistic fuzzy quasi-hyperideal* of $\mathbf{S}$ if, for any $x, y \in S$,

1. $(\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}) \leq \underline{\mathbf{f}}$,
2. $x \leq y$ implies $\underline{\mathbf{f}}(x) \geq \underline{\mathbf{f}}(y)$.

EXAMPLE 3.9. Let $S = \{a, b, c, d, e\}$. Define the hyperoperation $\circ$ on S by the following table:

$\circ$	a	b	c	d	e
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a, b\}$	$\{a\}$	$\{a, d\}$	$\{a\}$
c	$\{a\}$	$\{a, e\}$	$\{a, c\}$	$\{a, c\}$	$\{a, e\}$
d	$\{a\}$	$\{a, b\}$	$\{a, d\}$	$\{a, d\}$	$\{a, b\}$
e	$\{a\}$	$\{a, e\}$	$\{a\}$	$\{a, c\}$	$\{a\}$

Define an order on S as follows:

$$\leq := \{(a, b), (a, c), (a, d), (a, e)\} \cup \Delta_S.$$

Therefore, $\mathbf{S} := \langle S; \circ, \leq \rangle$ is an ordered semihypergroup. We define an interval valued intuitionistic fuzzy set $\underline{\mathbf{f}}$ of $\mathbf{S}$ by

$$\underline{\mathbf{f}}(x) := \begin{cases} ([0.7, 0.8], [0.1, 0.2]) & \text{if } x = a, \\ ([0.2, 0.3], [0.5, 0.6]) & \text{if } x = b, d, \\ ([0.6, 0.7], [0.2, 0.3]) & \text{if } x = c, \\ ([0.5, 0.6], [0.3, 0.4]) & \text{if } x = e, \end{cases}$$

for all $x \in S$. Therefore, $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$. By simple calculation, we can see that $\underline{\mathbf{f}}$ is not an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$. Indeed, there are $b, c \in S$, such that $\inf_{z \in c \circ b}\{\mathbf{f}_1(z)\} < \mathbf{f}_1(c)$. Moreover, $\underline{\mathbf{f}}$ is not an interval valued intuitionistic fuzzy left hyperideal of $\mathbf{S}$ since there are $c, d \in S$, such that $\inf_{z \in d \circ c}\{\mathbf{f}_1(z)\} < \mathbf{f}_1(c)$.

In this section, we provide some characterizations of interval valued intuitionistic fuzzy hyperideals of an ordered semihypergroup in term of their level subsets (as mentioned above).

LEMMA 3.10. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}$ *be an interval valued intuitionistic fuzzy set of* $\mathbf{S}$. *Then* $\underline{\mathbf{f}}$ *is an interval valued intuitionistic fuzzy right (resp., left) hyperideal of* $\mathbf{S}$ *if, and only if, for every* $\mathbf{s}, \mathbf{t} \in D[0,1]$, *the nonempty level subset* $\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ *of* $\underline{\mathbf{f}}$ *is a right (resp., left) hyperideal of* $\mathbf{S}$.

Proof. Let $x, y \in S$. We illustrate first that $\inf_{z \in x \circ y}\{\mathbf{f}_1(z)\} \geq \mathbf{f}_1(x)$. Let $\mathbf{s} = \mathbf{f}_1(x)$. By our presumption, $\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{1}))$ is a right hyperideal of $\mathbf{S}$. Thus, we have that $x \circ y \subseteq \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{1}))$, since $x \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{1}))$. Then for every, $z \in x \circ y$, we have that $\mathbf{f}_1(z) \geq \mathbf{s}$. This implies that $\inf_{z \in x \circ y}\{\mathbf{f}_1(z)\} \geq \mathbf{f}_1(x) = \mathbf{s}$. Next, we illustrate that $\sup_{z \in x \circ y}\{\mathbf{f}_2(z)\} \leq \mathbf{f}_2(x)$. Now, we put $\mathbf{t} = \mathbf{f}_2(x)$. By the hyperideality of $\text{lev}(\underline{\mathbf{f}}; (\mathbf{0}, \mathbf{t}))$ and $x \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{0}, \mathbf{t}))$, $x \circ y \subseteq \text{lev}(\underline{\mathbf{f}}; (\mathbf{0}, \mathbf{t}))$. Thus, for every $z \in x \circ y$, we have $\mathbf{f}_2(z) \leq \mathbf{t}$. This implies that $\sup_{z \in x o y}\{\mathbf{f}_2(z)\} \leq \mathbf{f}_2(x)$. Finally, we assume that $x \leq y$. We put $\mathbf{u} = \mathbf{f}_1(y)$ and $\mathbf{v} = \mathbf{f}_2(y)$. Then, it is not difficult to see that $y \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. Thus, $x \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. This shows that $\underline{\mathbf{f}}(x) \geq \underline{\mathbf{f}}(y)$. Altogether, we have that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$.

Conversely, assume that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$. Let $\mathbf{u}, \mathbf{v} \in D[0,1]$, such that $\text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v})) \neq \emptyset$. Let $z \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v})) \circ S$. Then, $z \in x \circ y$ for some $x \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$ and $y \in S$. Since $\inf_{z \in x o y}\{\mathbf{f}_1(z)\} \geq \mathbf{f}_1(x) \geq \mathbf{u}$ and $\sup_{z \in x \circ y}\{\mathbf{f}_2(z)\} \leq \mathbf{f}_2(x) \leq \mathbf{v}$, we have $\mathbf{f}_1(z) \geq \mathbf{u}$ and $\mathbf{f}_2(z) \leq \mathbf{v}$. Therefore, $z \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. This implies that $\text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v})) \circ S \subseteq \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. Lastly, we let $a \in S$ and $b \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$, such that $a \leq b$. Then $\mathbf{f}_1(a) \geq \mathbf{f}_1(b) \geq \mathbf{u}$ and $\mathbf{f}_2(a) \leq \mathbf{f}_2(b) \leq \mathbf{v}$. Thus, $a \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. This shows that $\text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$ is a right hyperideal of $\mathbf{S}$. □

One of the special elements of $\mathsf{IVIFS}(\mathbf{S})$ of an ordered semihypergroup $\mathbf{S}$ is that $\underline{\mathbf{f}}_I$ the *characteristic function of* I, where I is a nonempty subset of S. It is defined by

$$\underline{\mathbf{f}}_I(x) = \begin{cases} (\mathbf{1}, \mathbf{0}) & \text{if } x \in I, \\ (\mathbf{0}, \mathbf{1}) & \text{if } x \notin I, \end{cases}$$

for all $x \in S$. We observe that for any $\mathbf{s}, \mathbf{t} \in D[0,1]$, $\text{lev}(\underline{\mathbf{f}}_I; (\mathbf{s}, \mathbf{t})) = I$. Therefore, by Lemma 3.10, we obtain the following corollary.

Corollary 3.11. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}_I$ *be the characteristic function of* I. *Then* $\underline{\mathbf{f}}_I$ *is an interval valued intuitionistic fuzzy right (resp., left) hyperideal of* $\mathbf{S}$ *if and only if* I *is a right (resp., left) hyperideal of* $\mathbf{S}$.

Lemma 3.12. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}$ *be an interval valued intuitionistic fuzzy set of* $\mathbf{S}$. *Then* $\underline{\mathbf{f}}$ *is an interval valued intuitionistic fuzzy bi-hyperideal of* $\mathbf{S}$ *if and only if for every* $\mathbf{s}, \mathbf{t} \in D[0,1]$, *the nonempty level subset* $\mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ *of* $\underline{\mathbf{f}}$ *is a bi-hyperideal of* $\mathbf{S}$.

Proof. ($\Leftarrow$): Let $x, y, z \in S$. Firstly, we show that $\inf_{u \in x \circ y \circ z}\{\mathbf{f}_1(u)\} \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(z)\}$. We put $\mathbf{s} = \min\{\mathbf{f}_1(x), \mathbf{f}_1(z)\}$. By assumption, we have that $\mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{1}))$ is a bi-hyperideal of $\mathbf{S}$. As such, it is not difficult to see that $x, z \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{1}))$. This implies that $x \circ y \circ z \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{1}))$. Therefore, for every $u \in x \circ y \circ z$, we obtain that $\mathbf{f}_1(u) \geq \mathbf{s}$, that is, $\inf_{u \in x \circ y \circ z}\{\mathbf{f}_1(u)\} \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(z)\}$. Next, we show $\sup_{u \in x \circ y \circ z}\{\mathbf{f}_2(u)\} \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}$. Let $\mathbf{t} = \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}$. Since $\mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{0}, \mathbf{t}))$ is a bi-hyperideal of $\mathbf{S}$, $x \circ y \circ z \subseteq \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{0}, \mathbf{t}))$. This implies that for any $u \in x \circ y \circ z$, we have $\mathbf{f}_2(u) \leq \mathbf{t}$. Thus, $\sup_{u \in x \circ y \circ z}\{\mathbf{f}_2(u)\} \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}$. Finally, we suppose that $x \leq y$. Similar to Lemma 3.10, we put $\mathbf{u} = \mathbf{f}_1(y)$ and $\mathbf{v} = \mathbf{f}_2(y)$. Therefore, it is not difficult to see that $y \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. Thus, $x \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$, by the hyperideality of $\mathbf{S}$. This shows that $\underline{\mathbf{f}}(x) \geq \underline{\mathbf{f}}(y)$. Altogether, we have that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy bi-hyperideal of $\mathbf{S}$.

($\Rightarrow$): Assume that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy bi-hyperideal of $\mathbf{S}$. Let $\mathbf{s}, \mathbf{t} \in D[0,1]$, such that $\mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \neq \emptyset$. We illustrate that $\mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \circ S \circ \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \subseteq \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$. Let $u \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \circ S \circ \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$. Therefore, $u \in x \circ y \circ z$ for some $x, z \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ and $y \in S$. Since

$$\inf_{u \in x \circ y \circ z}\{\mathbf{f}_1(u)\} \geq \mathbf{f}_1(x) \geq \mathbf{s} \quad \text{and} \quad \sup_{u \in x \circ y \circ z}\{\mathbf{f}_2(u)\} \leq \mathbf{f}_2(z) \leq \mathbf{t},$$

we have $\mathbf{f}_1(u) \geq \mathbf{s}$ and $\mathbf{f}_2(u) \leq \mathbf{t}$. Thus, $u \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$. Finally, we let $a \in S$ and $b \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$, such that $a \leq b$. Then $\mathbf{f}_1(a) \geq \mathbf{f}_1(b) \geq \mathbf{s}$ and $\mathbf{f}_2(a) \leq \mathbf{f}_2(b) \leq \mathbf{t}$. Thus, $a \in \mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$. That is, $\mathrm{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ is a bi-hyperideal of $\mathbf{S}$. □

Similar to Corollary 3.11, we obtain the following result.

Corollary 3.13. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}_I$ *be the characteristic function of* I. *Then* $\underline{\mathbf{f}}_I$ *is an interval valued intuitionistic fuzzy bi-hyperideal of* $\mathbf{S}$ *if and only if* I *is a bi-hyperideal of* $\mathbf{S}$.

LEMMA 3.14. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}$ *be an interval valued intuitionistic fuzzy set of* $\mathbf{S}$*. Then* $\underline{\mathbf{f}}$ *is an interval valued intuitionistic fuzzy quasi-hyperideal of* $\mathbf{S}$ *if and only if for every* $\mathbf{s}, \mathbf{t} \in D[0,1]$*, the nonempty level subset* $\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ *of* $\underline{\mathbf{f}}$ *is a quasi-hyperideal of* $\mathbf{S}$*.*

Proof. ($\Leftarrow$): Let $x \in S$. We illustrate that $((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(x) \leq \underline{\mathbf{f}}(x)$. Clearly, the proof is achieved if $A_x = \emptyset$. Suppose that $A_x \neq \emptyset$. Then there are $a, b \in S$, such that $x \leq a \circ b$. We let $\mathbf{s} = \min\{\mathbf{f}_1(a), \mathbf{f}_1(b)\}$ and $\mathbf{t} = \max\{\mathbf{f}_2(a), \mathbf{f}_2(b)\}$. By our presumption, $\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ is a quasi-hyperideal of $\mathbf{S}$. It is clear that $a, b \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$. Then, we have $x \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ since $x \in (\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \circ S] \cap (S \circ \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))]$. Therefore, $\mathbf{f}_1(x) \geq \mathbf{s}$ and $\mathbf{f}_2(x) \leq \mathbf{t}$. This implies that for any $(u, v) \in A_x$, we have $\mathbf{f}_1(x) \geq \min\{\mathbf{f}_1(u), \mathbf{f}_1(v)\}$ and $\mathbf{f}_2(x) \leq \max\{\mathbf{f}_2(u), \mathbf{f}_2(v)\}$. Thus,

$$\begin{aligned}
&((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(x) \\
&= \min\{(\underline{\mathbf{f}} * \underline{\mathbf{1}})(x), (\underline{\mathbf{1}} * \underline{\mathbf{f}})(x)\} \\
&= (\min\{\sup_{(u,v)\in A_x} \{\min\{\mathbf{f}_1(u), \mathbf{1}(v)\}\}, \sup_{(u,v)\in A_x} \{\min\{\mathbf{1}(u), \mathbf{f}_1(v)\}\}\}, \\
&\qquad \max\{\inf_{(u,v)\in A_x} \{\max\{\mathbf{f}_2(u), \mathbf{0}(v)\}\}, \inf_{(u,v)\in A_x} \{\max\{\mathbf{0}(u), \mathbf{f}_2(v)\}\}\}) \\
&= (\min\{\sup_{(u,v)\in A_x} \{\min\{\mathbf{f}_1(u)\}\}, \sup_{(u,v)\in A_x} \{\min\{\mathbf{f}_1(v)\}\}\}, \\
&\qquad \max\{\inf_{(u,v)\in A_x} \{\max\{\mathbf{f}_2(u)\}\}, \inf_{(u,v)\in A_x} \{\max\{\mathbf{f}_2(v)\}\}\}) \\
&= (\sup_{(u,v)\in A_x} \{\min\{\mathbf{f}_1(u), \mathbf{f}_1(v)\}\}, \inf_{(u,v)\in A_x} \{\max\{\mathbf{f}_2(u), \mathbf{f}_2(v)\}\}) \\
&\leq (\mathbf{f}_1(x), \mathbf{f}_2(x)) \\
&= \underline{\mathbf{f}}(x).
\end{aligned}$$

Therefore, $(\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}) \leq \underline{\mathbf{f}}$. Next, we let $a, b \in S$, such that $a \leq b$. We suppose that $\mathbf{u} = \mathbf{f}_1(b)$ and $\mathbf{v} = \mathbf{f}_2(b)$. Then $b \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. Thus, $a \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$ by the hyperideality of $\text{lev}(\underline{\mathbf{f}}; (\mathbf{u}, \mathbf{v}))$. This shows that $\underline{\mathbf{f}}(a) \geq \underline{\mathbf{f}}(b)$. Altogether, we have that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$.

($\Rightarrow$): Assume that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$. Let $\mathbf{s}, \mathbf{t} \in D[0,1]$, such that $\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \neq \emptyset$. We illustrate that $(\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \circ S] \cap (S \circ \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))] \subseteq \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$. Let $x \in (\text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t})) \circ S] \cap (S \circ \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))]$. Then $x \leq a \circ u$ and $x \leq v \circ b$ for some $a, b \in \text{lev}(\underline{\mathbf{f}}; (\mathbf{s}, \mathbf{t}))$ and $u, v \in S$. This means that $A_x \neq \emptyset$, more precisely, $(a,u),(v,b)\in A_x$. Then

$$\begin{aligned}(\underline{\mathbf{f}}*\underline{\mathbf{1}})(x) &= (\sup_{(u,v)\in A_x}\{\min\{\mathbf{f}_1(u),\mathbf{1}(v)\}\}, \inf_{(u,v)\in A_x}\{\max\{\mathbf{f}_2(u),\mathbf{0}(v)\}\})\\ &\geq (\mathbf{f}_1(a),\mathbf{f}_2(a))\\ &\geq (\mathbf{s},\mathbf{t})\end{aligned}$$

and

$$\begin{aligned}(\underline{\mathbf{1}}*\underline{\mathbf{f}})(x) &= (\sup_{(u,v)\in A_x}\{\min\{\mathbf{1}(u),\mathbf{f}_1(v)\}\}, \inf_{(u,v)\in A_x}\{\max\{\mathbf{0}(u),\mathbf{f}_2(v)\}\})\\ &\geq (\mathbf{f}_1(b),\mathbf{f}_2(b))\\ &\geq (\mathbf{s},\mathbf{t}).\end{aligned}$$

Thus, $((\underline{\mathbf{f}}*\underline{\mathbf{1}})\cap(\underline{\mathbf{1}}*\underline{\mathbf{f}}))(x)\geq(\mathbf{s},\mathbf{t})$. Therefore, $\underline{\mathbf{f}}(x)\geq((\underline{\mathbf{f}}*\underline{\mathbf{1}})\cap(\underline{\mathbf{1}}*\underline{\mathbf{f}}))(x)\geq(\mathbf{s},\mathbf{t})$. This means that $x\in\mathrm{lev}(\underline{\mathbf{f}};(\mathbf{s},\mathbf{t}))$. Now, we let $a\in S$ and $b\in\mathrm{lev}(\underline{\mathbf{f}};(\mathbf{s},\mathbf{t}))$, such that $a\leq b$. Then $\mathbf{f}_1(a)\geq\mathbf{f}_1(b)\geq\mathbf{s}$ and $\mathbf{f}_2(a)\leq\mathbf{f}_2(b)\leq\mathbf{t}$. Thus, $a\in\mathrm{lev}(\underline{\mathbf{f}};(\mathbf{s},\mathbf{t}))$. Therefore, we have that $\mathrm{lev}(\underline{\mathbf{f}};(\mathbf{s},\mathbf{t}))$ is a quasi-hyperideal of $\mathbf{S}$. □

From Example 3.9, we can see that there exists an interval valued intuitionistic fuzzy quasi-hyperideal which is not an interval valued intuitionistic fuzzy right hyperideal nor an interval valued intuitionistic fuzzy left hyperideal. However, the following result shows that the converse is not true.

THEOREM 3.15. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then, every interval valued intuitionistic fuzzy right (resp., left) hyperideal of* $\mathbf{S}$ *is an interval valued intuitionistic fuzzy quasi-hyperideal of* $\mathbf{S}$.

Proof. Let $\underline{\mathbf{f}}$ be an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$. We show that $((\underline{\mathbf{f}}*\underline{\mathbf{1}})\cap(\underline{\mathbf{1}}*\underline{\mathbf{f}}))(a)\leq\underline{\mathbf{f}}(a)$ for all $a\in S$. Suppose that $(\underline{\mathbf{f}}*\underline{\mathbf{1}})=(\mathbf{c}_1,\mathbf{c}_2)$ and $(\underline{\mathbf{1}}*\underline{\mathbf{f}})=(\mathbf{d}_1,\mathbf{d}_2)$. That is, $(\underline{\mathbf{f}}*\underline{\mathbf{1}})\cap(\underline{\mathbf{1}}*\underline{\mathbf{f}})=(\min(\mathbf{c}_1,\mathbf{d}_1),\max(\mathbf{c}_2,\mathbf{d}_2))$. Let $a\in S$. The proof is achieved if $A_a=\emptyset$. As such, we let $(x,y)\in A_a$. Then $a\leq x\circ y$, that is, $a\leq z$ for some $z\in x\circ y$. This implies that:

$$\mathbf{f}_1(a)\geq\mathbf{f}_1(z)\geq\inf_{z\in x\circ y}\{\mathbf{f}_1(z)\}\geq\mathbf{f}_1(x)$$

and

$$\mathbf{f}_2(a)\leq\mathbf{f}_2(z)\leq\sup_{z\in x\circ y}\{\mathbf{f}_2(z)\}\geq\mathbf{f}_2(x).$$

Therefore, we obtain

$$\begin{aligned}\mathbf{f}_1(a) &\geq \sup_{z\in x\circ y}\{\mathbf{f}_1(x)\}\\ &= \sup_{z\in x\circ y}\{\min\{\mathbf{f}_1(x),\mathbf{1}(y)\}\}\\ &= \mathbf{c}_1(a)\\ &\geq \min(\mathbf{c}_1,\mathbf{d}_1)(a)\end{aligned}$$

and

$$\begin{aligned}\mathbf{f}_2(a) &\leq \inf_{z\in x\circ y}\{\mathbf{f}_2(x)\}\\ &= \inf_{z\in x\circ y}\{\max\{\mathbf{f}_1(x),\mathbf{0}(y)\}\}\\ &= \mathbf{c}_2(a)\\ &\leq \max(\mathbf{c}_2,\mathbf{d}_2)(a).\end{aligned}$$

This shows that $((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(a) \leq \underline{\mathbf{f}}(a)$ for all $a \in S$. Thus, the proof is completed. □

We observe in Example 3.7 that $\underline{\mathbf{f}}$ is not an interval valued intuitionistic fuzzy quasi-hyperideal because there exists $b \in S$, such that $\min(\mathbf{c}_1,\mathbf{d}_1)(a) = \mathbf{f}_1(a) > \mathbf{f}_1(b)$, where $\min(\mathbf{c}_1,\mathbf{d}_1)(a)$ is the degree of membership to $((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(a)$. This observation illustrates that an interval valued intuitionistic fuzzy bi-hyperideal need not to be an interval valued intuitionistic fuzzy quasi-hyperideal. The following theorem provides an explanation.

THEOREM 3.16. *Let* $\mathbf{S}$ *be an ordered semihypergroup. Then every interval valued intuitionistic fuzzy quasi-hyperideal of* $\mathbf{S}$ *is an interval valued intuitionistic fuzzy bi-hyperideal of* $\mathbf{S}$.

Proof. Let $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$. Let $x, y, z \in S$. Firstly, we illustrate that $\inf_{u\in x\circ y\circ z}\{\mathbf{f}_1(u)\} \geq \min\{\mathbf{f}_1(x),\mathbf{f}_1(z)\}$. Suppose that $\underline{\mathbf{f}} * \underline{\mathbf{1}} = (\mathbf{c}_1,\mathbf{c}_2)$ and $\underline{\mathbf{1}} * \underline{\mathbf{f}} = (\mathbf{d}_1,\mathbf{d}_2)$. Since $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$, we have that $\mathbf{f}_1 \geq \min(\mathbf{c}_1,\mathbf{d}_1)$ and $\mathbf{f}_2 \leq \max(\mathbf{c}_2,\mathbf{d}_2)$. For any $a \leq t$ for some $t \in x \circ y \circ z$, we consider

$$\begin{aligned}\mathbf{c}_1(a) &= \sup_{(u,v)\in A_a}\{\min\{\mathbf{f}_1(u), \mathbf{1}(v)\}\}\\ &\geq \min\{\mathbf{f}_1(x), \mathbf{1}(v) : v \in y \circ z\}\\ &= \mathbf{f}_1(x)\end{aligned}$$

and

$$\begin{aligned}\mathbf{d}_1(a) &= \sup_{(u,v)\in A_a}\{\min\{\mathbf{1}(u), \mathbf{f}_1(v)\}\}\\ &\geq \min\{\mathbf{1}(u), \mathbf{f}_1(z) : u \in x \circ y\}\\ &= \mathbf{f}_1(z).\end{aligned}$$

This implies that $\min(\mathbf{c}_1, \mathbf{d}_1)(a) \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(z)\}$. Thus,

$$\inf_{u\in x\circ y\circ z}\{\mathbf{f}_1(u)\} \geq \min(\mathbf{c}_1, \mathbf{d}_1)(a) \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(z)\}.$$

Lastly, we illustrate that $\sup_{u\in x\circ y\circ z}\{\mathbf{f}_2(u)\} \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}$. For any $a \leq t$ for some $t \in x \circ y \circ z$, we now consider

$$\begin{aligned}\mathbf{c}_2(a) &= \inf_{(u,v)\in A_a}\{\max\{\mathbf{f}_2(u), \mathbf{0}(v)\}\}\\ &\leq \max\{\mathbf{f}_2(x), \mathbf{0}(v) : v \in y \circ z\}\\ &= \mathbf{f}_2(x)\end{aligned}$$

and

$$\begin{aligned}\mathbf{d}_2(a) &= \inf_{(u,v)\in A_a}\{\max\{\mathbf{0}(u), \mathbf{f}_2(v)\}\}\\ &\leq \max\{\mathbf{0}(u), \mathbf{f}_2(z) : u \in x \circ y\}\\ &= \mathbf{f}_2(z).\end{aligned}$$

Then $\max(\mathbf{c}_2, \mathbf{d}_2)(a) \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}$. Thus, we obtain

$$\sup_{u\in x\circ y\circ z}\{\mathbf{f}_2(u)\} \leq \max(\mathbf{c}_2, \mathbf{d}_2)(a) \leq \max\{\mathbf{f}_2(x), \mathbf{f}_2(z)\}.$$

Therefore, we have that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy bi-hyperideal of $\mathbf{S}$. □

We now have a question arising from the previously illustrated results: how do the concepts of interval valued intuitionistic fuzzy quasi-hyperideals and interval valued intuitionistic fuzzy bi-hyperideals coincide?

Let us consider a class of ordered semihypergroups called regular ordered semihypergroups. Let $\mathbf{S}$ be an ordered semihypergroup. The element $a \in S$ is said to be *regular* if there exists $x \in S$ such that $a \leq a \circ x \circ a$. An ordered semihypergroup $\mathbf{S}$ is *regular* if every element in S is regular. We address our question by the following theorem.

THEOREM 3.17. *Let* $\mathbf{S}$ *be a regular ordered semihypergroup. Then* $\underline{\mathbf{f}}$ *is an interval valued intuitionistic fuzzy quasi-hyperideal of* $\mathbf{S}$ *if and only if* $\underline{\mathbf{f}}$ *is an interval valued intuitionistic fuzzy bi-hyperideal of* $\mathbf{S}$.

Proof. By Theorem 3.16, it remains to verify that any interval valued intuitionistic fuzzy bi-hyperideal is an interval valued intuitionistic fuzzy quasi-hyperideal. Let $\underline{\mathbf{f}}$ be an interval valued intuitionistic fuzzy bi-hyperideal of $\mathbf{S}$. It needs to be illustrated that $(\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}) \leq \underline{\mathbf{f}}$. Let $a \in S$. The proof is achieved whenever $A_a = \emptyset$. Assuming that $A_a \neq \emptyset$, let us suppose that $\underline{\mathbf{f}} * \underline{\mathbf{1}} = (\mathbf{c}_1, \mathbf{c}_2)$ and $\underline{\mathbf{1}} * \underline{\mathbf{f}} = (\mathbf{d}_1, \mathbf{d}_2)$. We consider the following cases.

1. $\mathbf{c}_1(a) \leq \mathbf{f}_1(a)$ and $\mathbf{c}_2(a) \geq \mathbf{f}_2(a)$.
2. $\mathbf{c}_1(a) \leq \mathbf{f}_1(a)$ and $\mathbf{c}_2(a) < \mathbf{f}_2(a)$.
3. $\mathbf{c}_1(a) > \mathbf{f}_1(a)$ and $\mathbf{c}_2(a) \geq \mathbf{f}_2(a)$.
4. $\mathbf{c}_1(a) > \mathbf{f}_1(a)$ and $\mathbf{c}_2(a) < \mathbf{f}_2(a)$.

We observe that the proof is achieved if $\mathbf{c}_1(a) \leq \mathbf{f}_1(a)$ and $\mathbf{c}_2(a) \geq \mathbf{f}_2(a)$. Indeed,

$$\begin{aligned}((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(a) &= (\min(\mathbf{c}_1, \mathbf{d}_1)(a), \max(\mathbf{c}_2, \mathbf{d}_2)(a)) \\ &\leq (\mathbf{c}_1(a), \mathbf{c}_2(a)) \\ &\leq (\mathbf{f}_1(a), \mathbf{f}_2(a)) \\ &= \underline{\mathbf{f}}(a).\end{aligned}$$

Thus, we assume that $\mathbf{c}_1(a) > \mathbf{f}_1(a)$ and $\mathbf{c}_2(a) < \mathbf{f}_2(a)$. We shall illustrate that $\mathbf{d}_1(a) \leq \mathbf{f}_1(a)$ and $\mathbf{d}_2(a) \geq \mathbf{f}_2(a)$. Let $(u, v) \in A_a$. Since $\mathbf{c}_1(a) > \mathbf{f}_1(a)$, there exists $(x, y) \in A_a$, such that $\min\{\mathbf{f}_1(x), \mathbf{1}(y)\} = \mathbf{f}_1(x) > \mathbf{f}_1(a)$. Since $a \in S$ and by the regularity of $\mathbf{S}$, there exists $t \in S$ such that $a \leq a \circ t \circ a$. Now, we know that $a \leq u \circ v$ and $a \leq x \circ y$. Since $\mathbf{S}$ is an ordered semihypergroup, we have $a \leq x \circ y \circ t \circ u \circ v$. That is, $a \leq k$ for some $k \in x \circ y \circ t \circ u \circ v$. Then

$$\mathbf{f}_1(a) \geq \mathbf{f}_1(k) \geq \inf_{z \in x \circ y \circ t \circ u \circ v} \{\mathbf{f}_1(z)\} \geq \min\{\mathbf{f}_1(x), \mathbf{f}_1(v)\}.$$

It is clear that $\min\{\mathbf{f}_1(x), \mathbf{f}_1(v)\} = \mathbf{f}_1(v)$, otherwise, it contradicts the fact that $\mathbf{c}_1(a) > \mathbf{f}_1(a)$. Thus, $\mathbf{f}_1(a) \geq \mathbf{f}_1(v) = \min\{\mathbf{1}(u), \mathbf{f}_1(v)\}$. Now, since (u,v) is an arbitrary element of A_1, we see that

$$\mathbf{f}_1(a) \geq \sup_{(u,v)\in A_a} \{\min\{\mathbf{1}(u), \mathbf{f}_1(v)\}\} = \mathbf{d}_1(a).$$

On the other hand, since $\mathbf{c}_2(a) < \mathbf{f}_2(a)$, there exists $(x', y') \in A_a$, such that $\max\{\mathbf{f}_2(x'), \mathbf{0}(y')\} = \mathbf{f}_2(x') < \mathbf{f}_2(a)$. Since $a \leq x' \circ y'$, $a \leq u \circ v$ and $a \leq a \circ t \circ a$ for some $t \in S$, we have $a \leq x' \circ y' \circ t \circ u \circ v$. That is, $a \leq k$ for some $k \in x' \circ y' \circ t \circ u \circ v$. Then:

$$\mathbf{f}_2(a) \leq \mathbf{f}_2(k) \leq \sup_{z\in x'\circ y'\circ t\circ u\circ v} \{\mathbf{f}_2(z)\} \leq \max\{\mathbf{f}_2(x'), \mathbf{f}_2(v)\}.$$

If $\max\{\mathbf{f}_2(x'), \mathbf{f}_2(v)\} = \mathbf{f}_2(x')$, then $\mathbf{f}_2(a) \leq \mathbf{f}_2(x')$, which is a contradiction. Thus, we get $\max\{\mathbf{f}_2(x'), \mathbf{f}_2(v)\} = \mathbf{f}_2(v)$. This implies that $\mathbf{f}_2(a) \leq \mathbf{f}_2(v) = \max\{\mathbf{0}(u), \mathbf{f}_2(v)\}$. This means that:

$$\mathbf{f}_2(a) \leq \inf_{(u,v)\in A_a} \{\max\{\mathbf{0}(u), \mathbf{f}_2(v)\}\} = \mathbf{d}_2(a).$$

Altogether, we obtain that

$$\begin{aligned}((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(a) &= (\min(\mathbf{c}_1, \mathbf{d}_1)(a), \max(\mathbf{c}_2, \mathbf{d}_2)(a)) \\ &\leq (\mathbf{d}_1(a), \mathbf{d}_2(a)) \\ &\leq (\mathbf{f}_1(a), \mathbf{f}_2(a)) \\ &= \underline{\mathbf{f}}(a).\end{aligned}$$

By these arguments, we can see that if $\mathbf{c}_1(a) > \mathbf{f}_1(a)$, then $\mathbf{d}_1(a) \leq \mathbf{f}_1(a)$, and if $\mathbf{c}_2(a) < \mathbf{f}_2(a)$, then $\mathbf{d}_2(a) \geq \mathbf{f}_1(a)$. This leads us that $((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(a) \leq \underline{\mathbf{f}}(a)$. Therefore, $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$. □

The final major result that we illustrate concerns interval valued intuitionistic fuzzy right, left and quasi-hyperideals of an ordered semihypergroup. That is, we aim to prove that for any interval valued intuitionistic fuzzy quasi-hyperideal, it is a minimal of an interval valued intuitionistic fuzzy right and left hyperideal. We also aim to prove the converse. Firstly, we illustrate that for any interval valued intuitionistic fuzzy quasi-hyperideal, it is a minimal of an interval valued intuitionistic fuzzy right and left hyperideal. To do so, we provide some elements as follows:

PROPOSITION 3.18. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}$ *an interval valued intuitionistic fuzzy set of* $\mathbf{S}$*, satisfying the property that* *for any* $a, b \in S$, $\underline{\mathbf{f}}(a) \geq \underline{\mathbf{f}}(b)$ *whenever* $a \leq b$. *Then the following statements hold:*

1. $\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}})$ *is an interval valued intuitionistic fuzzy right hyperideal of* $\mathbf{S}$.
2. $\underline{\mathbf{f}} \cup (\underline{\mathbf{1}} * \underline{\mathbf{f}})$ *is an interval valued intuitionistic fuzzy left hyperideal of* $\mathbf{S}$.

Proof. We illustrate only that $\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}})$ is an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$. To show that $\underline{\mathbf{f}} \cup (\underline{\mathbf{1}} * \underline{\mathbf{f}})$ is an interval valued intuitionistic fuzzy left hyperideal of $\mathbf{S}$, the same logic sequence applies. Suppose that $\underline{\mathbf{f}} * \underline{\mathbf{1}} = \underline{\mathbf{c}}$ and $\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}}) = \underline{\mathbf{d}}$. To complete the proof we have to illustrate that for any $x, y \in S$, we have $\inf_{z \in x \circ y}\{\mathbf{d}_1(z)\} \geq \mathbf{d}_1(x)$ and $\sup_{z \in x \circ y}\{\mathbf{d}_2(z)\} \leq \mathbf{d}_2(x)$. Moreover, for any $x, y \in S$, we must have $\mathbf{d}_1(x) \geq \mathbf{d}_1(y)$ and $\mathbf{d}_2(x) \leq \mathbf{d}_2(y)$ provided that $x \leq y$.

Let $x, y \in S$ and $z \in x \circ y$. Then

$$\begin{aligned}
\mathbf{d}_1(z) &= \max\{\mathbf{f}_1(z), \mathbf{c}_1(z)\} \\
&\geq \mathbf{c}_1(z) \\
&= \sup_{(x,y) \in A_z} \{\min\{\mathbf{f}_1(x), \mathbf{1}(y)\}\} \\
&= \sup_{(x,y) \in A_z} \{\mathbf{f}_1(x)\} \\
&\geq \mathbf{f}_1(x).
\end{aligned} \tag{3}$$

Let $(a, b) \in A_x$. Since $\mathbf{S}$ is an ordered semihypergroup and $(x, y) \in A_z$, we have that $z \leq a \circ u$ for some $u \in b \circ y$. Then $\mathbf{c}_1(z) \geq \min\{\mathbf{f}_1(a), \mathbf{1}(u)\} = \mathbf{f}_1(a) = \min\{\mathbf{f}_1(a), \mathbf{1}(b)\}$. Thus,

$$\mathbf{d}_1(z) \geq \mathbf{c}_1(z) \geq \sup_{(a,b) \in A_x} \{\min\{\mathbf{f}_1(a), \mathbf{1}(b)\}\} = \mathbf{c}_1(x). \tag{4}$$

Since z is an arbitrary element of $x \circ y$ and Inequalities (3) and (4), we obtain that $\inf_{z \in x \circ y}\{\mathbf{d}_1(z)\} \geq \mathbf{d}_1(x)$. We observe that $\inf_{z \in x \circ y}\{\mathbf{d}_1(z)\} \geq \mathbf{d}_1(x)$ also holds if $A_x = \emptyset$.

Let $x, y \in S$ and $x \in x \circ y$. Then

$$
\begin{aligned}
\mathbf{d}_2(z) &= \min\{\mathbf{f}_2(z), \mathbf{c}_2(z)\} \\
&\leq \mathbf{c}_2(z) \\
&= \inf_{(x,y)\in A_z} \{\max\{\mathbf{f}_2(x), \mathbf{0}(y)\}\} \\
&= \inf_{(x,y)\in A_z} \{\mathbf{f}_2(x)\} \\
&\leq \mathbf{f}_2(x).
\end{aligned} \tag{5}
$$

Let $(a, b) \in A_x$. Since $z \leq x \circ y$, we have $z \leq a \circ u$ for some $u \in b \circ y$. Then $\mathbf{c}_2(z) \leq \max\{\mathbf{f}_2(a), \mathbf{0}(u)\} = \mathbf{f}_2(a) = \max\{\mathbf{f}_2(a), \mathbf{0}(b)\}$. Thus,

$$
\mathbf{d}_2(z) \leq \mathbf{c}_2(z) \leq \inf_{(a,b)\in A_x} \{\max\{\mathbf{f}_2(a), \mathbf{0}(b)\}\} = \mathbf{c}_2(x). \tag{6}
$$

By Inequalities (5) and (6), we obtain that $\sup_{z\in x\circ y}\{\mathbf{d}_2(z)\} \leq \mathbf{d}_2(x)$. It is not difficult to see that if $A_x = \emptyset$, then $\sup_{z\in x\circ y}\{\mathbf{d}_2(z)\} \leq \mathbf{d}_2(x)$.

Lastly, let $x, y \in S$ such that $x \leq y$. We illustrate that $\mathbf{d}_1(x) \geq \mathbf{d}_1(y)$ and $\mathbf{d}_2(x) \leq \mathbf{d}_2(y)$. It is clear that $A_y \subseteq A_x$ since $x \leq y$. This implies that

$$
\begin{aligned}
\mathbf{c}_1(x) &= \sup_{(a,b)\in A_x} \{\min\{\mathbf{f}_1(a), \mathbf{1}(b)\}\} \\
&\geq \sup_{(a,b)\in A_y} \{\min\{\mathbf{f}_1(a), \mathbf{1}(b)\}\} \\
&= \mathbf{c}_1(y)
\end{aligned} \tag{7}
$$

and

$$
\begin{aligned}
\mathbf{c}_2(y) &= \inf_{(a,b)\in A_y} \{\max\{\mathbf{f}_2(a), \mathbf{0}(b)\}\} \\
&\leq \inf_{(a,b)\in A_x} \{\max\{\mathbf{f}_2(a), \mathbf{0}(b)\}\} \\
&= \mathbf{c}_2(x).
\end{aligned} \tag{8}
$$

By our presumption and Inequalities (7) and (8), we have that:

$$
\mathbf{d}_1(x) = \max\{\mathbf{f}_1(x), \mathbf{c}_1(x)\} \geq \max\{\mathbf{f}_1(y), \mathbf{c}_1(y)\} = \mathbf{d}_1(y)
$$

and

$$
\mathbf{d}_2(x) = \min\{\mathbf{f}_2(x), \mathbf{c}_2(x)\} \leq \min\{\mathbf{f}_2(y), \mathbf{c}_2(y)\} = \mathbf{d}_2(y).
$$

That is, $(\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}}))(x) \geq (\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}}))(y)$.

Altogether, we obtain that $\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}})$ is an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$. □

Now, we introduce a useful lemma given by Kehayopulu and Tsingelis.

LEMMA 3.19 ([8, Lemma 9]). *If a, b and c are real numbers, then*
1. $\min\{a, \max\{b, c\}\} = \max\{\min\{a, b\}, \min\{a, c\}\}$,
2. $\max\{a, \min\{b, c\}\} = \min\{\max\{a, b\}, \max\{a, c\}\}$.

By the above lemma, we obtain a similar result in term of elements in $D[0, 1]$.

LEMMA 3.20. *If $\mathbf{f}, \mathbf{g}, \mathbf{h} \in D[0, 1]$, then*
1. $\min\{\mathbf{f}, \max\{\mathbf{g}, \mathbf{h}\}\} = \max\{\min\{\mathbf{f}, \mathbf{g}\}, \min\{\mathbf{f}, \mathbf{h}\}\}$,
2. $\max\{\mathbf{f}, \min\{\mathbf{g}, \mathbf{h}\}\} = \min\{\max\{\mathbf{f}, \mathbf{g}\}, \max\{\mathbf{f}, \mathbf{h}\}\}$.

Consequently, we obtain the following result:

PROPOSITION 3.21. *Let $\mathbf{S}$ be an ordered semihypergroup and $\underline{\mathbf{f}}, \underline{\mathbf{g}}, \underline{\mathbf{h}} \in \mathsf{IVIFS}(\mathbf{S})$. Then we have*

$$\underline{\mathbf{f}} \cap (\underline{\mathbf{g}} \cup \underline{\mathbf{h}}) = (\underline{\mathbf{f}} \cap \underline{\mathbf{g}}) \cup (\underline{\mathbf{f}} \cap \underline{\mathbf{h}}).$$

Proof. Let $x \in S$. By Lemma 3.20, we have

$$\min\{\mathbf{f}_1(x), \max\{\mathbf{g}_1(x), \mathbf{h}_1(x)\}\}$$
$$= \max\{\min\{\mathbf{f}_1(x), \mathbf{g}_1(x)\}, \min\{\mathbf{f}_1(x), \mathbf{h}_1(x)\}\}$$

and

$$\max\{\mathbf{f}_2(x), \min\{\mathbf{g}_2(x), \mathbf{h}_2(x)\}\}$$
$$= \min\{\max\{\mathbf{f}_2(x), \mathbf{g}_2(x)\}, \max\{\mathbf{f}_2(x), \mathbf{h}_2(x)\}\}.$$

That is, $\underline{\mathbf{f}} \cap (\underline{\mathbf{g}} \cup \underline{\mathbf{h}}) = (\underline{\mathbf{f}} \cap \underline{\mathbf{g}}) \cup (\underline{\mathbf{f}} \cap \underline{\mathbf{h}})$. □

This will allow us to prove our theorem.

THEOREM 3.22. *Let $\mathbf{S}$ be an ordered semihypergroup, and $\underline{\mathbf{f}}$ be an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$. Then $\underline{\mathbf{f}} = \underline{\mathbf{g}} \cap \underline{\mathbf{h}}$ for some interval valued intuitionistic fuzzy right hyperideal $\underline{\mathbf{g}}$ of $\mathbf{S}$, and interval valued intuitionistic fuzzy left hyperideal $\underline{\mathbf{h}}$ of $\mathbf{S}$.*

Proof. By Proposition 3.18, $\underline{\mathbf{g}} := \underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}})$ is an interval valued intuitionistic fuzzy right hyperideal of $\mathbf{S}$ and $\underline{\mathbf{h}} := \underline{\mathbf{f}} \cup (\underline{\mathbf{1}} * \underline{\mathbf{f}})$ is an interval valued intuitionistic fuzzy left hyperideal of $\mathbf{S}$. Since $(\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}) \subseteq \underline{\mathbf{f}}$, $\underline{\mathbf{f}} \cap (\underline{\mathbf{f}} * \underline{\mathbf{1}}) \subseteq \underline{\mathbf{f}}$ and $\underline{\mathbf{f}} \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}) \subseteq \underline{\mathbf{f}}$, by Proposition 3.21, $\underline{\mathbf{f}} = [\underline{\mathbf{f}} \cup (\underline{\mathbf{f}} * \underline{\mathbf{1}})] \cap [\underline{\mathbf{f}} \cup (\underline{\mathbf{1}} * \underline{\mathbf{f}})] = \underline{\mathbf{g}} \cap \underline{\mathbf{h}}$. □

Of course, it is natural to ask whether the converse of Theorem 3.22 is true or not. Fortunately, following theorem shows that it is true.

Theorem 3.23. *Let* $\mathbf{S}$ *be an ordered semihypergroup,* $\underline{\mathbf{g}}$ *be an interval valued intuitionistic fuzzy right hyperideal of* $\mathbf{S}$ *and* $\underline{\mathbf{h}}$ *be an interval valued intuitionistic fuzzy left hyperideal of* $\mathbf{S}$*. Then* $\underline{\mathbf{g}} \cap \underline{\mathbf{h}}$ *is an interval valued intuitionistic fuzzy quasi-hyperideal of* $\mathbf{S}$*.*

Proof. Suppose that $\underline{\mathbf{f}} = \underline{\mathbf{g}} \cap \underline{\mathbf{h}}$. We show that $\underline{\mathbf{f}}$ is an interval valued intuitionistic fuzzy quasi-hyperideal of $\mathbf{S}$. Let $a \in S$. We have that $((\underline{\mathbf{f}} * \underline{\mathbf{1}}) \cap (\underline{\mathbf{1}} * \underline{\mathbf{f}}))(a) \leq \underline{\mathbf{f}}(a)$ if $A_a = \emptyset$. Let $(x, y) \in A_a$. Then $a \leq t$ for some $t \in x \circ y$. Thus, we have

$$\mathbf{g}_1(a) \geq \mathbf{g}_1(t) \geq \inf_{z \in x \circ y} \{\mathbf{g}_1(z)\} \geq \mathbf{g}_1(x) \geq \mathbf{f}_1(x).$$

Therefore,

$$(\mathbf{f}_1 * \mathbf{1})(a) = \sup_{(x,y) \in A_a} \{\min\{\mathbf{f}_1(x), \mathbf{1}(y)\}\} = \sup_{(x,y) \in A_a} \{\min\{\mathbf{f}_1(x)\}\} \leq \mathbf{g}_1(a).$$

Similarly, we have $(\mathbf{1} * \mathbf{f}_1)(a) \leq \mathbf{h}_1(a)$. This implies that

$$\begin{aligned} ((\mathbf{f}_1 * \mathbf{1}) \cap (\mathbf{1} * \mathbf{f}_1))(a) &= \min\{(\mathbf{f}_1 * \mathbf{1})(a), (\mathbf{1} * \mathbf{f}_1)(a)\} \\ &\leq \min\{\mathbf{g}_1(a), \mathbf{h}_1(a)\} \\ &= (\mathbf{g}_1 \cap \mathbf{h}_1)(a). \end{aligned}$$

On the other hand, it is not difficult to show that $((\mathbf{f}_1 * \mathbf{1}) \cap (\mathbf{1} * \mathbf{f}_1))(a) \geq (\mathbf{g}_1 \cap \mathbf{h}_1)(a)$. Since $\underline{\mathbf{g}}(x) \leq \underline{\mathbf{g}}(y)$ and $\underline{\mathbf{h}}(x) \leq \underline{\mathbf{h}}(y)$ whenever $x \leq y$, we also have $\underline{\mathbf{f}}(x) \leq \underline{\mathbf{f}}(y)$. Altogether, the theorem and its converse is sound. □

Combining Theorem 3.22 and 3.23 we have a characterization of interval valued intuitionistic fuzzy quasi-hyperideals of an ordered semihypergroup as follows.

Corollary 3.24. *Let* $\mathbf{S}$ *be an ordered semihypergroup and* $\underline{\mathbf{f}}$ *be an interval valued intuitionistic fuzzy set of* $\mathbf{S}$*. Then* $\underline{\mathbf{f}}$ *is an interval valued intuitionistic fuzzy quasi-hyperideal of* $\mathbf{S}$ *if, and only if,* $\underline{\mathbf{f}} = \underline{\mathbf{g}} \cap \underline{\mathbf{h}}$ *for some interval valued intuitionistic fuzzy right hyperideal* $\underline{\mathbf{g}}$ *of* $\mathbf{S}$ *and interval valued intuitionistic fuzzy left hyperideal* $\underline{\mathbf{h}}$ *of* $\mathbf{S}$*.*

4. Concluding remarks

In this paper, we apply the concept of interval valued intuitionistic fuzzy sets to an algebraic structure, also known as ordered semihypergroups. It is clear that the notion of interval valued intuitionistic fuzzy sets is a generalization of interval valued fuzzy sets. Moreover, every ordered semigroup can be viewed as an ordered semihypergroup. In fact, we let $\langle S; \cdot, \leq \rangle$ be an ordered semigroup. Then we define $\circ\colon S \times S \to \mathrm{Sb}^*(S)$ by $x \circ y := \{x \cdot y\}$ for all $x, y \in S$, and $A \leq B$ if for any $a \in A$, there exists $b \in B$ such that $a \leq b$ for all $A, B \in \mathrm{Sb}^*(S)$. Then we have that $\langle S; \circ, \leq \rangle$ is an ordered semihypergroup. Therefore, the results we obtain in this paper will always hold in terms of interval valued fuzzy ideals in ordered semigroups.

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