KYUNGPOOK Math. J. 60(2020), 821-830
https://doi.org/10.5666/KMJ.2020.60.4.821
pISSN 1225-6951 eISSN 0454-8124

Some Relativistic Properties of Lorentzian Para-Sasakian Type Spacetime

Krishnendu De
Department of Mathematics, Kabi Sukanta Mahavidyalaya, Bhadreswar, P. O. - Angus, Hooghly, Pin 712221, West Bengal, India
e-mail : krishnendu.de@outlook.in

Abstract. The object of the present paper is to classify a special type of spacetime, called Lorentzian para-Sasakian type spacetime (4-dimensional LP-Sasakian manifold with a coefficient α) satisfying certain curvature conditions.

1. Introduction

In 2002, De et.al [5] introduced the notion of Lorentzian Para-Sasakian manifolds with a coefficient α which generalizes the idea of LP-Sasakian manifolds introduced by Matsumoto [9]. Hereafter, we refer to Lorentzian Para-Sasakian manifolds with a coefficient α as LP-Sasakian manifolds with a coefficient α. Mihai and Rosca [11] also introduced the same notion of LP-Sasakian manifolds independently and obtained several results in this manifold. LP-Sasakian manifolds with a coefficient α have been studied in [3, 4]. Ikawa and his co-authors [6, 7]) investigated Sasakian manifolds with a Lorentzian metric and obtained several interesting results. Motivated by the above works, we study some relativistic properties of LP-Sasakian manifolds with a coefficient α.

The basic difference between the Riemannian and the semi-Riemannian (signature of the metric tensor g is $(+,+,+,...+,+,+)$ and $(-,-,-,...+,+,+)$ respectively) geometry is the existence of a null vector, that is, a vector v satisfying $g(v,v)=0$. A non-zero vector $v \in T_pM$ is said to be *timelike* (resp; *non-spacelike, null, spacelike*) if it satisfies $g(v,v)<0$ (resp; $\leq 0. = 0, > 0$) [13]. A Lorentzian manifold is a special case of a semi-Riemannian manifold. Spacetime means a four dimensional connected semi-Riemannian manifold (M^4, g) with Lorentz metric g of signature $(-,+,+,+)$. Here we consider a special type of spacetime which is called *Lorentzian para-Sasakian type spacetime*(4-dimensional LP-Sasakian manifold with

Received November 14, 2019; revised July 16, 2020; accepted July 21, 2020.
2010 Mathematics Subject Classification: 53B30, 53C50, 53C80.
Key words and phrases: LP-Sasakian manifold with a coefficient α, ξ-conformally flat manifold, η-Einstein manifold.

a coefficient α).

A tensor C of type $(1,3)$ that remains invariant under conformal transformation for an n-dimensional Riemannian manifold M^n, is defined by [16]

$$\begin{aligned} C(X,Y)U = R(X,Y)U - \frac{1}{n-2}\{S(Y,U)X - S(X,U)Y + g(Y,U)QX \\ - g(X,U)QY\} + \frac{r}{(n-1)(n-2)}\{g(Y,U)X - g(X,U)Y\} \end{aligned} \tag{1.1}$$

for all X, Y, $U \in TM$, where r is the scalar curvature of M and Q is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor S. The equation (1.1) is known as *Weyl conformal curvature tensor*. The same defination is valid in a Lorentzian manifold. Identically, if the conformal curvature tensor of a manifold of dimension > 3 vanishes then the manifold is called *conformally flat*. However for a 3-dimensional Riemannian manifold $C = 0$.

A Riemannian or a semi-Riemannian manifold is said to be *semisymmetric* [15] if its curvature tensor R satisfies the condition

$$R(X,Y)\cdot R = 0,$$

for all X, $Y\in \chi(M)$, where $R(X,Y)$ acts as a derivation on the curvature tensor R. Locally symmetric spaces and all two-dimensional Riemannian spaces are trivial examples of semisymmetric spaces. But a semisymmetric space is not necessarily locally symmetric. A fundamental study on such manifolds was made by Szabó [14, 15] and Kowalski [8]. In this connection we can mention the book of Boeckx, Kowalski and Vanhecke [2] and the references therein.

Also, a Riemannian or a semi-Riemannian manifold is said to be *Ricci semisymmetric* [12] if the Ricci tensor S of type (0,2) satisfies the condition

$$R(X,Y)\cdot S = 0,$$

for all X, $Y\in \chi(M)$.

In 1995, Alias, Romero and Sánchez [1] introduced the notion of *generalized Robertson-Walker (GRW) spacetimes*. A Lorentzian manifold M of dimension $n \geq 3$ is named generalized Robertson-Walker (GRW) spacetime if it is the warped product $M = I \times_{q^2} M^*$ with base $(I, -dt^2)$, warping function q and the fibre (M^*, g^*) is an (n-1)-dimensional Riemannian manifold. If M^* is a 3-dimensional Riemannian manifold of constant curvature, the spacetime is called a *Robertson-Walker* (RW) *spacetime*. Therefore, GRW spacetimes are a wide generalization of RW spacetimes.

In general relativity, the perfect fluid spacetime is of special interest. Lorentzian manifolds with Ricci tensor of the form

$$S(X,Y) = \alpha g(X,Y) + \beta A(X)A(Y), \tag{1.2}$$

where α, β are scalar fields and U is a unit timelike vector field corresponding to the 1-form A (that is, $g(U,U) = -1$), are called *perfect fluid spacetimes*.

Einstein's Field equations without cosmological constant are given by

$$S(X,Y) - \frac{r}{2}g(X,Y) = kT(X,Y), \tag{1.3}$$

for all vector fields X, Y where k is the gravitational constant and T is the energy momentum tensor of type $(0,2)$.

The energy momentum tensor T is said to describe a *perfect fluid* [13] if

$$T(X,Y) = (\mu + p)A(X)A(Y) + pg(X,Y), \tag{1.4}$$

where μ is the energy density function, p is the isotropic pressure function of the fluid.

The paper is organized as follows. In Section 2, some preliminary results are recalled. After preliminaries in Section 3, we prove that a ξ-conformally flat Lorentzian para-Sasakian type spacetime is a Robertson-Walker spacetime. Then we study ϕ-Weyl semisymmetric Lorentzian para-Sasakian type spacetime and prove that a ϕ-Weyl semisymmetric Lorentzian para-Sasakian type spacetime is a perfect fluid spacetime, provided $\alpha^2 = \sigma$ and a state equation is derived. In the next section, we shown that a Ricci-semisymmetric Lorentzian para-Sasakian type spacetime is an Einstein spacetime and the state equation is given by $p + \mu =$ scalar.

2. Preliminaries

A differentiable manifold M^n of dimension n is called *Lorentzian almost para-contact manifold* [9] if it admits a $(1,1)$ tensor field ϕ, a contravariant vector field ξ, a covariant vector field η and a Lorentzian metric g of type $(0,2)$ (for each point $p \in M$, the tensor g_p: $T_pM \times T_pM \to \mathbb{R}$ inner product of signature $(-,+,+,.....,+)$, where T_pM denotes the tangent vector space of M at p and $\mathbb{R}$ is the real number space) satisfy the following

$$\phi^2(X) = X + \eta(X)\xi, \quad \eta(\xi) = -1, \tag{2.1}$$

$$g(X,\xi) = \eta(X), \quad g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y) \tag{2.2}$$

for all vector fields X, Y. Also in the Lorentzian almost paracontact manifold M^n, the following relations hold [9] :

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \tag{2.3}$$

$$\Omega(X,Y) = \Omega(Y,X), \tag{2.4}$$

where $\Omega(X,Y) = g(X,\phi Y)$.

If the relations

$$\begin{aligned}(\nabla_Z\Omega)(X,Y) = \alpha[&(g(X,Z) + \eta(X)\eta(Z))\,\eta(Y) \\ &+ (g(Y,Z) + \eta(Y)\eta(Z))\eta(X)],\end{aligned} \tag{2.5}$$

$$\Omega(X,Y)=\frac{1}{\alpha}(\nabla_X\eta)(Y), \tag{2.6}$$

hold for a Lorentzian almost paracontact manifold M^n where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g, and α is a non-zero scalar function then M^n is called an *LP-Sasakian manifold with a coefficient* α [5]. If we take $\alpha = 1$, then it reduces to an *LP-Sasakian manifold* [9].

If a vector field V satisfies the equation of the following form:

$$\nabla_X V = \beta X + T(X)V,$$

where β is a non-zero scalar function and T is a covariant vector field, then V is called a *torse-forming vector field* [16].

In the Lorentzian manifold M^n, if we assume that ξ is a unit torse-forming vector field, then we have the equation:

$$(\nabla_X\eta)(Y)=\alpha[g(X,Y)+\eta(X)\eta(Y)], \tag{2.7}$$

where α is a non-zero scalar function. It is to be noted that in Lorentzian para-Sasakian type spacetime ξ is a unit timelike vector.

Let us consider an LP-Sasakian manifold M^4 (ϕ,ξ,η,g) with a coefficient α. Then we have the following relations [5]:

$$\eta(R(X,Y)Z)=(\alpha^2-\sigma)[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)], \tag{2.8}$$

$$S(X,\xi)=3(\alpha^2-\sigma)\eta(X), \tag{2.9}$$

$$R(X,Y)\xi=(\alpha^2-\sigma)[\eta(Y)X-\eta(X)Y], \tag{2.10}$$

$$(\nabla_X\phi)(Y)=\alpha[g(X,Y)\xi+2\eta(X)\eta(Y)\xi+\eta(Y)X], \tag{2.11}$$

$$S(\phi X,\phi Y)=S(X,Y)+3(\alpha^2-\sigma)\eta(X)\eta(Y), \tag{2.12}$$

for all vector fields X, Y, Z, where R, S denote respectively the curvature tensor and the Ricci tensor of the manifold.

Now we state the following results which will be needed in the later section.

Lemma 2.1.([5]) *In a Lorentzian almost paracontact manifold $M^n(\phi,\xi,\eta,g)$ with its structure (ϕ,ξ,η,g) satisfying $\Omega(X,Y)=\frac{1}{\alpha}(\nabla_X\eta)(Y)$, where α is a non-zero scalar, the vector field ξ is torse-forming if and only if $\psi^2=(n-1)^2$ holds good.*

Lemma 2.2. *In an LP-Sasakian manifold with a coefficient α the curvature tensor R satisfies the relation*

$$\begin{aligned}R(X,Y)\phi Z-\phi R(X,Y)Z&=(\alpha^2-\sigma)[g(X,Z)\eta(Y)-g(Y,Z)\eta(X)]\xi\\&\quad+(\alpha^2-\sigma)\eta(Z)[X\eta(Y)-Y\eta(X)].\end{aligned} \tag{2.13}$$

3. ξ-conformally Flat Lorentzian Para-Sasakian Type Spacetime

Definition 3.1. A 4-dimensional Lorentzian para-Sasakian type spacetime is said to be ξ-*conformally flat* [18] if

$$C(X,Y)\xi = 0, \qquad \text{for any } X, Y \in TM.$$

In this section we characterize ξ-conformally flat Lorentzian para-Sasakian type spacetime. Let M be a ξ-conformally flat Lorentzian para-Sasakian type spacetime. Putting $U = \xi$ in (1.1) and using $C(X,Y)\xi = 0$, we have

$$\begin{aligned} R(X,Y)\xi = {} & \frac{1}{2}\{S(Y,\xi)X - S(X,\xi)Y + \eta(Y)QX - \eta(X)QY\} \\ & - \frac{r}{6}[\eta(Y)X - \eta(X)Y]. \end{aligned} \tag{3.1}$$

Using (2.9) and (2.10) in the above equation, we obtain

$$\eta(Y)QX - \eta(X)QY = \left(\frac{r}{3} - (\alpha^2 - \sigma)\right)[\eta(Y)X - \eta(X)Y]. \tag{3.2}$$

Replacing Y by ξ in the foregoing equation yields

$$QX = \left(\frac{r}{3} - (\alpha^2 - \sigma)\right)X - \left(\frac{r}{3} - 4(\alpha^2 - \sigma)\right)\eta(X)\xi. \tag{3.3}$$

From (3.3), we get

$$S(X,Y) = \left(\frac{r}{3} - (\alpha^2 - \sigma)\right)g(X,Y) - \left(\frac{r}{3} - 4(\alpha^2 - \sigma)\right)\eta(X)\eta(Y). \tag{3.4}$$

That is, the spacetime is a perfect fluid spacetime. In view of the above discussions we state the following:

Theorem 3.1. *A ξ-conformally flat Lorentzian para-Sasakian type spacetime is a perfect fluid spacetime.*

Since conformally flat implies ξ-conformally flat, therefore we can state:

Corollary 3.1. *A conformally flat Lorentzian para-Sasakian type spacetime is a perfect fluid spacetime.*

Replacing Y by ξ in the equation (3.4) yields

$$S(X,\xi) = \left[\frac{2r}{3} - 5(\alpha^2 - \sigma)\right]\eta(X),$$

which implies ξ is an eigenvector of the Ricci tensor.

Corollary 3.2. *In a ξ-conformally flat Lorentzian para-Sasakian type spacetime, the characteristic vector field ξ is an eigenvector of the Ricci tensor.*

It is known [5] that if an LP-Sasakian manifold with a coefficient α is η- Einstein, then the Ricci tensor S is of the form

$$S(Y,Z) = \left[\frac{r}{3} - \alpha^2 - \frac{\psi\sigma}{3}\right] g(Y,Z) + \left[\frac{r}{3} - 4\alpha^2 - \frac{4\psi\sigma}{3}\right] \eta(Y)\eta(Z). \tag{3.5}$$

By virtue of (3.4) and (3.5), we get

$$\left[\sigma + \frac{\psi\sigma}{3}\right] g(X,Z) + \left[4\sigma + \frac{4\psi\sigma}{3}\right] \eta(X)\eta(Z) = 0. \tag{3.6}$$

Putting $Z = \xi$ in (3.6), we obtain

$$\eta(X)\sigma(\psi + 3) = 0, \tag{3.7}$$

which gives

$$\psi^2 = 3^2. \tag{3.8}$$

Hence by Lemma 2.1 we conclude that ξ is torse-forming. Thus we can state the following:

Theorem 3.2. *In a ξ-conformally flat Lorentzian para-Sasakian type spacetime, the characteristic vector field ξ is a torse-forming vector field.*

In [10], the authors proved the following:

Theorem 3.3.([10, Proposition 3.7]) *A Lorentzian manifold of dimension $n \geq 3$ is a GRW spacetime if and only if it admits a unit timelike torse-forming vector, $\nabla_X V = \phi(X + \omega(X)V)$, where $\omega(X) = g(X,V)$ for every vector field X, that is also an eigenvector of the Ricci tensor.*

In view of the above Theorem we conclude the following:

Theorem 3.4. *A ξ-conformally flat Lorentzian para-Sasakian type spacetime is a GRW spacetime.*

Also in [10] the following result was stated *" A 4-dimensional GRW spacetime is perfect fluid if and only if it is a RW spacetime."* Therefore we can state the following:

Theorem 3.5. *A ξ-conformally flat Lorentzian para-Sasakian type spacetime is a RW spacetime.*

4. ϕ-Weyl Semisymmetric Lorentzian Para-Sasakian Type Spacetime

In 2012, Yildiz and De [17] introduced the notion of ϕ-Weyl semisymmetric (k,μ)-contact metric manifold.

Definition 4.1. A (k, μ)-contact metric manifold is said to be *ϕ-Weyl semisymmetric* if

$$C(X, Y).\phi = 0$$

for arbitrary vector fields X, Y.

We adopt the same definition in an LP-Sasakian manifolds with a coefficient α. Let M be a ϕ-Weyl semisymmetric Lorentzian para-Sasakian type spacetime. The condition $C(X, Y).\phi = 0$ turns into

$$(C(X, Y).\phi)Z = C(X, Y).\phi Z - \phi C(X, Y)Z = 0, \tag{4.1}$$

for any vector field X, Y, Z.

Using (1.1) in (4.1), we have

$$\begin{aligned} & R(X, Y)\phi Z - \frac{1}{2}\{S(Y, \phi Z)X - S(X, \phi Z)Y + g(Y, \phi Z)QX \\ & - g(X, \phi Z)QY\} + \frac{r}{6}\{g(Y, \phi Z)X - g(X, \phi Z)Y\} \\ & - \phi(R(X, Y)Z) + \frac{1}{2}\{S(Y, Z)\phi X - S(X, Z)\phi Y + g(Y, Z)\phi(QX) \\ & - g(X, Z)\phi(QY)\} - \frac{r}{6}\{g(Y, Z)\phi X - g(X, Z)\phi Y\} = 0. \end{aligned} \tag{4.2}$$

Putting $Z = \xi$ and using (2.13), we have

$$\begin{aligned} & - (\alpha^2 - \sigma)[X\eta(Y) - Y\eta(X)] \\ & + \frac{1}{2}\{\eta(Y)\phi(QX) - \eta(X)\phi(QY) + S(Y, \xi)\phi X \\ & - S(X, \xi)\phi Y\} - \frac{r}{6}\{\eta(Y)\phi X - \eta(X)\phi Y\} = 0. \end{aligned} \tag{4.3}$$

Taking the inner product on both sides by W and symmetry property of ϕ, we obtain

$$\begin{aligned} & - (\alpha^2 - \sigma)[g(X, W)\eta(Y) - g(Y, W)\eta(X)] \\ & + \frac{1}{2}\{\eta(Y)S(\phi X, W) - \eta(X)S(\phi Y, W) + S(Y, \xi)S(X, W) \\ & - S(X, \xi)S(Y, W)\} - \frac{r}{6}\{\eta(Y)g(\phi X, W) - \eta(X)g(\phi Y, W)\} = 0. \end{aligned} \tag{4.4}$$

Replacing X by ξ in the foregoing equation yields

$$\begin{aligned} & - (\alpha^2 - \sigma)[\eta(W)\eta(Y) + g(Y, W)] + \frac{1}{2}\{S(\phi Y, W) \\ & + S(Y, \xi)S(W, \xi) - S(\xi, \xi)S(Y, W)\} - \frac{r}{6}g(\phi Y, W) = 0. \end{aligned} \tag{4.5}$$

Replacing W by ϕW and using (2.2), we obtain

$$S(X,Y)=\frac{r}{3}g(X,Y)+\left(\frac{r}{3}-3\right)\eta(X)\eta(Y), \tag{4.6}$$

provided $\alpha^2=\sigma$. That is, the spacetime is a perfect fluid spacetime. Thus we have the following:

Theorem 4.1. *A ϕ-Weyl semisymmetric Lorentzian para-Sasakian type spacetime is a perfect fluid spacetime, provided $\alpha^2=\sigma$.*

From (1.3) and (4.6), we obtain

$$kT(X,Y)=-\frac{r}{6}g(X,Y)+\left(\frac{r}{3}-3\right)\eta(X)\eta(Y). \tag{4.7}$$

The above equation is of the form of a perfect fluid spacetime, where $kp=-\frac{r}{6}$ and $k(p+\mu)=(\frac{r}{3}-3)$ from which it follows that $p=-\frac{r}{6k}$ and $\mu=-\frac{1}{k}(3-\frac{r}{2})$.

Therefore the state equation is $p-\mu=\frac{1}{k}(\frac{r}{3}-3)$, that is, $p=\mu+$ scalar. If the scalar curvature is constant, then the state equation reduces to $p=\mu+$ constant.

5. Ricci-semisymmetric Lorentzian Para-Sasakian Type Spacetime

In this section we deal with Lorentzian para-Sasakian type spacetime satisfying

$$R.S=0. \tag{5.1}$$

From (5.1) it follows that

$$S(R(X,Y)Z,W)+S(Z,R(X,Y)W)=0. \tag{5.2}$$

Replacing Z by ξ and using (2.9) and (2.10), we get

$$\begin{aligned}&(\alpha^2-\sigma)\{\eta(Y)S(X,W)-\eta(X)S(Y,W)\}\\&+3(\alpha^2-\sigma)^2\{g(Y,W)\eta(X)-g(X,W)\eta(Y)\}=0.\end{aligned} \tag{5.3}$$

Now putting $Y=\xi$ in the foregoing equation, we obtain

$$S(X,W)=3(\alpha^2-\sigma)g(X,W), \tag{5.4}$$

which implies that the manifold under consideration is an Einstein spacetime.

Now we consider perfect fluid spacetime obeying Einstein's equation whose velocity vector field is the characteristic vector field of the manifold. Then we get from Einstein's field equation

$$S(X,Y)-\frac{r}{2}g(X,Y)=kT(X,Y)=k[(\mu+p)\eta(X)\eta(Y)+pg(X,Y)]. \tag{5.5}$$

Using (5.4) in (5.5) and putting $Y=\xi$ infers

$$3(\alpha^2-\sigma)\eta(X)-\frac{r}{2}\eta(X)=k[-(\mu+p)\eta(X)+p\eta(X)]$$

$$(5.6) \qquad or, \quad \mu = \frac{r - 6(\alpha^2 - \sigma)}{2k}.$$

Now taking a frame field and contracting X and Y in (5.5), we obtain by using (5.6)

$$(5.7) \qquad p = \frac{-r - 6\alpha^2 + 6\sigma}{6k}.$$

Equation (5.6) and (5.7) together yields that

$$(5.8) \qquad p + \mu = scalar.$$

Thus we have :

Theorem 5.1. *A Ricci-semisymmetric Lorentzian para-Sasakian type spacetime is an Einstein spacetime and the state equation is given by* $p + \mu =$ *scalar.*

Acknowledgements. The author is thankful to the referee and the Editor in Chief for their valuable suggestions towards the improvement of the paper.

References

[1] L. Alías, A. Romero and M. Sánchez, *Uniqueness of complete spacelike hypersurfaces of constant mean curvature in generalized Robertson-Walker spacetimes*, Gen. Relativity Gravitation, **27(1)**(1995), 71–84.

[2] E. Boeckx, O. Kowalski and L. Vanhecke, *Riemannian manifolds of conullity two*, Singapore World Sci. Publishing, 1996.

[3] U. C. De and K. Arslan, *Certain curvature conditions on an LP-Sasakian manifold with a coefficient* α, Bull. Korean Math. Soc., **46**(2009), 401–408.

[4] U. C. De, J. B. Jun and A. A. Shaikh, *On conformally flat LP-Sasakian manifolds with a coefficient* α, Nihonkai Math. J., **13**(2002), 121–131.

[5] U. C. De, A. A. Shaikh and A. Sengupta, *On LP-Sasakian manifolds with acoefficient* α, Kyungpook Math. J., **42**(2002), 177–186.

[6] T. Ikawa and M. Erdogan, *Sasakian manifolds with Lorentzian metric*, Kyungpook Math. J., **35**(1996), 517–526.

[7] T. Ikawa and J. B. Jun, *On sectional curvatures of a normal contact Lorentzian manifold*, Korean J. Math. Sciences, **4**(1997), 27–33.

[8] O. Kowalski, *An explicit classification of 3-dimensional Riemannian spaces satisfying* $R(X,Y) \cdot R = 0$, Czechoslovak Math. J., **46**(1996), 427–474.

[9] K. Matsumoto, *On Lorentzian paracontact manifolds*, Bull. Yamagata Univ. Nat. Sci., **12**(1989), 151–156.

[10] K. Matsumoto and I. Mihai, *On a certain transformation in a Lorentzian para-Sasakian manifold*, Tensor (N.S.), **47**(1988), 189–197.

[11] I. Mihai and R. Rosca, *On Lorentzian P-Sasakian manifolds*, Classical Analysis, 155-169, World Sci. Publ., Singapore, 1992.

[12] V. A. Mirzoyan, *Ricci semisymmetric submanifolds(Russian)*, Itogi Nauki i Tekhniki. Ser. Probl. Geom. **23**, VINITI, Moscow, 1991, 29–66.

[13] B. O'Neill, *Semi Riemannian geometry*, Academic press, New york, 1983.

[14] Z. I. Szabó, *Classification and construction of complete hypersurfaces satisfying* $R(X,Y)\cdot R=0$, Acta Sci. Math. (Szeged), **47**(1981), 321–348.

[15] Z. I. Szabó, *Structure theorems on Riemannian spaces satisfying* $R(X,Y)\cdot R=0$*., I*, The local version, J. Diff. Geom., **17**(1982), 531–582.

[16] K. Yano, *On the torse-forming direction in Riemannian spaces*, Proc. Imp. Acad. Tokyo, **20**(1944), 340–345.

[17] A. Yildiz and U. C. De, *A classification of* (k,μ)*-contact metric manifolds*, Commun. Korean Math. Soc., **27**(2012), 327–339.

[18] G. Zhen, *On Conformal Symmetric K-contact manifolds*, Chinese Quart. J. of Math., **7**(1992), 5–10.