KYUNGPOOK Math. J. 60(2020), 753-765
https://doi.org/10.5666/KMJ.2020.60.4.753
pISSN 1225-6951 eISSN 0454-8124

The Multi-step Adomian Decomposition Method for Approximating a Fractional Smoking Habit Model

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Abstract. Smoking is one of the main causes of health problems and continues to be one of the world's most significant health challenges. In this paper, we use the multi-step Adomian decomposition method (MSADM) to obtain approximate analytical solutions for a mathematical fractional model of the evolution of the smoking habit. The proposed MSADM scheme is only a simple modification of the Adomian decomposition method (ADM), in which ADM is treated algorithmically with a sequence of small intervals (i.e. time step) for finding accurate approximate solutions to the corresponding problems. A comparative study between the new algorithm and the classical Runge-Kutta method is presented in the case of integer-order derivatives. The solutions obtained are also presented graphically. The results reveal that the method is effective and convenient for solving linear and nonlinear differential equations of fractional order.

1. Introduction

Fractional order ordinary differential equations, as generalizations of classical integer order ordinary differential equations, are increasingly used to model problems in fluid flow, mechanics, viscoelasticity, biology, physics, engineering and other applications [17, 9]. Fractional differential equations are the result of mathematical modeling of complex processes and phenomena. The most fundamental character-

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Received June 10, 2015; accepted Jun 21, 2019.
2010 Mathematics Subject Classification: 11Y35, 65L05.
Key words and phrases: fractional differential equations, multi-step Adomian decomposition method, smoking model, numerical analysis; mathematical fractional model.

istic of these models is their nonlocal characteristic which does not exist in the differential operators of integer order. This property means that the next aspect of a model relates not only upon its present state but also upon all of its history. As a consequence, there is an already very large, and still growing, number of scientific and engineering problems involving fractional derivatives. The fractional calculus (that is, derivatives and integrals of any real or complex order) will perhaps be the calculus of the twenty-first century [3, 12, 22, 24]. The solutions of fractional differential equations are quite involved. In general, there exists no method that yields exact solutions to fractional differential equations. Only approximate solutions can be derived. Several methods have been used to solve fractional differential equations, such as Laplace transform method [18, 20], Fourier transform method [14], homotopy perturbation method [19, 25], homotopy analysis method [2, 28, 27], Adomian decomposition method [1, 5, 13, 21] and differential transform method [6, 7, 8]. In this paper, we investigate the applicability and effectiveness of ADM when treated as an algorithm with a sequence of intervals (i.e. time step) for finding accurate approximate solutions to a time-fractional epidemic model for the habit of smoking in a community in Spain. This modified method is named as the multi-step Adomian decomposition method. It can be found that the corresponding numerical solutions obtained by using ADM are valid only for a short time. While the ones obtained by using MSADM are valid and more accurate over a longer time. The agree closely with the RK4-5 numerical solutions in the case of integer-order systems.

Smoking is the leading cause of preventable death, and is estimated to kill more than 5 million people worldwide each year. This number is expected to grow. Smoking or tobacco is a known or probable cause of cancers of the oral cavity, larynx, lung, oesophagus, bladder, pancreas, renal pelvis, stomach, and cervix. Smoking is also a cause of heart disease, strokes, peripheral vascular diseases, chronic obstructive lung diseases and other respiratory diseases, and low-birth weight babies [15].

There have been several attempts [16, 26], since the 2000s, to mathematically model the effort to give up smoking. In 2000, Castillo-Garsow et al. [4] proposed a simple mathematical model for giving up smoking. The fact that epidemic models consist of a system of non-linear differential equations underlines the importance of having reliable methods for solving them. This type of model can be integrated using any standard numerical method. However, it is known that these algorithms are subject to problems such as numerical instabilities, oscillations or false equilibrium states. This means that the numerical solution may not correspond to the real solution of the original system of differential equations. This is the reason why we are interested in obtaining a continuous solution in the form of an analytical approximation to the real solution. The epidemic model for smoking is a system of non-linear differential equations without closed solution. The interest of this fractional model is that it has been able to describe the general evolution of the spread of smoking. It was constructed using real data for the initial values and for the parameters of the system. Constant population is assumed by taking birth

and death rates equal and different from zero. The structure of this paper is as follows. In Section 2, we present some necessary definitions and notation related to fractional calculus. In Section 3, we construct the general form of MSADM for a system of fractional differential equations. In Section 4, we describe the MSADM of system of a time-fractional epidemic model for smoking in a community and numerical simulations are presented graphically. Finally, we make our conclusions in Section 5

2. Preliminaries

In this section, we introduce the linear operators of fractional integration and fractional differentiation in the framework of the Riemann-Liouville and Caputo fractional calculus.

Definition 2.1. A real function $f(t), t > 0$, is said to be *in the space* C_μ, $\mu \in R$ if there exists a real number $p > \mu$ such that $f(t) = x^p f_1(t)$, where $f_1(t) \in C[0, \infty)$. Clearly $C_\mu \subset C_\beta$ if $\beta \leq \mu$.

Definition 2.2. A function $f(t), t > 0$, is said to be *in the space* C_μ^m, $m \in N \cup \{0\}$, if $f^{(m)} \in C_\mu$.

Definition 2.3. The *left sided Riemann-Liouville fractional integral operator of order* $\alpha \geq 0$ of a function $f \in C_\mu$, $\mu \geq -1$, is defined as

$$J^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad \alpha > 0, \quad t > 0,$$
$$J^0 f(t) = f(t).$$

Definition 2.4. Let $f \in C_{-1}^m$ for $m \in N \cup \{0\}$. The *Caputo fractional derivative* of $f(t)$ is defined by

$$D_*^\alpha f(t) = \begin{cases} \left[J^{m-\alpha} f^{(m)}(t)\right], & m-1 < \alpha < m, \quad m \in N, \\ \frac{d^m f(t)}{dt^m}, & \alpha = m. \end{cases}$$

Hence, we have the following properties [9, 17]

$$J^\alpha J^\nu f = J^{\alpha+\nu} f, \qquad \alpha, \nu \geq 0, \quad f \in C_\mu, \quad \mu \geq -1.$$
$$J^\alpha t^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} t^{\gamma+\alpha}, \quad \alpha > 0, \quad \gamma > -1, \quad t \geq 0. \tag{2.1}$$
$$J^\alpha D_*^\alpha f(t) = f(t) - \sum_{k=0}^{m-1} f^{(k)}(0^+) \frac{t^k}{k!}, \quad t > 0, \quad m-1 < \alpha \leq m.$$

3. Multi-step Adomian Decomposition Method

The ADM is used to provide approximate solutions for nonlinear problems in the form of convergent series with easily computable components. It has been shown that the approximated solution obtained by this method are not always valid for large t. Therefore we use the MSADM, which offers an accurate solution over a greater range of t than does the ADM [13]. In this section we construct the general form of the MSADM. For this purpose, consider the fractional system of differential equations

$$D^{\alpha_i} x_i(t) = N_i(t, x_1(t), ..., x_r(t)), \quad t \geq 0, \quad 0 < \alpha_i \leq 1, \tag{3.1}$$

subject to the initial conditions

$$x_i(0) = d_i, \quad i = 1, 2, ..., r, \tag{3.2}$$

where $(N_i(t, x_1(t), ..., x_r(t)),\ i = 1, 2, ..., r)$ are linear/nonlinear functions of $t, x_1(t), ..., x_r(t)$. We employ MSADM to solve the system of equations (3.1), (3.2). Let $[0, T]$ be the interval over which we want to find the solution of the initial value problem (3.1), (3.2). Assume that the interval $[0, T]$ is divided into n subintervals $\Delta t,\ [t_0, t_1],\ [t_1, t_2],\ [t_2, t_3], ..., [t_{n-1}, t_n]$ of equal length with $t_0 = 0$ and $t_n = T$. Let t^* be the initial value for each subintervals and let $x_{i,j}(t)$ for $i = 1, 2, ..., r$ be approximate solutions in the subinterval $[t_{j-1}, t_j]$ for $j = 1, 2, ..., n$, with initial guesses

$$x_{i,1}(t^*) = d_i \text{ and } x_{i,j}(t^*) = X_{i,j}(t_{j-1}) = X_{i,j-1}(t_{j-1}) \tag{3.3}$$

where $X_{i,j}(t)$ will be constructed later in equatiuon (3.15). Now, we can construct deformation equations of the system (3.1) by setting

$$D^{\alpha_i} x_{i,j}(t) = N_{i,j}(t, x_{1,j}(t), \ldots, x_{r,j}(t)) \tag{3.4}$$

for all $0 < \alpha_i \leq 1$, $i = 1, 2, \ldots, r$, and $j = 1, 2, ..., n$. Applying J^{α_i} to both the sides of (3.4), we get

$$x_{i,j}(t) = x_{i,j}(t^*) + J^{\alpha_i} N_{i,j}(t, x_{1,j}(t), \ldots, x_{r,j}(t)). \tag{3.5}$$

We employ the Adomian decomposition method to solve the systems of equations (3.4) [13]. For all $i = 1, 2, \ldots, r$ and $j = 1, 2, ..., n$, let

$$x_{i,j}(t) = x_{i,j}(t^*) + \sum_{m=1}^{\infty} x_{i,j,m}(t), \tag{3.6}$$

and

$$N_{i,j}(t, x_{1,j}(t), \ldots, x_{r,j}(t)) = \sum_{m=0}^{\infty} A_{i,j,m}, \tag{3.7}$$

where $A_{i,j,m}$ is an Adomian polynomial which depends on $x_{i',j,m'}$ for $i' = 1, \ldots, r$ and $m' = 0, \ldots, m$. In view of Eqs. (3.6) and (3.7), the equation (3.5) takes the form

$$\sum_{m=0}^{\infty} x_{i,j,m}(t) = x_{i,j}(t^*) + J^{\alpha_i} \sum_{m=0}^{\infty} A_{i,j,m}(x_{1,j,0}, ..., x_{1,j,m}, x_{2,j,0}, ...$$

$$, x_{2,j,m}, ..., x_{r,j,0}, ..., x_{r,j,m}), i = 1, 2, \ldots, r, \quad j = 1, 2, ..., n, \tag{3.8}$$

We set

$$x_{i,j,0}(t) = x_{i,j}(t^*),$$

and

$$x_{i,j,m+1}(t) = J^{\alpha_i} A_{i,j,m}(x_{1,j,0}, ..., x_{1,j,m}, x_{2,j,0}, ..., x_{2,j,m}, ..., x_{n,j,0}, ...$$

$$, x_{n,j,m}), \ i = 1, 2, \ldots, r, \quad j = 1, 2, ..., n, \quad m = 0, 1, ... \tag{3.9}$$

In order to determine the Adomian polynomials, we introduce a parameter λ, so (3.7) becomes

$$N_{i,j}(t, \sum_{m=0}^{\infty} x_{1,j,m}\lambda^m, \ldots, \sum_{m=0}^{\infty} x_{r,j,m}\lambda^m) = \sum_{m=0}^{\infty} A_{i,j,m}\lambda^m, \tag{3.10}$$

Let $x_{i,j\lambda}(t) = \sum\limits_{m=0}^{\infty} x_{i,j,m}\lambda^m$, then

$$A_{i,j,m} = \frac{1}{m!}\frac{d^m}{d\lambda^m} N_{i,j\lambda}(t, x_{1,j\lambda}, ..., x_{r,j\lambda})|_{\lambda=0}, \tag{3.11}$$

where

$$N_{i,j,\lambda}(t, x_{1,j}, ..., x_{r,j}) = N_{i,j}(t, x_{1,j\lambda}, ..., x_{r,j\lambda}). \tag{3.12}$$

In view of (3.11), (3.12) we get

$$\begin{aligned} A_{i,j,m} &= \frac{1}{m!}\frac{d^m}{d\lambda^m}[N_{i,j\lambda}(t, x_{1,j\lambda}(t), ..., x_{r,j\lambda}(t))]_{\lambda=0} \\ &= \frac{1}{m!}\frac{d^m}{d\lambda^m}[N_{i,j}(t, \sum_{m=0}^{\infty} x_{1,j,m}\lambda^m, ..., \sum_{m=0}^{\infty} x_{r,j,m}\lambda^m)]_{\lambda=0} \\ &= [\frac{1}{m!}\frac{d^m}{d\lambda^m} N_{i,j}(t, \sum_{m=0}^{\infty} x_{1,j,m}\lambda^m, ..., \sum_{m=0}^{\infty} x_{r,j,m}\lambda^m)]_{\lambda=0}. \end{aligned} \tag{3.13}$$

Hence (3.9), (3.13) lead to the following recurrence relations:

$$x_{i,j,0}(t) = x_{i,j}(t^*),$$

$$x_{i,j,m+1}(t) = J^{\alpha_i}[\frac{1}{m!}\frac{d^m}{d\lambda^m}N_{i,j}(t, \sum_{m=0}^{\infty} x_{1,j,m}\lambda^m, ..., \tag{3.14}$$

$$\sum_{m=0}^{\infty} x_{r,j,m}\lambda^m)]_{\lambda=0}, m = 0, 1, 2, ...$$

We can approximate the solution $x_{i,j}(t)$ by the truncated series

$$f_{i,j,k} = \sum_{m=0}^{k-1} x_{i,j,m}(t), \lim_{k\longrightarrow\infty} f_{i,j,k} = x_{i,j.}$$

For the convergence of the above method, if system (3.1) admits a unique solution, then the method will produce the unique solution and If the system (3.1) does not admit a unique solution, the decomposition method will give a solution among many (possible) other solutions [1]. The solutions of system (3.1) in each subinterval $[t_{j-1}, t_j],\ j = 1, 2, ..., n,$ has the form

$$X_{i,j}(t) = \sum_{m=0}^{K} x_{i,j,m}(t - t_{j-1}),\ i = 1, 2, \ldots, r,\ j = 1, 2, ..., n, \tag{3.15}$$

and the solution of system (3.1) in the interval $[0, T]$ is given by

$$x_i(t) = \sum_{j=1}^{n} \chi_v X_{i,j}(t),\ i = 1, 2, \ldots, r, \tag{3.16}$$

where

$$\chi_v = \begin{cases} 1, & t \in [t_{j-1}, t_j], \\ 0, & t \notin [t_{j-1}, t_j]. \end{cases}$$

4. Numerical Results

This model was presented in [10, 11, 23] to describe and predict the evolution of the habit of smoking in Spain and to quantify the impact of Spanish smoke-free policies. Smoking has traditionally been modeled by ordinary differential equations. The following system of ordinary differential equations models the dynamics

between the different subpopulations considered.

$$\begin{aligned} \frac{dn}{dt} - \mu(1-n) + \beta n(s+c) &= 0, \\ \frac{ds}{dt} - \beta n(s+c) - \rho e - \eta c + (\gamma+\lambda+\mu)s &= 0, \\ \frac{dc}{dt} - \gamma s + (\eta+\delta+\mu)c &= 0, \\ \frac{de}{dt} - \lambda s - \delta c + (\rho+\mu)e &= 0. \end{aligned} \tag{4.1}$$

The subpopulations included in the model are: n is the proportion of the total population who has never smoked, s is the proportion of people who smoke less than 20 cigarettes per day, c is the proportion of individuals who smoke more than 20 cigarettes per day and e is the proportion of ex-smokers. The parameter μ denotes birth rate in the community; β denotes the transmission rate due to the social pressure to adopt smoking habit; ρ expresses the rate at which ex-smokers return to smoking; η is the rate at which an excessive smoker becomes a normal smoker by decreasing the number of cigarettes per day; γ is the rate at which normal smokers become excessive smokers by increasing the number of cigarettes per day; λ denotes the rate at which normal smokers stop smoking and δ is the rate at which excessive smokers stop smoking. The population is constant and it has been normalized to unity, so

$$n + s + c + e = 1, \tag{4.2}$$

for any instant of time. The objective of the present paper is to use the MSADM to obtain the approximate solution of the following time-fractional epidemic model for smoking:

$$\begin{aligned} D_*^{\alpha_1} n(t) - \mu(1-n(t)) + \beta n(t)(s(t)+c(t)) &= 0, \\ D_*^{\alpha_2} s(t) - \beta n(t)(s(t)+c(t)) - \rho e(t) - \eta c(t) + (\gamma+\lambda+\mu)s(t) &= 0, \\ D_*^{\alpha_3} c(t) - \gamma s(t) + (\eta+\delta+\mu)c(t) &= 0, \\ D_*^{\alpha_4} e(t) - \lambda s(t) - \delta c(t) + (\rho+\mu)e(t) &= 0, \end{aligned} \tag{4.3}$$

subject to the initial conditions

$$n(0) = n_0,\ s(0) = s_0,\ c(0) = c_0,\ e(0) = e_0, \tag{4.4}$$

In order to perform the resumption methods, we set the values of the parameters as in [4]: $\mu = 0.01$, $\beta = 0.0381$, $\rho = 0.0425$, $\eta = 0.1244$, $\gamma = 0.1175$, $\lambda = 0.0498$ and $\delta = 0.0498$. Moreover, The initial conditions are chosen as: $n(0) = 0.5045,\ s(0) = 0.2059,\ c(0) = 0.1559$ and $e(0) = 0.1337$.

In order to solve the system (4.3), (4.4), we define the nonlinear terms by

$$\begin{aligned}
N_{1,j}(\lambda) &= \mu(1-n_j(\lambda)) - \beta n_j(\lambda)(s_j(\lambda)+c_j(\lambda)) \\
&= \sum_{m=0}^{\infty} A_{1,j,m}, \\
N_{2,j}(\lambda) &= \beta n_j(\lambda)(s_j(\lambda)+c_j(\lambda)) + \rho e_j(\lambda) + \alpha c_j(\lambda) \\
-(\gamma+\breve{\lambda}+\mu)s_j(\lambda) &= \sum_{m=0}^{\infty} A_{2j,m}, \\
N_{3,j}(\lambda) &= \gamma s_j(\lambda) - (\alpha+\delta+\mu)c_j(\lambda)) \\
&= \sum_{m=0}^{\infty} A_{3,j,m}, \\
N_{4,j}(\lambda) &= \breve{\lambda} s_j(\lambda) + \delta c_j(\lambda) - (\rho+\mu)c_j(\lambda)) \\
&= \sum_{m=0}^{\infty} A_{4,j,m}, \qquad\qquad j = 1,2,3,...,n,
\end{aligned} \tag{4.5}$$

where

$$\begin{aligned}
n_j(\lambda) &= \sum_{m=0}^{K} n_{j,m}(t)\lambda^m, \\
s_j(\lambda) &= \sum_{m=1}^{K} s_{j,m}(t)\lambda^m, \\
c_j(\lambda) &= \sum_{m=1}^{K} c_{j,m}(t)\lambda^m, \\
e_j(\lambda) &= \sum_{m=1}^{K} e_{j,m}(t)\lambda^m,
\end{aligned}$$

and

$$A_{i,j,m} = \frac{1}{m!}\frac{d^m}{d\lambda^m}[N_{i,j}(\lambda)]_{\lambda=0},\ i = 1,2,3,4,\ j = 1,2,...,n,\ m = 1,2,...,K.$$

So in this case we have to satisfy the initial condition at each of the subintervals. Accordingly, the initial values will be changed for each subinterval, i.e.

$$\begin{aligned}
&n_1(t^*) = 0.5045,\ n_j(t^*) = n_j(t_{j-1}) = n_{j-1}(t_{j-1}), \\
&s_1(t^*) = 0.2059,\ s_j(t^*) = s_j(t_{j-1}) = s_{j-1}(t_{j-1}) \\
&c_1(t^*) = 0.1559,\ c_j(t^*) = c_j(t_{j-1}) = c_{j-1}(t_{j-1}) \\
&e_1(t^*) = 0.1337,\ e_j(t^*) = e_j(t_{j-1}) = e_{j-1}(t_{j-1}), \\
&\qquad j = 1,2,3,...,n.
\end{aligned} \tag{4.6}$$

Where t^* is the initial value for each subintervals. The Adomain decomposition series (3.9) leads to the following scheme:

$$\begin{aligned} n_{j,0} &= n_j(t^*),\ n_{j,m+1} = J^{\alpha_1} A_{1,j,m}, \\ s_{j,0} &= s_j(t^*),\ s_{j,m+1} = J^{\alpha_2} A_{2,j,m}, \\ c_{j,0} &= c_j(t^*),\ c_{j,m+1} = J^{\alpha_3} A_{3,j,m}, \\ e_{j,0} &= e_j(t^*),\ e_{j,m+1} = J^{\alpha_4} A_{4,j,m}, \\ j &= 1, 2, 3, ..., n,\ m = 0, 1, 2, ...K. \end{aligned}$$

The solutions of system (4.3), (4.4) in each subinterval $[t_{j-1}, t_j]$, $j = 1, 2, ..., n,$ has the form

$$\begin{aligned} N_j(t) &= \sum_{m=0}^{K} n_{j,m}(t - t_{j-1}), \\ S_j(t) &= \sum_{m=0}^{K} s_{j,m}(t - t_{j-1}), \\ C_j(t) &= \sum_{m=0}^{K} c_{j,m}(t - t_{j-1}), \\ E_j(t) &= \sum_{m=0}^{K} e_{j,m}(t - t_{j-1}),\ j = 1, 2, ..., n, \end{aligned} \tag{4.7}$$

and the solution in the interval $[0, T]$ is given by

$$\begin{aligned} n(t) &= \sum_{j=1}^{n} \chi_v N_j(t), \\ s(t) &= \sum_{j=1}^{n} \chi_v S_j(t), \\ c(t) &= \sum_{j=1}^{n} \chi_v C_j(t), \\ e(t) &= \sum_{j=1}^{n} \chi_v E_j(t). \end{aligned} \tag{4.8}$$

System (4.3) with transformed initial conditions were solved analytically using the MSADM and numerically using the classical Runge Kutta method in the case of integer-order derivative. To demonstrate the effectiveness of the proposed algorithm as an approximate tool for solving the nonlinear system of fractional differential equations (4.3) with initial conditions (4.4) for larger t, we use a small time step. We assume that the optimal campaign continues for 200 days, so apply the proposed

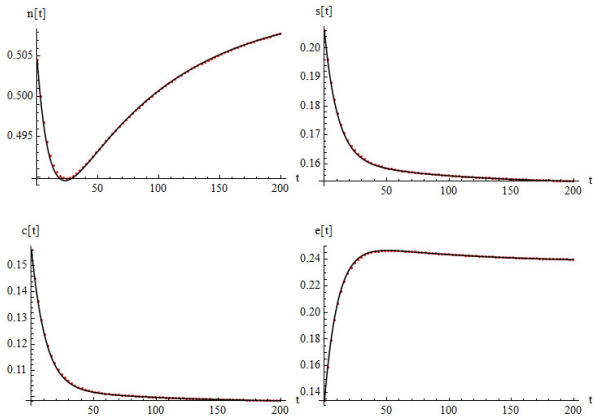

Figure 1: Comparing the result of the MSADM; dotted line, the RK4; solid line, the numerical solution of the system with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$

algorithm on the interval $[0, 200]$. We choose to divide the interval $[0,\ 200]$ into subintervals with time step $\Delta t = 1$ and we get the ADM solution at each subinterval. All the results are calculated using the computer algebra package Mathematica. Fig 1 shows the phase portrait for the classical model of smoking using the MSADM (when $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$) and the fourth-order Runge–Kutta method (RK4). From the graphical results in Fig.1, it can be seen the results obtained using the MSADM match the results of the RK4 very well, which implies that the MSADM can predict the behavior of these variables accurately for the region under consideration. Figs. 2 and 3 show the phase portrait for the fractional-order model of smoking obtained for different values of $\alpha(\ \alpha_1, \alpha_2, \alpha_3, \alpha_4)$ using the MSADM.

4. Conclusions

The analytical approximations to the solutions of the model for the habit of smoking habit are reliable and confirm the power and ability of the MSADM as an easy device for computing the solution of nonlinear problems. In this paper, a fractional differential model of smoking is studied and its approximate solution is presented using a MSADM. Comparisons of the results obtained by using the MSADM with that obtained by the classical Runge–Kutta method in the integer case reveal that the approximate solutions obtained by ADM are only valid for a small time, while the ones obtained by MSADM are highly accurate and valid for a long time. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability. Finally, the recent appearance of nonlinear fractional differential equations as models in fields such as science and engineering motivates investigating various solutions methods for such equations.

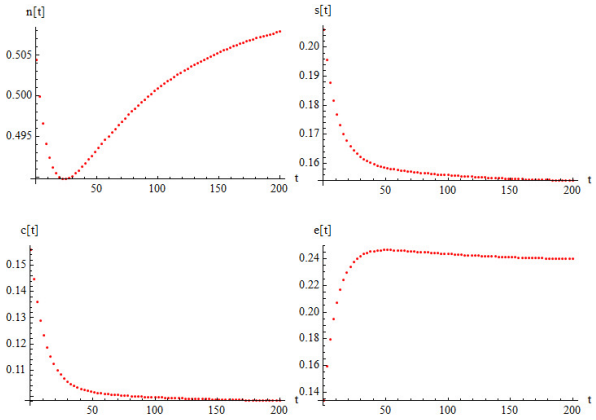

Figure 2: Phase plot of $n(t), s(t), c(t)$ and $e(t)$, with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.95$

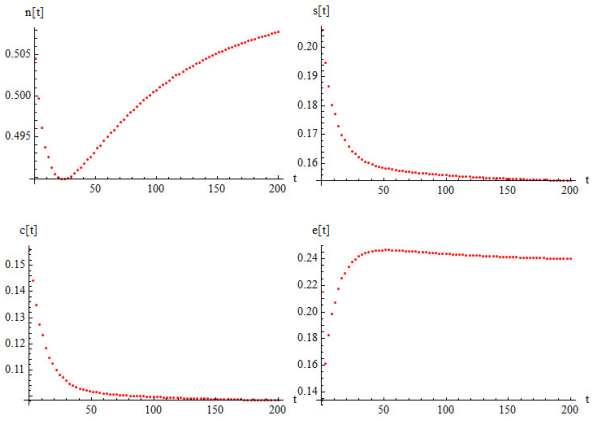

Figure 3: Phase plot of $n(t), s(t), c(t)$, and $e(t)$, with $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.001$

We hope that this work is a step in this direction.

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