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Observing Sites for the Central Solar Eclipses in Ancient Chinese History

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Abstract: We determine the observing sites for eclipses of large magnitude recorded in ancient Chinese chronicles from 200 BCE to 900 CE, by adopting the difference between terrestrial time and universal time, ΔT, given by Morrison & Stephenson (2004). The records of solar eclipses with large magnitude are divided into four groups in accordance with the historical variations of the capital cities of ancient Chinese dynasties. We determine areas in which all the eclipses in each group, with an eclipse magnitude larger than a certain threshold value, could be observed. We find that these areas coincide with the historical capitals, which agrees with the general idea that the solar eclipses were observed at the capital of each dynasty. This result also verifies the ΔT values during the period from 100 BCE to 400 CE, during which historical records of eclipses are so rare that the ΔT values can only be obtained by interpolating the long-term data. Moreover, we show that the eclipses described by the term *Ji* in East-Asian history are not all total eclipses; their mean magnitude is 0.96 ± 0.04. We find that complementary expressions, such as dark daytime and appearance of stars during the eclipse, strengthen the possibility that eclipses described by the term *Ji* were total. We also provide quantitative definitions for expressions such as 'being not complete and like a hook', 'being almost complete', 'visibility of stars during the eclipse', and 'darkness during an eclipse.' The literal meanings of these expressions are in agreement with the recent physical modeling of sky brightness during total eclipses provided by Können & Hinz (2008).

Key words: eclipses — ephemerides — history and philosophy of astronomy

1. Introduction

A solar eclipse occurs when the Moon casts its shadow on the Earth's surface, such that the Sun is observed to be totally or partially obscured. Total or annular solar eclipses are so spectacular that large numbers of eclipse records from many civilizations are found. In particular, these events were not only regarded as important omens by ancient East Asians, but also considered to be useful for establishing and maintaining calendar systems. Thus, eclipses were systematically observed by astronomers in those civilizations.

Historical records of solar and lunar eclipses are useful for measuring both the long-term changes in the Earth's rate of rotation and the tidal deceleration of lunar orbital motion. The misalignment of the tidal bulge of the Earth relative to the line connecting Earth and Moon causes a secular torque between them. It produces a secular acceleration of the lunar longitude $\dot{n} = -25.8''/\mathrm{cy}^2$, a well-known displacement of the Moon's barycenter that corresponds to an increase in the Earth–Moon distance of 3.8 cm/year, and a slow-down in the Earth's spin (Stephenson 1997, pp. 35–36).

Variations in the Earth's rate of rotation or the rate of change in the length of day (LOD) of 2.3 ± 0.1 ms/cy produce the cumulative effect represented by ΔT, which is defined as the difference between Terrestrial Time (TT) and Universal Time (UT). UT is defined by the variable rotation of the Earth and TT is an ideally uniform time-scale computed from the positions of solar system bodies theoretically expected from the laws of gravitation. The value of ΔT in the epoch 500 BCE is as much as 5 hours, and that in the epoch 1000 CE is as much as 30 minutes (Morrison & Stephenson 2004). Thus, accurate knowledge of the ΔT value is crucial for accurate calculations of ancient eclipses (Meeus 1998a).

Values of ΔT over the telescopic era, between 1620 CE and 1955 CE, are usually determined from a comparison of observed and computed positions of the Moon derived from lunar occultation of stars (Stephenson & Morrison 1984). For the pre-telescopic era, eclipses provide the most relevant data for determining ΔT. Approximately 400 ancient and medieval solar and lunar eclipse records from East Asia, Babylonia, Greece, Europe, and Arabia were compiled and analyzed by comparing individual values of UT derived from observations with the computed equivalent of TT, and ΔT curves were fitted using spline curves with several nodes (Stephenson et al. 1995; Stephenson & Morrison 1995; Stephenson 1997; Morrison & Stephenson 2001, 2004, 2005). We can also find a tabular form of ΔT values (Stephenson & Morrison 2005). However, ΔT values obtained from timed solar and lunar eclipses, as can be seen in Figure 3 of Morrison & Stephenson (2004), in Figure 9 of Stephenson & Morrison (2005), or in Figure 9 of Stephenson et al. (2016), show a period of sparse data

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from 100 BCE to 400 CE. For this period, we have to rely interpolated values of ΔT, which still need to be verified independently. In order to verify the consistency of ΔT values given by Morrison & Stephenson (2004, 2005), we will check the observing sites determined from eclipse records described as *Ji*[1] eclipses.

In order to derive ΔT values from historical eclipses, the following requirements must be met (Stephenson & Morrison 1995): (i) the geographical position of the observer must be known accurately, (ii) the exact date of the event must be known or calculable from other circumstances associated with the event, and (iii) the value of UT, or its equivalent, must be given or calculable from the reported circumstances. The requirements related to the dates can be met with good accuracy. All dates from the beginning of the *Han* dynasty (202 BCE) can be converted to Julian calendar dates (Stephenson & Morrison 1995). However, most of these records are brief, giving no more than the dates of occurrences, and hence those few observations that contain important details are useful. Another kind of observation that is useful in determining ΔT are eclipse records whose timing was expressed relative to the local horizon, as the time-interval from sunrise or sunset, or in terms of the altitude of the Sun, the Moon, or a selected bright star (Stephenson & Morrison 1995).

Measurements of ΔT require the accurate geographical location of the observer. However, historical records of East Asian eclipses do not include the observers' explicit location. When historical eclipses are studied, the observers' location is usually presumed to be the capital city of each dynasty. The geographical location of the capital is presumed to be known from historical and archaeological studies. However, these presumptions need to be verified, and the self-consistency of the resultant ΔT table also needs to be verified. Thus, in this work, we check whether the historical eclipses of large magnitude could have been observed in the dynastic capitals of Chinese history, assuming that the latest value of ΔT given by Morrison & Stephenson (2004, 2005) is correct.

Watanabe (1979) concluded that a *Ji* eclipse was judged to be total if its magnitude was greater than approximately 0.9. Saito & Ozawa (1992) found that none of the eclipses termed *Ji* in Chinese history books from the 5^{th} century to the 9^{th} century was total. In many previous investigations, it has been assumed that *Ji* eclipses are the same as present-day total eclipses. However, this assumption needs to be verified. Furthermore, additional expressions describing large magnitude eclipses in Chinese history (and the histories of other East Asian countries), e.g., 'almost complete', 'not complete and like a hook', 'visibility of stars', and 'dark daytime', were used. In this paper, we will provide quantitative definitions for all these phenomena.

The basic records are compiled from the solar eclipse data cataloged in *Zhongguo Gudai Tianxiang Jilu Zongji* (General Compilation of Astronomical Records in Ancient Chinese Histories) from the *Han* dynasty to the end of the *Tang* dynasty (Beijing Observatory 1988). There are approximately 30 *Ji* solar eclipses mentioned for this period. Section 2.3 describes the Chinese records of eclipses, whose translations are given in Appendix A. In order to calculate historical eclipses, the coordinates of the Sun and the Moon need to be calculated with sufficient accuracy. Either VSOP87/ELP2000/85 or DE406/LE406 is widely used, but here we use the simple and practical method given in Meeus (1998b), which is sufficiently accurate for historical astronomy. A detailed discussion on the calculation methods can be found in Section 2.1 of this paper. We adopt the spline-fit values of ΔT given in Morrison & Stephenson (2004, 2005), after correcting for the difference in the lunar acceleration parameter $\dot{n}$ between the ΔT table and Meeus' method. Our results are given in Section 3, and the conclusion follows in Section 4.

[1] A Chinese character meaning 'complete' or 'already'.

2. METHOD

2.1. Calculation of Solar Eclipses

Here, we discuss the calculation of a solar eclipse. For a given geographical location of an observer, we make time-sequence calculations of eclipse magnitude. We then find the magnitude and time of the eclipse maximum. We repeat these calculations for other geographical locations.

We first compute the positions of the Sun and the Moon. The precise coordinates of the Sun can be calculated with the VSOP87 model of Bretagnon & Francou (1988), and the precise coordinates of the Moon can be calculated by using the ELP2000/82 model (Chapront-Touzé & Chapront 1983). The Five Millennium Canon of Solar Eclipses from 2000 BCE to 3000 CE, calculated by Espenak & Meeus (2006) and serviced by NASA/JPL, adopts these theories and the mean arguments, such as L', D, M, M', and F given by Chapront et al. (2002). The arguments were obtained by applying the lunar tidal acceleration $\dot{n} = 25.858''/\text{cy}^2$ measured from Lunar Laser Ranging data. We also used the DE-series packages to generate the Planetary and Lunar Ephemeris, including DE405 (Standish 1998) which encompasses the time span from 9 December 1599 to 20 February 2201. While the VSOP/ELP models are semi-analytic, the DE-series packages make use of numerical integrations of the differential equations of celestial mechanics which determine the lunar motion.

In order to meet the needs for deriving ΔT from historical research, the VSOP/ELP and DE-series were modified to be less accurate but more stable over a longer time span. DE406 is the same ephemeris as DE405, but the accuracy of the interpolating polynomials was degraded. For DE406/LE406, the interpolating accuracy is no worse than 1 meter for the moon. DE406/LE406 is able to encompass a longer time span, from 3000 BCE to 3000 CE. ELP-2000/85 was obtained from ELP-2000/82 by disregarding a large number of small terms and by adding third and fourth powers of time to the arguments, including the Delaunay arguments. These modifications make ELP2000-85 less precise than ELP-2000/82 by a factor of 50 over the time span of a few centuries, but more stable over a longer time span. A detailed description of ELP 2000-85 can be found in Chapront-Touzé & Chapront (1991). Although these two kinds of computational methods are accurate, we use the simplified version described in Meeus (1998b), because it is simple and sufficiently accurate to be used for historical work.

In this paper, the geocentric position of the Sun is computed by using the method given in Chapter 25 of Meeus (1998b); this comprises the most important terms from VSOP87, allowing the accuracy in the measurement of the Sun's position to have an error not exceeding $1''$ between the years 2000 BCE and 6000 CE (Meeus 1998b). The $1''$ error in the Sun's position is approximately equal to a 1.5 km displacement of the Moon's shadow projected on the Earth's surface.

The geocentric position of the Moon is calculated using the abridged solution of ELP 2000-85 (Chapront-Touzé & Chapront 1988) given in Chapter 47 of Meeus (1998b). There, the lunar orbital acceleration is assumed to be $\dot{n} = -23.8946''/\mathrm{cy}^2$ (Chapront-Touzé & Chapront 1988). The solution can be used to compute lunar positions with a precision of approximately $20''$ and is valid over the time span from 1500 BCE to 2000 CE. According to Meeus (1998b), the results are accurate to within approximately $10''$ of the longitude of the Moon and $4''$ of its latitude. The $20''$ error in the Moon's position approximately equals a 30 km displacement of the Moon's shadow projected on the Earth's surface. In the 2nd edition of his book, Meeus (1998b) adopted the improved Delaunay arguments in Chapront et al. (1998). These arguments were also based on a lunar orbital acceleration $\dot{n} = -23.8946''/\mathrm{cy}^2$. The arguments have since been improved further (Chapront et al. 2002). However, we do not take them into account in this paper, and merely calculate the lunar ephemeris by adopting the arguments in Meeus (1998b).

Accurate knowledge of ΔT is crucial for calculating the local circumstances of solar eclipses. Morrison & Stephenson (2004, 2005) obtained hundreds of ΔT values from historical records and derived the best fit that describes the ΔT values from 700 BCE to 1600 CE. In this paper we use the spline-fits of the ΔT values listed in Table 1 of Morrison & Stephenson (2004, 2005). These values somewhat differ from those in Stephenson (1997, pp. 515–516), but the lunar tidal acceleration, $\dot{n} = -26.0''/\mathrm{cy}^2$, is the same. Since the method in Meeus (1998b) is based on $\dot{n}_{\mathrm{Meeus}} = -23.8946''/\mathrm{cy}^2$, we adjust the ΔT_{MS} values of Morrison & Stephenson (2004, 2005) to be consistent with the lunar acceleration used in Meeus (1998b) by using the relationship $\Delta T_{\mathrm{Meeus}} = \Delta T_{\mathrm{MS}} - 0.91072(\dot{n}_{\mathrm{Meeus}} + 26.0)u^2$, where $u = (\mathrm{year} - 1955.5)/100$ (Lieske 1987). The corrections amount to a movement of the eclipse zone to the west by 3.1° (1 CE), 1.7° (500 CE), and 0.7° (1000 CE).

We obtained the geocentric longitude, latitude, and the distances to both the Sun and the Moon. (Note that the following explanation is for the Sun. The coordinates of the Moon can be calculated using similar procedures.) The geocentric coordinates are transformed into Cartesian coordinates $\vec{r}_s$ in an Earth-fixed geocentric, right-handed coordinate system, with the xy-plane being the Earth's equatorial plane, the xz-plane being the Greenwich meridian plane, and the z-axis pointing towards the north celestial pole. The geocentric position vector $\vec{R}_o$ of the observer on the Earth's surface in this coordinate system is computed, considering both the Earth's shape parameters and the observer's latitude, longitude, and height above the mean sea level. A detailed explanation is found in Seidelmann (1992). In this work, the geographical longitudes are measured positively eastward from the meridian of Greenwich and the effects of ΔT are considered.

The Earth's shape parameters include the equatorial radius a = 6,378.137 km and the flattening of the reference ellipsoid $f = 1/298.257223563$. We also assume 1 au = 14,959,787 km. In order to compute the geocentric position vector $\vec{R'}_o$ of the observer for a given time and geographical location, we rotate the vector $\vec{R}_o$ by an angle equivalent to the sidereal time. The sidereal time is likewise computed using the method of Meeus (1998b). The position vector of the Sun, $\vec{R}_s$, for an observer on the Earth's surface is $\vec{R}_s = \vec{r}_s - \vec{R'}_o$. The position vector of the Moon, $\vec{R}_m$, can be calculated in a similar manner. In this step, we can compute the altitude of the Sun. If the Sun is very close to the horizon, we take atmospheric refraction into account by using Saemundsson's formula (Meeus 1998b).

The apparent angular radius of the Sun, π_s, can be obtained from $\pi_s = D_s/|\vec{R}_s|$, where D_s = 695,989 km is the radius of the Sun. Similarly, the apparent angular radius of the Moon is $\pi_m = D_m/|\vec{R}_m|$, where $\vec{R}_m$ is the position vector of the Moon and the radius of the Moon is D_m = 1738.091 km. From the position vectors of the Sun and the Moon, we can obtain the relative angular distance θ by assuming a spherical geometry. The eclipse magnitude μ, which is defined as the fraction of the obscured part in units of the diameter of the Sun, follows from $\mu = (\pi_s + \pi_m - \theta)/2\pi_s$. These computations are repeated at steps of 30 seconds during a time span of $|t - T_{\max}| < 5$ hours, where $T_{\max}$ is the time of the greatest eclipse. Although the time of the greatest eclipse can be found by considering the eclipse conditions described in Chapter 54 of Meeus (1998b), we adopt the values in the Five Millennium Canon of Solar Eclipses (Espenak & Meeus 2006) for simplicity.

From the results of the above calculations for a given geographical location of an observer, the time of the greatest eclipse, the maximum magnitude of the eclipse, and the contact times can be obtained accurately by interpolation. The contact times can be found by considering the fact that the contact of the solar disk and the lunar disk occurs when the sign of eclipse magnitude changes. If the greatest eclipse occurs when the Sun is below the horizon, we obtain the maximum magnitude possible during daytime. Repeating the same procedures for each geographical point on 0.25° × 0.25° grid points on the surface of the Earth, we obtain a 2-D visibility map of eclipse magnitudes, i.e. an eclipse map.

2.2. Determination of Observing Sites

A solar eclipse is observed in a specific manner by a certain observer on the Earth's surface. The specific parameters are usually referred to as local circumstances. In turn, we can determine the observer's location from the known local circumstances, which are given by observables such as the eclipse magnitude, the time for each phase of the solar eclipse, and the direction of the obscuration. Here, the eclipse magnitude μ is the fraction of the solar diameter that is obscured by the Moon. The time can be either the time of maximum obscuration or one of the contact times. The direction of obscuration denotes which part of the solar disk is obscured by the Moon. Unfortunately, such observables are usually not given in the records of historical eclipses. The eclipse magnitudes were rarely recorded, except for *Ji* eclipses. Even when the magnitude was given in the records, its measurement error is approximately 0.05 because the angular size of the Sun/Moon is $30'$ and the angular resolution of bare eyes is roughly $1'$–$2'$. Likewise, the eclipse times are rarely given. Even when the eclipse times are given, they are regarded as the time of the greatest eclipse. The measurement error of time is usually one hour and at best 15 minutes. Therefore, in order to determine the observing sites, we must make full use of the available incomplete information.

The most useful records appear to be those of central eclipses, because central eclipses have no ambiguity or degeneracy resulting from the direction of obscuration. Here, 'central eclipse' is a generic term for total, annular, or hybrid eclipses. The observing site for a central eclipse can be determined if the observing time is known. The time information can be either the time of the greatest eclipse, contact times, or the position of the Sun relative to the horizon. The observing site can be determined in principle by two records of central eclipses that were observed at the same place, even if the records have no detailed time information other than the date; it is highly probable that the observing site is located in the intersecting zone of the paths of the central eclipses. In this case, the more records we have, the narrower the observation zone becomes.

Several previous studies have located observing sites of historical eclipses (Park & La 1994; Park 1996). In those works, the area showing the maximum mean magnitude of eclipses is regarded as the most probable observing site. Apparently, these studies assumed implicitly that the observability of eclipses is proportional to the eclipse magnitude. However, that assumption appears dubious because we usually experience that, even though the magnitudes of eclipses are small, the eclipses can be observed with bare eyes if the Sun was extincted at low altitudes or the Sun is screened by clouds. Moreover, we can recognize the eclipsed solar disk at a glance during the eclipse even when the altitude of the Sun is relatively high. Our approach does not rely on implicit assumptions since we only use central eclipses. Furthermore, our work aims at a quantitative understanding of the characteristics of historical eclipses.

2.3. Chinese Historical Eclipses

In historical chronicles, there are records of solar eclipses that were reported as total. In the cases of East Asian historical eclipses, it is believed that a total eclipse was described by using the term *Ji*. *Ji* was sometimes replaced by the term *Jin*.[2] Both mean 'complete,' but it is possible that *Ji* (or *Jin*) does not always indicate totality. Based upon his own computations of Chinese historical eclipses, Watanabe (1979) asserted that a historical eclipse was judged to be a *Ji* eclipse if the eclipse magnitude was greater than approximately 0.9. Stephenson (1997, pp. 262–265) analyzed the records of the *Ji* eclipses in *Goryeosa* (Annals of the *Goryeo* dynasty) and concluded that at least some of the records may merely represent abbreviated descriptions of eclipses that were originally described as being 'almost complete.'

Various historical eclipses have been described as 'almost complete.' The expression appears to refer to a partial eclipse of a large magnitude that is less than 1.0. Ambiguity may occur in the cases of annular eclipses. It is not known whether the term *Ji* was also applied to annular eclipses. Hence, the expression 'being almost complete' could imply either the ring phase of annular eclipses or a total eclipse that was observed as a large partial eclipse (Stephenson 1997, p. 233). Tanikawa & Soma (2004) investigated the 'almost complete' solar eclipse of 17 July 188 BCE in *Hanshu* (the Annals of the *Han* dynasty). Since this eclipse was also observed in Rome as an obvious total eclipse, they could obtain a reliable magnitude $\mu > 0.93$ at *Chang'an*, the present-day *Xian*. Thus, we can regard this value as a guideline for 'almost complete' eclipses.

The term *Ji* is sometimes accompanied by allusions to the visibility of stars in daytime; for example, the eclipses of 429 CE, 454 CE, and 761 CE in Table 1 are described in this manner. According to the historical records of 429 CE, 'star(s) was(were) seen during the solar eclipse'. During the eclipse of 454 CE, 'stars and asterisms were very brilliant'. During the solar eclipse of 761 CE, 'all the bright stars could be seen'. Können & Hinz (2008) investigated the visibility of stars and planets during the partial and total phases of a solar eclipse. They gave limiting magnitudes for the visibility of stars during a solar eclipse, as listed in Table 2 of their paper. Here, the limiting magnitude is the stellar magnitude of the faintest star that can be seen. According to Table 2 of their paper, many stars can be seen only when the magnitude of eclipse is at least 0.99. The 'large stars' in the record of 761 CE might be first magnitude stars and bright planets. According to Können & Hinz (2008), all the planets visible to the unaided eye can be seen when the eclipse magnitude is larger than 0.95, and first magnitude stars become visible in the sky when the eclipse magnitude is greater than approximately 0.98.

Some records of large partial eclipses are accompanied by allusions to darkness. The records of the eclipses of 181 BCE, 120 CE, and 429 CE are examples of such records. An 'almost complete' eclipse occurred in 120

[2] This is a Chinese character meaning 'complete' or 'exhausted.'

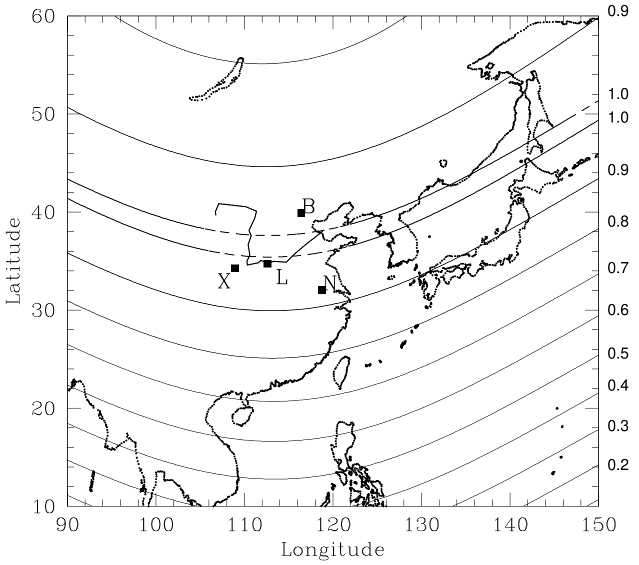

Figure 1. Eclipse map of 429 CE. Continuous lines are contours of equal eclipse magnitude. The dashed lines represent the boundaries of the total eclipse. The magnitudes are given to the right of the diagram. The wiggly line represents *Huanghe* or the Yellow River. N represent *Nanjing* or *Jian-kang*; X represents *Xian* or the capital city of both the *Han* and *Tang* dynasties; L represents *Luoyang*; B represents *Beijing*.

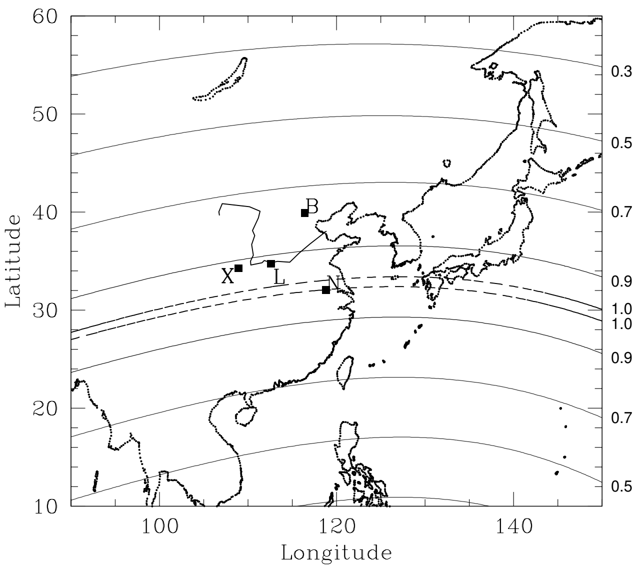

Figure 2. Eclipse map of 454 CE. The central zone is marked by dashed lines. X represents *Xian*; N represents *Nanjing*; L represents *Luoyang*; B represents *Beijing*.

CE, when 'the ground became as dark as in the evening dusk.' The term 'dusk' in ancient astronomy of East Asian dynasties is roughly equivalent to nautical twilight (Ahn & Park 2004). The sky brightness during the total eclipse of 29 March 2006 was observed to be equivalent to the sky twilight brightness at a solar altitude of approximately $-6°$ (Nawar et al. 2007), which is the boundary between civil twilight and nautical twilight. The limiting magnitude of the sky at altitudes between 10° to 30° during nautical twilight is approximately $m_l = +2^{\mathrm{m}}.5$ (Nawar 1983), corresponding to an eclipse magnitude of 0.995 in Können & Hinz (2008). Thus, the eclipse of 120 CE was close to being total.

The eclipse of 429 CE is interesting because there are records of multiple observations. One observation was made at the capital *Jian-kang*, present-day *Nanjing*, and the other at the north of the Yellow River. The first observation stated that a star[3] could be seen during the eclipse that was 'not complete and like a hook.' It is difficult to estimate the abundance of observed stars because classical Chinese characters have no plural forms. Our calculation finds an eclipse magnitude $\mu = 0.93$ in *Nanjing*, which corresponds to the appearance of bright planets, such as Venus, Mercury, and Jupiter (Können & Hinz 2008). Note that Saito & Ozawa (1992, p. 203) regarded the star as Venus, because the magnitude of Venus was $-3.^{\mathrm{m}}8$ and it was located 45° west of the Sun on that day.

[3]Note that the character *Xing* meaning 'star' has the same form for both singular and plural. In addition, this character can mean both planets and stars.

The second observation stated that the area north of the Yellow River became dark during the eclipse. Our calculation for this eclipse is shown in Figure 1, where the Yellow River is depicted as a wiggly line. The boundaries of the totality zone are roughly parallel to the latitude lines, which implies that the ΔT value does not affect our interpretation. Since the totality zone is largely located north of the Yellow River, we see that the darkness implies an eclipse as strong as or even stronger than eclipses described as both 'visibility of stars' and 'almost complete and like a hook.'

For the eclipse of 181 BCE, it was said that the day became as dark as dusk during the *Ji* eclipse. Our calculation shows that this eclipse actually occurred with magnitude $\mu = 0.99$ at *Chang'an*. A continuous decrease in the sky brightness can be observed when the eclipse magnitude exceeds 0.9 (Können & Hinz 2008). In addition, the darkness during an eclipse implies that it becomes as dark as a nautical twilight. This happens only when the limiting magnitude of the sky is larger than $2^{\mathrm{m}}.5$, which corresponds to an eclipse magnitude of 0.995 (Können & Hinz 2008).

Another expression for large eclipses is that 'the Sun looked like a hook during the eclipse.' Several examples can be found in Table 1 and Appendix A. This expression implies that the non-eclipsed part solar disk at the maximum phase looked like a hook. Such a crescent Sun can be seen during either an annular eclipse or a large partial eclipse, which Stephenson (1997, p. 62) has discussed in detail. We also note that ancient people could see the solar disk, which can be seen when the Sun's altitude is very low. Ancient people also used primitive tools to diminish the sunlight (Stephenson 1997, p. 463).

The eclipse of 429 CE discussed above is an example of hook-like eclipses. These eclipses are usually accompanied by various expressions, such as 'not complete', 'almost complete', and 'visible stars'. These are unusually large partial eclipses. The additional expressions discussed above imply that the magnitudes of hook-like eclipses can be diverse, and greater than 0.8, but less than 1.0.

Table 1 lists the Chinese historical solar eclipses that were reported as being more than large partial eclipses. Here, we neglect those eclipse records that were obviously predictions; for example, the eclipse of 794 CE (see the records in Appendix A). In column 7 of Table 1 we show eclipse magnitudes $\mu_{\rm est}$ estimated by comparing the descriptions with the results of Können & Hinz (2008). In column 8, $\mu_{\rm obs}$ is the computed value of the magnitude at the eclipse maximum, observed at the capital of each dynasty on a given date. $T_{\rm max}$ is the time of maximum eclipse in local time, and $A_{\rm Sun}$ is the altitude of the Sun at maximum eclipse in Babylonian degrees.

3. RESULTS

3.1. Observer Locations

We grouped large historical eclipses in Chinese history from 200 BCE to 900 CE into four groups in accordance with the historical capitals. The Western *Han* dynasty (206 BCE to 8 CE) established its capital city at *Chang'an* or present-day *Xian*, the Eastern *Han* dynasty (25 to 220 CE) at *Luoyang*, the Southern Dynasties (420 to 589 CE) at *Jian-kang*, or the present-day *Nanjing*, and the *Sui* (589 to 618 CE) and *Tang* (618 to 907 CE) dynasties at *Chang'an*. In Table 2, we list the capitals and their geographical coordinates and altitudes, which are used in our calculations.

Eclipse maps with equi-magnitude contours are shown in Figures 1 and 2. The dashed lines mark an eclipse magnitude of 1.0. Figure 2 is the eclipse map for the historical eclipse of 454 CE, which was recorded in the chronicles of the *Liu-Song* dynasty (420–479 CE). The capital city of the dynasty was *Jian-kang* or present-day *Nanjing*. According to the chronicle, 'the eclipse was a *Ji* eclipse, and all the stars in lunar lodges were brightly lit'. This description implies that the magnitude of the eclipse was large. Our calculation shows that the eclipse was a total eclipse in *Nanjing*.

3.2. Western *Han* Dynasty

There are ten central eclipses recorded in the chronicles of the Western *Han* dynasty (206 BCE to 8 CE). Among these, the eclipse of 34 BCE did not occur on the dates given in the record. According to the chronicles, the eclipses of 147 BCE and 80 BCE were not *Ji* eclipses but merely 'almost complete' eclipses. In addition, the eclipse of 2 BCE was 'not complete and like a hook.' Our calculations demonstrate that these three eclipses were relatively small-magnitude eclipses with $\mu < 0.8$ at *Chang'an*. Hence, Pankenier (2012) suggested that the eclipse record of 147 BCE was inherited from a provincial report of near totality, and pointed out the fact that the eclipse records of 80 BCE contain inconsistent descriptions of a 'total' and 'almost total' eclipse. Our calculation shows that the eclipse of 2 BCE occurred at sunrise and that its maximum magnitude at *Chang'an* was 0.85. Hence, although its eclipse magnitude was relatively small, a hook-shaped solar disk could be observed during the sunrise. However, this eclipse cannot be regarded as a central eclipse. Therefore, since these three eclipses were not recorded as being central eclipses, we discard them and just use six records to draw a visibility map, which is shown in Figure 3. We see that at the historical capital city of the Western *Han* dynasty (*Chang'an* or present-day *Xian*, all the historical eclipses of magnitudes larger than 0.85 could be observed.

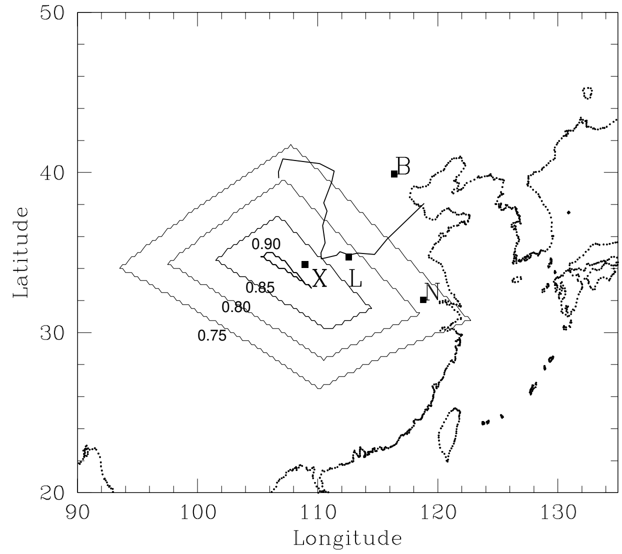

Figure 3. Visibility map for the historical eclipses of the Western *Han* dynasty. The contours show the areas within which eclipses of a certain magnitude larger than a certain threshold value were visible. The threshold values are given next to the contours. Here X, L, N, and B represent *Xian*, *Luoyang*, *Nanjing*, and *Beijing*, respectively. The historical capital during the Western *Han* dynasty was *Chang'an*, i.e., present-day *Xian*. Its location approximately coincides with the common area where all the central eclipses were visible.

3.3. Eastern *Han* Dynasty and Northern Dynasties

A group of eclipses were recorded as having been witnessed at *Luoyang* or the common capital of the Eastern *Han* dynasty (25–220 CE), the *Wei* dynasty (220–265 CE) of the Three Kingdoms, and the Western *Jin* dynasty (265–420 CE). There are only four records of large solar eclipses in these periods. Among these, the eclipse of 243 CE was observable at *Luoyang*, but the central zone was far away from *Luoyang*. Hence, we discard this record, and draw the visibility map shown in Figure 4. The permitted area is relatively large, due to the small number of historical eclipses, but we see that all three eclipses could be observed as large eclipses of magnitudes larger than 0.95 at *Luoyang*.

3.4. Southern Dynasties

The *Jin* dynasty unified China in 265 CE, its capital was *Luoyang* until 311 CE. However, only half a century

Table 1
Solar eclipses greater than large partial eclipses in Chinese history

Number	Reality	Date	ΔT	Place	Description	μ_{est}	μ_{obs}	Type	$T_{\max}$	A_{Sun}
Western *Han* Dynasty										
1	Yes (Yes)	−197-08-07	12754	X	Ji	0.9	0.97	A	09:03	47
2	Yes (Yes)	−187-07-17	12633	X	Ji, almost	0.9	0.88	T	15:26	44
3	Yes (Yes)	−180-03-04	12550	X	darkness, Ji	0.9	0.99	T	15:10	31
4	No (Yes)	−146-11-10	12155	X	almost	0.93	0.79	T	11:03	38
5	Yes (Yes)	−88-09-29	11509	X	hook	0.93	0.90	H	15:38	26
6	No (Yes)	−79-09-20	11410	X	almost	0.93	0.84	T	13:12	53
7	No (No)	−33-08-23	—	X	hook	0.8	—	—	—	—
8	Yes (Yes)	−27-06-19	10858	X	Ji, hook	0.9	0.96	T	09:38	58
9	No (Yes)	−1-02-05	10590	X	hook	0.9	0.85	H	08:04	11
10	Yes (Yes)	2-11-22	10560	X	Ji	0.9	0.93	H	08:18	16
Eastern *Han* Dynasty										
1	Yes (Yes)	65-12-16	9938	L	Ji	0.9	0.98	T	09:50	24
2	Yes (Yes)	120-01-18	9408	L	Ji; all stars; almost, evening	0.99	1.01	T	13:50	30
3	No (Yes)	243-06-05	8228	L	Ji	0.9	0.63	A	06:44	23
4	Yes (Yes)	306-07-27	7622	L	Ji	0.9	0.96	A	15:50	38
Southern Dynasties										
1	Yes (Yes)	360-08-28	7094	N	hook; Ji; almost	0.9	0.91	A	10:54	63
2	Yes (Yes)	429-12-12	6165	N	hook, stars	0.9	0.93	T	12:15	34
3	No (No)	453-08-20	—	N	Ji, all stars	0.99	—	—	—	—
4	Yes (Yes)	454-08-10	6165	N	Ji, stars	0.9	1.00	T	08:42	42
5	Yes (Yes)	516-04-18	5553	N	Ji	0.9	0.94	A	07:07	21
6	Yes (Yes)	522-06-10	5494	N	Ji	0.9	1.00	T	08:09	40
7	No (Yes)	523-11-23	5484	N	Venus	0.95	0.89	A	11:35	37
***Sui* and *Tang* Dynasties**										
1	No (Yes)	562-10-14	5105	X	Ji	0.9	0.33	A	07:21	14
2	Yes (Yes)	616-05-21	4588	L	Ji	0.9	0.93	A	15:55	36
3	Yes (Yes)	702-09-26	3792	X	hook; almost	0.9	0.97	T	15:17	29
4	No (No)	703-10-16	—	X	Ji	0.9	—	—	—	—
5	Yes (Yes)	729-10-27	3555	X	hook	0.9	0.92	T	06:58	7
6	No (Yes)	754-06-25	3340	X	hook; almost	0.9	0.84	T	11:42	79
7	Yes (Yes)	756-10-28	3324	Nx	Ji	0.9	0.94	T	17:05	sunset
8	Yes (Yes)	761-08-05	3281	X	Ji, bright stars	0.98	1.00	T	09:46	54
9	No (Yes)	822-04-25	2784	B	Ji	0.9	1.03	T	12:53	62
10	No (No)	879-04-25	—	X	Ji	0.9	—	—	—	—
11	No (Yes)	888-04-15	2285	X	Ji	0.9	0.69	A	10:56	63

'Reality' (column 2) has the form A(B), where B describes whether an eclipse occurred while A describes whether it has been used for analysis. Dates (YYY-MM-DD) are Julian; note that there is no year zero. ΔT is an interpolated value based on the table given by Morrison & Stephenson (2004). Sites are the capitals of each dynasty, where X means *Chang'an* or present-day *Xian*; L means *Luoyang*; N means *Jian-kang* or present-day *Nanjing*; Nx means *Ningxian*; B means *Beijing*. 'Description '(column 6) provides a short description of each eclipse: *Ji* means a *Ji* eclipse; 'almost' means an eclipse that is almost complete; 'darkness' means an eclipse where darkness appeared; 'evening' means an eclipse during which it was as dark as evening; 'hook' means an eclipse that is 'not complete and like a hook'; 'stars' means an eclipse where stars appeared. See Appendix A for detailed records. μ_{est} is the eclipse magnitude estimated from the descriptions given in the records; μ_{obs} is the eclipse magnitude calculated by assuming that the observations were made at the capital of each dynasty. 'Type' (column 9) is the eclipse type: A means annular, T means total, H means hybrid. $T_{\max}$ is the time of maximum eclipse, and A_{Sun} is the altitude of the Sun at the maximum eclipse

later, the northern half of the country was overrun by nomads from the north, and *Luoyang* was destroyed. The remnants of the *Jin* court fled to the east and reconstructed the court in *Jian-kang*, which corresponds to modern-day *Nanjing*. This dynasty is called the Eastern *Jin* dynasty (317–420 CE). After the dynasty declined in 420 CE, a series of relatively short-lived dynasties, such as the *Liu-Song* (420–479 CE), Southern *Qi* (479–502 CE), *Liang* (502–557 CE), and *Chen* (557–589 CE) dynasties followed. These dynasties are called the Southern Dynasties. The capital of the Southern Dynasties was still *Jian-kang*, and it was seldom moved. In the north, the major dynasty was the Northern *Wei* dynasty (386–535 CE), whose capital was initially present-day *Datong*, and *Luoyang* later.

All the historical records of large eclipses during this period happen to be those of the Southern dynasties. Thus, we combine these records into one group, and

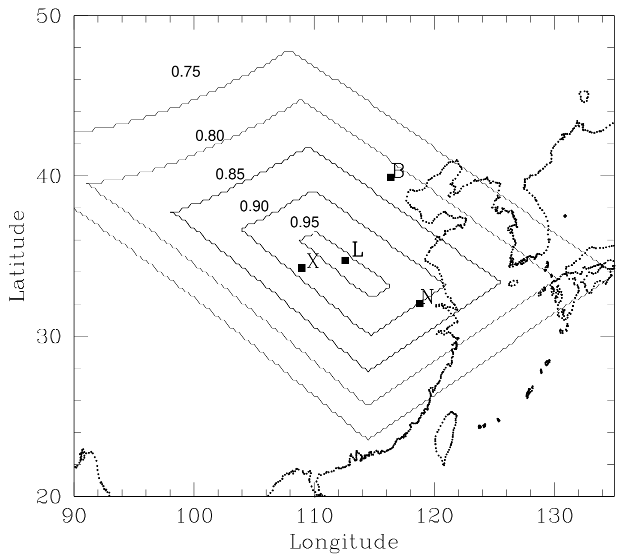

Figure 4. Visibility map similar to Figure 3, except for the historical eclipses of the Eastern *Han* dynasty. The innermost contour represents the boundary for a threshold magnitude of 0.95. The historical capital city of the dynasty was *Luoyang*, which is denoted by L. The historical center of the dynasty coincides with the common area where all the central eclipses were visible.

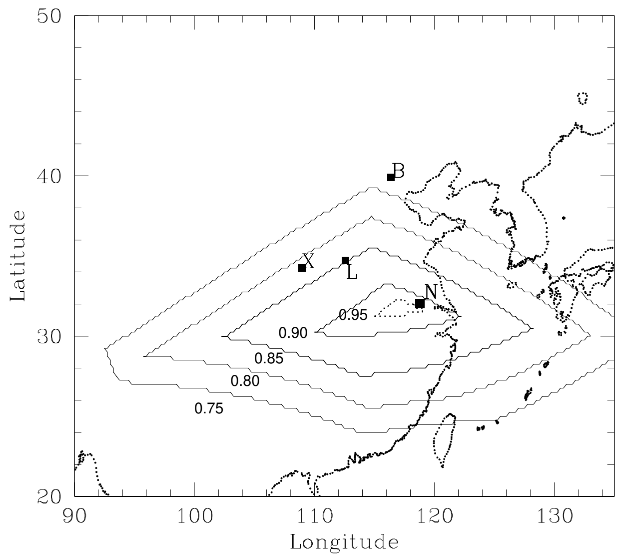

Figure 5. Visibility map, similar to Figures 3 and 4, except for the historical eclipses of the Southern Dynasties, such as the *Liu-Song*, Southern *Qi*, *Liang*, and *Chen* dynasties. The thick central dotted line represents the boundary for a threshold magnitude of 0.95. The historical capital during this period was *Jian-kang* or present-day *Nanjing*, which is denoted by N. The historical center of the dynasties coincides with the common area where all the central eclipses were visible.

look for a common place of observation. We have seven records, but our calculation shows that the eclipse of 453 CE did not occur on the date given in the historical record. As Stephenson (1997, p. 243) and Saito & Ozawa (1992) pointed out, the chronist noted down an incorrect calendar date. This record is probably the same as that of 454 CE. Another event that must be remarked upon is the eclipse of 523 CE. The eclipse seems to be described as being accompanied by the appearance of stars during daytime. However, the two phenomena were not described as having a cause-and-effect relationship. Likewise, Saito & Ozawa (1992) regarded these two phenomena as being independent. According to them, Venus was 44° east of the Sun and as bright as $-4^{\mathrm{m}}.3$ on the given date. Thus, we neglect this record in our analysis. We draw a visibility map, as shown in Figure 5, for the five eclipses. We note that all five eclipses could be observed as eclipses of magnitude as large as 0.95 at the historical capital of the Southern Dynasties or *Jian-kang* (modern-day *Nanjing*).

3.5. *Sui* and *Tang* Dynasties

The Southern and Northern Dynasties were unified by the *Sui* dynasty in 589 CE. The capital of the *Sui* dynasty was built close to the capital site of the Western *Han*, or *Chang'an*, and was called *Daxingcheng* (the City of Great Prosperity). However, after the death of the first emperor in 605 CE, *Luoyang* became the new capital. In 618 CE, the *Sui* dynasty was overtaken by the *Tang* dynasty. Throughout most of the duration of the *Tang* dynasty, the capital was *Chang'an*. The present city of *Xian* is centered around *Chang'an* of the *Tang* dynasty. It is located a few kilometers south-east of the capital of the Western *Han* dynasty. In this work, we neglect this short displacement.

Table 1 lists the eleven records of large solar eclipses during the *Sui* and *Tang* dynasties. Among these, the two eclipses of 703 CE and 879 CE did not occur on the dates given in the records. Another three eclipses, 562 CE, 754 CE, and 888 CE, did occur, but the central zones were far away from the capital. For example, the central zone of the eclipse of 562 CE spanned across Southeast Asia, and the zone of annularity did not reach any further north than 30° northern latitude in the longitude range covered by China, a fact that was also pointed out by Stephenson (1997, p. 244). The eclipse of 888 CE is not considered in Stephenson (1997, p. 245) because the account of the total obscuration is very brief and is cited only in the Annals of the *Xin-Tangshu*. However, it cannot be justified to exclude

Table 2
Geographical positions of the capital cities and other important places in Chinese history

Place	Latitude	Longitude	Altitude
Chang'an	34.264°	108.945°	410m
Luoyang	34.725°	112.592°	160m
Nanjing	32.046°	118.793°	10m
Ningxian	35.503°	107.919°	1,000m
Chengdu	30.667°	104.070°	500m
Beijing	39.904°	116.391°	50m
Handan	36.603°	114.474°	70m

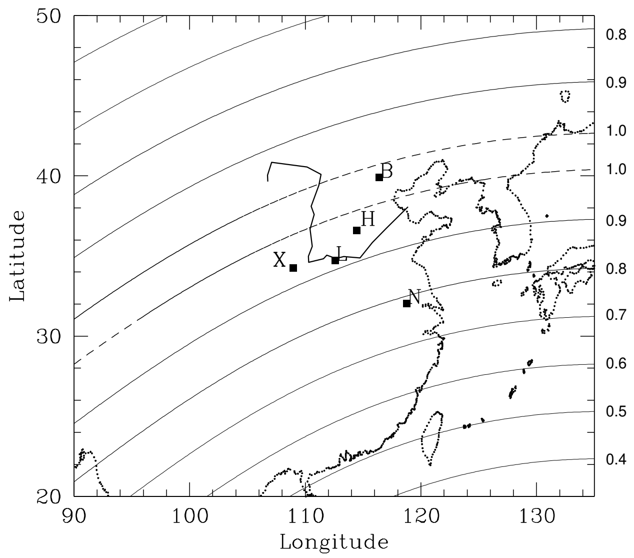

Figure 6. Eclipse map of 822 CE. According to the records, the eclipse was not total at *Chang'an*, denoted by X in the figure, but it was total in the area between *Beijing* and *Handan*, denoted by B and H, respectively.

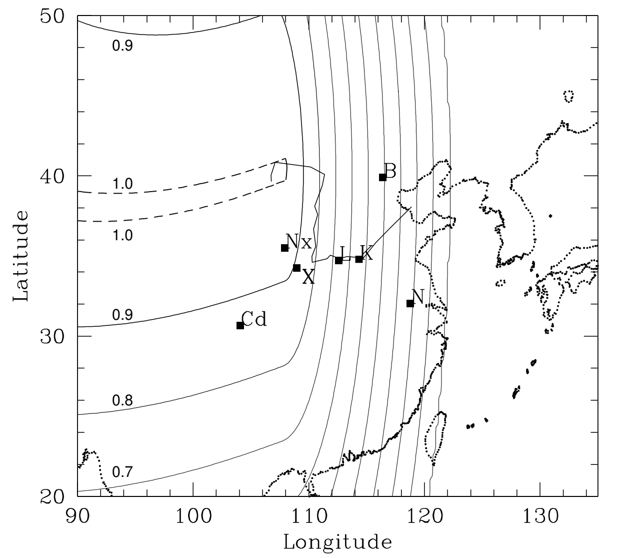

Figure 7. Eclipse map of 756 CE. Here, Nx represents *Ningxian*, where the new emperor *Suzong* resided at the time of eclipse, and Cd represents *Chengdu*, the place that the previous emperor *Xuanzong* fled to. The dashed line represents the eclipse magnitude $\mu = 1.0$.

the eclipse of 888 CE, because there are many other historical eclipses that likewise are described using only the term *Ji*. Our calculation shows that the eclipse of 888 CE did occur. However, the central zone spanned across southern China and the Korean peninsula, which does not agree with the record. Hence, we excluded this record.

Another three eclipses, 616 CE, 756 CE, and 822 CE, were observed in places other than *Chang'an*, the capital. The eclipse of 616 CE was observed in *Luoyang*, which was the capital of the *Sui* dynasty at that time. The eclipse of 822 CE is interesting, because the record describes the effects of the eclipse observed at two different places. Translated into English, the record reads: 'The Sun was eclipsed. The Sun was at 12 *du*[4] of (the lunar lodge of) *Wei*; a quarter of it was not completed. In (the regions of) *Yan* state and *Zhao* state, it was observed to be a *Ji* eclipse'.

This eclipse was investigated by both Stephenson (1997, p. 248) and Tanikawa & Soma (2004). According to Stephenson (1997), the eclipse was presumably partial with a magnitude of 3/4 at the capital, and the central zone spanned across the northern region of *Chang'an*. However, Tanikawa & Soma (2004) assumed that 'a quarter' describes the unobscured portion of the circumference of the solar disk. According to our calculation, the eclipse magnitude was 0.95 at *Chang'an*. Hence, the argument of Stephenson is not valid. However, we do not consider this eclipse in the analysis, because the central eclipse did not occur in *Chang'an*.

The capital city of the *Yan* state was *Beijing*, and the capital of the *Zhao* state was *Handan* since the Warring States period (403–221 BCE). Tanikawa & Soma (2004) made a minor mistake by regarding *Shijiazhuang* as the capital of *Zhao*. Since the difference in latitude between *Shijiazhuang* and *Handan* is merely 1.5°, their result is not significantly affected. We show the eclipse map of the 822 CE eclipse in Figure 6. We see that the central eclipse zone spanned across the area between Beijing and Handan. Furthermore, it is evident that the magnitude of the eclipse observed at *Chang'an* was not 3/4. In our calculation, it was 0.95. An eclipse magnitude of 0.75 at *Xian* can be obtained by adjusting ΔT, but the resulting ΔT value is unreasonably large.

We must be careful when investigating the eclipse of 28 October 756 CE. As Stephenson (1997, pp. 245–246) pointed out, the place of observation seems to be uncertain, because only a few months earlier, the capital of *Chang'an* had been occupied by rebels led by *An Lushan*; the Emperor *Xuanzong* (685–762 CE) and his court had fled to present-day *Chengdu*. His third son, *Suzong* (711–762 CE), fled to *Lingzhou* (present-day *Lingwu* in *Ningxia* province), where he formally ascended to the throne. Since the eclipse is recorded in *Suzong*'s chronicle in *Jiu-Tangshu*, we conclude that the eclipse was observed at the place where the new emperor resided. According to the chronicle, the place was present-day *Ningxian* in *Gansu* province, which is approximately 170 km north of *Chang'an*. We see in Figure 7 that both *Xian* and *Ningxian* are within the area in which the eclipse was observed as a large partial eclipse with a magnitude larger than 0.9.

Excluding the eight eclipses mentioned above, we draw the visibility map for the three remaining eclipses in the lower panel of Figure 8. The contours of the visibility map represent boundaries within which all eclipses were observed to have a magnitude larger than a certain

[4]In this paper, *du* refers to Chinese degrees. 365.25 *du* are equivalent to 360 Babylonian degrees (360°).

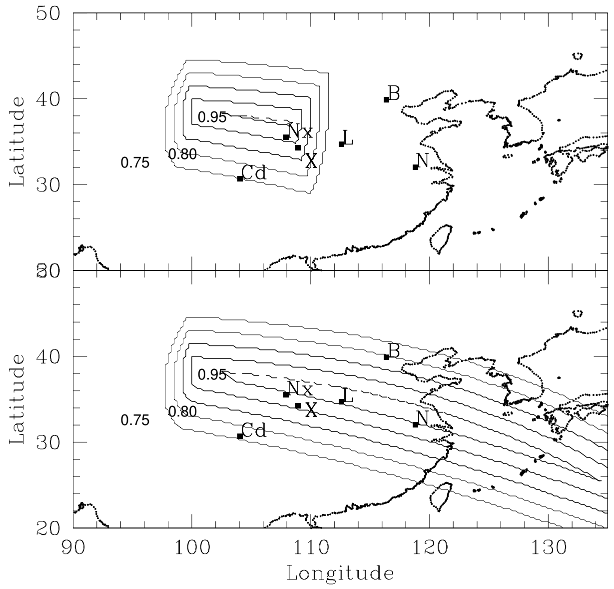

Figure 8. Visibility map for the central eclipses of the *Sui* and *Tang* dynasties recorded in history books. The innermost contour marks the threshold value of eclipse magnitude $\mu = 0.95$. The *lower panel* shows the results for the eclipses of 702 CE, 729 CE, and 761 CE, which were witnessed at *Chang'an*, the capital city of the *Tang* dynasty. The *upper panel* shows the visibility map after including the eclipses of both 616 CE and 756 CE, which were observed in other cities close to *Xian*. We see that eclipses of magnitude larger than 0.9 were visible at *Chang'an* (present-day *Xian*, denoted by X).

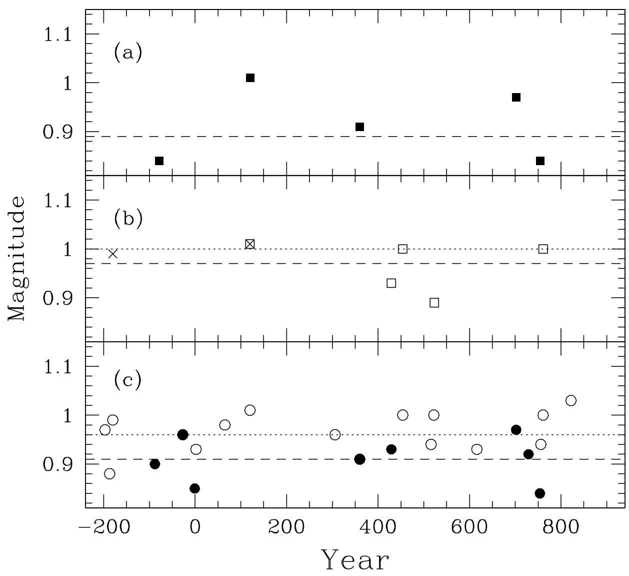

Figure 9. Eclipse magnitudes corresponding to various expressions that were used to describe large eclipses. The open circles in the *lower panel* represent the *Ji* eclipses; the solid circles represent eclipses that are 'not complete and like a hook'; the crosses in the *middle panel* represent daytime darkness; the open squares represent visibility of stars; the solid squares in the *upper panel* represent 'almost complete' eclipses.

value. We see that all three eclipses could be observed as eclipses of a magnitude larger than 0.9 at *Chang'an* or *Xian*. However, the permitted area is so wide that it seems impossible to pinpoint the observer locations. Among the three records that were observed in places other than the capital, those of 616 CE and 756 CE can be included in our analysis, because they were observed at places relatively close to the capital *Chang'an*. In the case of the eclipse of 822 CE, the observer location was explicitly given to be *Beijing*, which is very far away from *Chang'an*. Hence, the eclipse of 822 CE is omitted from our analysis. The visibility map for the five eclipses is shown in the upper panel of Figure 8. The historical capital of the *Tang* dynasty, *Chang'an*, coincides with the area where all five eclipses were observed as eclipses of magnitude larger than 0.85. We also note that the visibility map is mostly unchanged, even though the eclipse of 822 CE is included in the analysis. Therefore, we conclude that the observers of large eclipses during this period were located in the capital city.

4. CONCLUSIONS

In this work, we have calculated the central eclipses recorded in Chinese history, from the Western *Han* dynasty (206 BCE to 8 CE) to the *Tang* dynasty (618–907 CE), and determined the observing sites by obtaining an area of intersection within which all the central eclipses could be witnessed. This work can be used to verify the change in the Earth's rate of rotation, ΔT, from historical eclipses by assuming that the observer locations were the capital cities, which are well known from rich historical and archaeological records.

We have analyzed the records of large solar eclipses collected from *Zhongguo Gudai Tianxiang Jilu Zongji* or 'General Compilation of Astronomical Records in Ancient Chinese Histories' (Beijing Observatory 1988). The position of both the Sun and the Moon are calculated by using the method in Meeus (1998b) for the date given by each record. We have adopted ΔT values interpolated from the values in Morrison & Stephenson (2004, 2005). We have divided the records into four groups, considering the changes in capitals between dynasties. For each group of eclipses we have constructed a visibility map which constrains the area for the possible location of the observer. The four visibility maps show that the observing site for each group agrees with the historical capital of each dynasty.

When calculating the historical eclipses, ΔT values are usually adopted from those in Morrison & Stephenson (2004, 2005). However, the historical eclipse records for a period from 1 CE to 400 CE are so sparse that ΔT values for this period have to be interpolated from ΔT values in adjacent periods. Our work confirms the validity of the ΔT values for this period given by Morrison & Stephenson (2004, 2005).

The historical eclipses of large magnitude recorded in the chronicles of East Asian dynasties were described using various expressions. The most common term is *Ji*, which means 'exhausted'. A number of historical eclipses were described by expressions such as being 'complete',

Table 3
Observational and theoretical limiting magnitudes for expressions used to describe historical eclipses

Expressions	Magnitude	KH08[a]
Ji or 'complete'	0.96 ± 0.04	> 0.9
visible stars	0.97 ± 0.05	$0.95 \sim 0.98$
daytime darkness	1.00	0.99
'not complete and like a hook'	0.91 ± 0.04	$0.8 \sim 1.0$
'almost complete'	0.89 ± 0.08	$0.8 \sim 0.93$

[a]Können & Hinz (2008)

'not complete and like a hook', and 'almost complete'; 'dark daytime' and 'visibility of stars' were also used for emphasizing the large magnitude of *Ji* eclipses. We have discussed the physical meaning of each expression in Section 2.3 and estimated eclipse magnitudes, given in Table 1.

We have tested the idea that the term *Ji* does not always describe a total eclipse (Watanabe 1979; Stephenson 1997; Ahn 2008). The lower panel of Figure 9 shows the computed magnitudes of historical *Ji* eclipses assuming the observers were at the capital of each dynasty, as well as the average magnitude of the *Ji* eclipses. All the *Ji* eclipses, except for three abnormal ones, have magnitudes $\mu = 0.96 \pm 0.04$. According to our calculations, the three abnormal eclipses indeed occurred, but their magnitudes were too small to be regarded as total eclipses. This discrepancy is most likely due to the reports being provincial reports of (nearly) total eclipses (Pankenier 2012).

The magnitudes of the eclipses that were recorded as being 'not complete but like a hook' are shown in the lower panel of Figure 9. Their average magnitude is $\mu = 0.91 \pm 0.04$ which is slightly smaller than that of the *Ji* eclipses. The large magnitudes of *Ji* eclipses were frequently emphasized by adding remarks on either a visibility of stars or daytime darkness. The calculated magnitudes for these eclipses are plotted in the middle panel of Figure 9. The five records with visible stars had eclipse magnitudes $\mu = 0.97 \pm 0.05$, and the two records with daytime darkness were effectively total eclipses of magnitude $\mu = 1.00$. We conclude that both visible stars and daytime darkness imply an eclipse magnitude $\mu \geq 0.97$; however, additional data are required to strengthen this conclusion. A further category is formed by 'almost complete' eclipses. According to our calculations, this expression is used to describe eclipses with magnitudes $\mu = 0.89 \pm 0.08$ (see the upper panel of Figure 9). Our results are summarized in Table 3, where the magnitude values for the expressions show good agreement with those theoretically estimated by Können & Hinz (2008).

Overall, we arrive at the following conclusions:

- Solar eclipses of very large magnitude were observed at the capital city of each dynasty. This fact supports the general idea that records of celestial phenomena in chronicles of East Asian dynasties imply observations at the capital.
- The eclipse records described by the term *Ji* are not always total eclipses; the magnitude of *Ji* eclipses is 0.96 ± 0.04. Complementary expressions, such as 'daytime darkness' or 'visibility of stars' during the eclipses, strengthen the possibility of the *Ji* eclipses to be total, as well as the reliability of them being results of actual observations rather than being results of ephemeris calculations.
- Our calculations give quantitative definitions to such expressions as 'not complete but like a hook', 'almost complete', 'visibility of stars', and 'daytime darkness'. We summarize these results, which are in agreement with the theoretical calculations given by Können & Hinz (2008), in Table 3.

Remarkably, the concept of *Ji* eclipses is not equivalent to the modern definition of total eclipses, which is hinted at by the following conversation between a king and his court recorded in a chronicle called *Seungjungwon-Ilgi* (Diaries of the Court Secretariat of the *Joseon* dynasty) in 1751 CE (Ahn 2008).

> [On the 1st day of the 5th month in the 27th year of the reign of King *Yeongjo* (25 May 1751 CE)]
>
> The King asked 'Were there *Ji* eclipses in my kingdom? What is the meaning of the term *Ji*?' *Jo Wunkyu* (the Third Royal Secretary) replied 'According to the Spring and Autumn Annals of Mr. Zuo's Commentary, *Ji* means being exhausted or complete. Therefore, it means that the Sun is completely eaten.' *Kim Taeseo* (a royal astronomer) added the comment 'If the magnitude is greater than 0.8, then it is called *Ji*.' [Two sentences are omitted by the author of this paper.] The King asked again, 'From where did you know that?' *Kim Taeseo* replied, 'From astronomical books.'
>
> The King asked, 'Is it the same for lunar eclipses?' All replied, 'Yes, it is.'
>
> The King asked, 'A magnitude larger than 0.8? Are there any eclipses then whose magnitude is greater than 0.8?' All said, 'When the eclipse magnitude is greater than 0.8, the Sun appears to be completely eaten for an observer on the Earth.'

According to the above dialogue, eclipses of magnitude greater than 0.8 were regarded as *Ji* eclipses by a Korean astronomer of the 18th century *Joseon* dynasty. This notion originated from Chinese pre-modern calendrical methods that had been used for calculating the solar and lunar eclipses. According to those methods, the *Ji* eclipses, whose magnitude is greater than 0.8, have their first contacts at the western part of the solar disk and fourth contacts at the eastern part of the solar disk. Ahn (2008) showed, using simple geometrical considerations, that in order to meet the above-mentioned definition for the *Ji* eclipse, defined in terms of contact directions, the eclipse should have a magnitude larger than 0.8.

Therefore, we need to be careful when interpreting historical records of *Ji* eclipses for the purpose of determining ΔT values. It appears better to use the eclipse band where the magnitude is greater than 0.8, rather than to use the totality band. If the *Ji* eclipse records have additional expressions such as daytime darkness and appearance of stars, they result from real observations and are closer to totality. For those eclipses, we can use the totality band to determine the ΔT values.

Acknowledgments

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Appendix A. Chinese Historical Eclipses

198 BCE, Aug 7: The Sun was eclipsed [*Hanshu* vol. 1]. The Sun was eclipsed. It was *Ji*.[5] The Sun was at 13 *du*[6] of (the lunar lodge of) *Zhang* (υ Hya[7]) [*Hanshu* vol. 27].

188 BCE, Jul 17: The Sun was eclipsed. It was *Ji* [*Hanshu* vol. 2]. The sun was eclipsed. It was 'almost complete'. The Sun was at 1 *du* of (the lunar lodge of) *Qi-Xing* (α Hya) [*Hanshu* vol. 27].

181 BCE Mar 4: The Sun was eclipsed. It was *Ji*. The Sun was at 9 *du* of (the lunar lodge of) *Ying-Shi* (α Peg), which represents the interior of the Palace chambers. At that time the (Dowager) Empress of Kao-[tzu] was upset by it and said, 'This is on my account'. The next year the omen was satisfied [*Hanshu* vol. 27]. On the day *ji chou*, the Sun was eclipsed and it became 'dark during daytime'. The Empress Dowager was upset by it and her heart was ill at ease. Turning to those around he she said, 'This is on my account' [*Shiji* vol. 9].

147 BCE, Nov 10: The Sun was eclipsed. It was 'almost complete'. The Sun was at 9 *du* of the lunar lodge of) *Wei* (μ Sco) [*Hanshu* vol. 27]. The Sun was eclipsed [*Shiji* vol. 11].

89 BCE, Sep 29: The Sun was eclipsed. It was 'not complete and like a hook.' The Sun was at 2 *du* of *Kang* (κ Vir). At the hour of *fu* (=15–17 h) the eclipse began from the north-west. Towards the hour of sunset it recovered [*Hanshu* vol. 27]. The Sun was eclipsed [*Hanshu* vol. 6].

80 BCE, Sep 20: The Sun was eclipsed. It was 'almost complete'. The Sun was at 12 *du* of (the lunar lodge of) *Zhang* (υ Hya). [*Hanshu* vol. 27] The Sun was eclipsed. It was *Ji*. [*Hanshu* vol. 7]

34 BCE, Aug 23: The Sun was eclipsed. It was 'not complete and like a hook'. Then it set [*Hanshu* vol. 27]. The Sun was eclipsed [*Hanshu* vol. 9].

28 BCE, Jun 19: The Sun was eclipsed. It was *Ji* [*Hanshu* vol. 10]. The Sun was eclipsed. It was 'not complete and like a hook'. The Sun was at 6 *du* of (the lunar lodge of) *Dong-Jing* (μ Gem)... When the eclipse first began, it started from the south-west [*Hanshu* vol. 27].

[5] Here *Ji* is a Chinese letter meaning 'exhausted'.
[6] Remember that *du* is the Chinese degree.
[7] This star determines each of the 28 Chinese lunar lodges.

2 BCE, Feb 5: The Sun was eclipsed. It was 'not complete and like a hook'. The Sun was at 10 *du* of (the lunar lodge of) *Ying-shi* (α Peg) [*Hanshu* vol. 27]. The Sun was eclipsed [*Hanshu* vol. 11, vol. 81].

2 CE, Nov 22 The Sun was eclipsed [*Hanshu* vol. 12]. The Sun was eclipsed. It was *Ji* [*Hanshu* vol. 27].

65 CE, Dec 16 The Sun was eclipsed. It was *Ji* [*Hou-Hanshu* vol. 2]. The Sun was eclipsed. It was *Ji*. The Sun was at 11 *du* of (the lunar lodge of) Dou (ψ Sgr) [*Hou-Hanshu, Wuxingji*].

120 CE, Jan 18: The Sun was eclipsed. It was 'almost complete'. It was 'like evening on Earth'. The Sun was at 11 *du* of (the lunar lodge of) *Xu-Nu* (ϵ Aqr). According to *Gujinzhu*, 'stars were seen during the daytime' [*Hou-Hanshu, Wuxingji*]. The Sun was eclipsed. It was *Ji* [*Hou-Hanshu* vol. 5].

243 CE, Jun 5 The Sun was eclipsed. It was *Ji* [*Sanguozhi, Weishu* vol. 4].

306 CE, Jul 27 The Sun was eclipsed. It was *Ji*. [*Songshu* vol. 34 *Wuxing*] The Sun was eclipsed [*Jinshu* vol. 4, vol. 12].
♠ This eclipse was an annular eclipse. Hence, Saito & Ozawa (1992) suggested that both annular and total eclipses were described as *Ji* eclipses.

360 CE, Aug 28: The Sun was eclipsed. It was 'not complete and like a hook' [*Songshu* vol. 34 *Wuxing*]. The Sun was eclipsed. It was *Ji* [*Jinshu* vol. 8]. The Sun was eclipsed. It was 'almost *Ji*' and was in *Jiao* (α Vir) [*Jinshu* vol. 12 *Tianwenji*].

429 CE, Dec 12: The Sun was eclipsed; it was 'not complete and like a hook'. During the eclipse, 'star(s) were(was) seen'. At the hour of *fu*(=15–17^h), when the Sun was about to set, 'the Earth was in darkness in the north region of the Yellow River' [*Songshu* vol. 34 *Wuxingji*]. The Sun was eclipsed. 'Stars were seen in daytime' [*Nanshi* vol. 2 *Songbenji*]. The Sun was eclipsed [*Songshu* vol.5].
♠ Here *Hebei* means the northern area of *Huanghe* or the Yellow River. Stephenson (1997, p. 242) misunderstood this as the present-day Hebei province. This region belonged to the Northern *Wei* dynasty. Hence the totality was not seen at *Nanjing*, the capital of the *Liu-Song*, but it could be seen at the area north of *Huanhe*. Saito & Ozawa (1992) suggested that the star might be Venus because it was bright at that time.

453 CE, Aug 20: The Sun was eclipsed. It was *Ji*. 'All the stars were seen' [*Songshu* vol. 34 *Wuxingji*]. The Sun was eclipsed [*Songshu* vol. 6, *Nanshi* vol. 2].

454 CE, Aug 10: The Sun was eclipsed. It was *Ji*. 'All the lunar lodges were brightly lit' [*Songshu* vol. 34 *Wuxingji*]. The Sun was eclipsed. It was *Ji* [*Nanshi* vol. 2 *Songbenji*]. The Sun was eclipsed [*Weishu* vol. 6].
♠ According to our calculations, the record of 453 CE was not real, while that of 454 CE was real. Both are in *Songshu Wuxingji*. The compilers of *Wuxingji* misplaced one calendar year with the cyclical day adjusted

retroactively (Stephenson 1997, pp. 242–243).

516 CE, Apr 18: The Sun was eclipsed. It was *Ji* [*Nanshi* vol. 6 *Liangbenji*]. The Sun was eclipsed [*Liangshu* vol. 2].

522 CE, Jun 10: The Sun was eclipsed. It was *Ji* [*Liangshu* vol. 3, *Nanshi* vol. 7 *Liangbenji*].

523 CE, Nov 23: The Sun was eclipsed. 'Venus was seen in daytime' [*Liangshu* vol. 3, *Suishu* vol. 21 *Tianwenzhi*]. The Sun was eclipsed [*Nanshi* vol. 7 *Lianbenji*, *Weishu* vol. 105 *Tianxiangzhi*, *Beishi* vol. 4 *Weibenji*].
♠ This was an annular eclipse, and the appearance of Venus could be independent of the eclipse.

562 CE, Oct 14: The Sun was eclipsed. It was *Ji* [*Suishu* vol. 21 *Tianwenzhi*]. The Sun was eclipsed [*Zhoushu* vol. 5]. The Sun was eclipsed [*Chenshu* vol. 3, *Nanshi* vol. 10 *Chenbenji*].
♠ Our calculation shows that this eclipse was not total at *Chang'an*, where the capital city of the *Sui* dynasty was located. According to the record in *Chenshu*, the eclipse seems to have been merely partial. In our calculation, the track of annularity never reached further north than 20° north. Stephenson (1997, p. 244) argued that this record seems to have been a predicted eclipse.

616 CE, May 21: The Sun was eclipsed. It was *Ji* [*Suishu*, vol. 4, vol. 21; *Beishi* vol. 12 *Suibenji*].

702 CE, Sep 26: The Sun was eclipsed; it was 'not complete and like a hook'. It was observed at the capital and provinces in all four directions [*Jiu-Tangshu* vol. 6]. The Sun was eclipsed; it was 'almost *Ji*'. The Sun was at 1 *du* of (the lunar lodge of) *Jiao* (α Vir) [*Xin-Tangshu* vol. 32 *Tianwenzhi*].

703 CE, Oct 16 The Sun was eclipsed [*Jiu-Tangshu* vol. 36 *Tianwenzhi*]. The Sun was eclipsed. The Sun was at 7 *du* of (the lunar lodge of) Kang (κ Vir) [*Xin-Tangshu* vol. 32 *Tianwenzhi*].
♠ This record is not reproduced by our calculations. The original history books do not mention totality; only the private literature book mentions it, making the record dubious.

729 CE, Oct 27: The Sun was eclipsed. It was 'not complete and like a hook' [*Jiu-Tangshu* vol. 8]. The Sun was eclipsed. It was 'not complete and like a hook'. The Sun was at 9 *du* of (the lunar lodge of) *Di* (α Lib) [*Xin-Tangshu* vol. 32 *Tianwenzhi*]. The Sun was eclipsed [*Xin-Tangshu* vol. 5].

754 CE Jun 25: The Sun was eclipsed. It was 'not complete and like a hook' [*Jiu-Tangshu* vol. 9]. The Sun was eclipsed. It was 'almost complete'. The Sun was at 19 *du* of (the lunar lodge of) *Dong-Jing* (μ Gem) [*Xin-Tangshu* vol. 32 *Tianwenzhi*]. The Sun was eclipsed [*Jiu-Tangshu* vol. 36, *Xin-Tangshu* vol. 5].
♠ According to our calculation, the track of totality never reached further north than 30° N, regardless of the value of ΔT. The eclipse could not be witnessed to be a great eclipse at *Chang'an*.

756 CE, Oct 28: The Sun was eclipsed. It was *Ji* [*Jiu-Tangshu* vol. 10]. The Sun was eclipsed. The Sun was at 10 *du* of (the lunar lodge of) *Di* (α Lib) [*Xin-Tangshu* vol. 32 Tianwenzhi]. The Sun was eclipsed [*Jiu-Tangshu* vol. 36, *Xin-Tangshu* vol. 6].
♠ The place of observation for this eclipse cannot be established, because only a few months earlier the capital of *Chang'an* had been captured by rebels led by *An Lu-Shan*; the Emperor and his court had fled to the provinces, and *Chang'an* was not recovered until the following year (Stephenson 1997, p. 246). At the time of the *An-Shi* Rebellion, the Emperor, *Xuanzong*, fled to present-day *Chengdu*. His third son fled to *Lingzhou* (present-day *Lingwu* in *Ningxia* province), where the third son, *Suzong*, formally ascended to the throne. The eclipse record is recorded in *Suzong*'s chronicle in *Jiu-Tangshu*, and so the eclipse must have been observed at the place where the new emperor resided. According to the chronicle, that place was present-day *Ningxian* in *Gansu* province, approximately 170 km north of *Chang'an*.

761 CE, Aug 5: The Sun was eclipsed. It was *Ji*. 'All the bright stars could be seen' [*Jiu-Tangshu* vol. 10]. The Sun was eclipsed; 'all the bright stars could be seen'. The Astronomer-Royal *Qutan* (Gotama) reported (to the Emperor): 'On day *gui-wei*, the Sun diminished. The loss began at 6 *ke* after the hour of *chenzheng* (7–8^h). At 1 *ke* after the hour of *sizheng* (9–10^h) it was *Ji*. At 1 mark before the hour of *wu* (11–13^h) it was restored to fullness. The Sun was at 4 *du* of (the lunar lodge of) *Zhang* (υ Hya)' [*Jiu-Tangshu* vol. 36]. The Sun was eclipsed. It was *Ji*. 'All the bright stars were seen'. The Sun was at 4 *du* of (the lunar lodge of) *Zhang* (υ Hya) [*Xin-Tangshu* vol. 32]
♠ In this record, the local times of the various phases are given in units of *ke*, which is one hundredth of a day.

794 CE May 4: Astronomical officers reported that the Sun should be eclipsed. 5 *ke* after *si zheng* (9–10^h) a *Ji* eclipse should have occurred. 5 *ke* after *wei zheng* (13–14^h), it was fully restored. *Taichang* (Minister of Ceremonies) said, 'According to rites, the cabinet should be closed'. On that day, the eclipse was not seen due to thick clouds. All the court members congratulated themselves on that [*Jiu-Tangshu Tianwenji*].

822 CE, Apr 25 : The Sun was eclipsed. The Sun was at 12 *du* of (the lunar lodge of) *Wei* (35 Ari). A quarter was not exhausted. It was seen to be *Ji* in the *Yan* state and the *Zhao* state [*Jiu-Tangshu* vol. 36 *Tianwenzhi*].
♠ The *Yan* and *Zhao* regions correspond roughly to the *Hebei* and *Shanxi* provinces in northern China. This eclipse was investigated in detail by Tanikawa & Soma (2004).

879 CE, Apr 25: The Sun was eclipsed [*Xin-Tangshu* vol. 9]. The Sun was eclipsed. It was *Ji*. The Sun was at 8 *du* of (the lunar lodge of) *Wei* (35 Ari) [*Xin-Tangshu* vol. 32 *Tianwenzhi*].
♠ No eclipse occurred at the given date. Stephenson (1997, p. 246) regarded this record as a failed predic-

tion; direct evidence of solar eclipse prediction during the *Tang* dynasty is found in the chronicles. There are historical eclipses that have been predicted but not seen. Saito & Ozawa (1992) pointed out that this eclipse record does not appear in *Jiu-Tangshu* that became the source of *Xin-Tangshu*. He suggested this record might be an incorrect prediction.

888 CE, Apr 15: The Sun was eclipsed. It was *Ji* [*Xin-Tangshu* vol. 9]. The Sun was eclipsed. The Sun was at 8 *du* of (the lunar lodge of) *Wei* (35 Ari) [*Xin-Tangshu* vol. 32 *Tianwenzhi*].

♠ Stephenson (1997, p. 245) did not investigate the eclipse of 888 CE because the account of the total obscuration is very brief and only cited in the Annals of the *Xin-Tangshu*, although there is no actual reason to exclude that record. However, the eclipse was annular partial, and the eclipse magnitude could not be greater than 0.7 at *Chang'an*, unless an unreasonably large value of ΔT is adopted.

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