Received: 16 August 2018 | Revised: 16 November 2018 | Accepted: 10 December 2018

DOI: 10.4218/etrij.2018-0455

ORIGINAL ARTICLE

ETRI Journal WILEY

Array pattern synthesis using semidefinite programming and a bisection method

Jong-Ho Lee[1] | **Jeongsik Choi**[2] | **Woong-Hee Lee**[3] | **Jiho Song**[4]

[1]School of Electronic Engineering, Soongsil University, Seoul, Rep. of Korea

[2]Intel Labs, Intel Corporation, Santa Clara, California

[3]Department of Communication Systems, KTH Royal Institute of Technology, Stockholm, Sweden

[4]School of Electrical Engineering, University of Ulsan, Ulsan, Rep. of Korea

Correspondence
Jiho Song, School of Electrical Engineering, University of Ulsan, Ulsan, Korea.
Email: jihosong@ulsan.ac.kr

Funding information
This research was supported by the Basic Science Research Program through the National Research Foundation of Korea, which is funded by the Ministry of Education (NRF-2016R1D1A1B03930599).

In this paper, we propose an array pattern synthesis scheme using semidefinite programming (SDP) under array excitation power constraints. When an array pattern synthesis problem is formulated as an SDP problem, it is known that an additional rank-one constraint is generated inevitably and relaxed via semidefinite relaxation. If the solution to the relaxed SDP problem is not of rank one, then conventional SDP-based array pattern synthesis approaches fail to obtain optimal solutions because the additional rank-one constraint is not handled appropriately. To overcome this drawback, we adopted a bisection technique combined with a penalty function method. Numerical applications are presented to demonstrate the validity of the proposed scheme.

KEYWORDS
array pattern synthesis, bisection method, semidefinite programming, semidefinite relaxation

1 | INTRODUCTION

Recently, the synthesis of planar arrays has gained significant attention because it enables three-dimensional beamforming, which is one of the most promising techniques for next-generation wireless communication systems [1,2]. It is known that a three-dimensional beam pattern specified by elevation and azimuthal planes can be obtained by using antenna elements arranged in a two-dimensional grid. The objective of array pattern synthesis in linear and planar arrays is to determine the excitation weights for antenna elements to generate a specific beam pattern [3]. In [4–6], the desired beam pattern was generated by using an iterative adaptation scheme to adjust current beam patterns in an iterative manner. The authors of [7–9] demonstrated that certain types of synthesis issues can be formulated as convex optimization problems and solved using well-established interior-point methods. Additionally, genetic algorithm–based optimization was adopted to solve the array pattern synthesis problem in [10] and [11].

In this study, we focused on the application of semidefinite programming (SDP) to array pattern synthesis [12–14]. Array pattern synthesis problems can be formulated as SDP problems with an additional rank-one constraint. Although the additional rank-one constraint can be relaxed using the concept of semidefinite relaxation [15], the solution to the original problem must be of rank one. In both [13] and [14], no operations were used to handle the rank-one constraint and the array excitation was obtained by using the maximum eigenvalue and corresponding eigenvector of the solution to the SDP problem directly, even though the solution may not be of rank one. Therefore, there is no guarantee that the array excitations in [13] and [14] are optimal. To address this issue, we adopted a bisection technique [16] combined with a penalty function method (PFM) [17].

Furthermore, we also consider two different constraints on the power consumed to excite the antenna array: the overall power constraint (OPC) and individual power constraint (IPC). The OPC indicates that the total power consumed for exciting all antenna elements is limited. In contrast, the peak excitation power at each antenna element is restricted by the IPC. It is noteworthy that [14] did not consider any constraints on the power consumed for array excitation, whereas [13] imposed a restriction on the magnitude of each excitation weight.

The remainder of this paper is organized as follows. In Section 2, we derive an array pattern synthesis problem with array excitation power constraints. In Section 3, we use the SDP to solve the array pattern synthesis problem and demonstrate that the bisection technique combined with the PFM can handle the additional rank-one constraint. Section 4 presents numerical results and compares the array response characteristics of the proposed scheme with those of conventional schemes to validate the proposed scheme. Concluding remarks are provided in Section 5.

2 | PROBLEM FORMULATION

Consider a planar array with M elements, where the mth element has a pattern of $g_m(\theta, \varphi)$ and is fed by a complex excitation of w_m. The far-field response of the planar array can be expressed as [8,9]

$$f(\theta, \varphi) = \sum_{m=1}^{M} g_m(\theta, \varphi) w_m e^{j2\pi(ux_m + vy_m)}, \tag{1}$$

where x_m and y_m denote the location of the mth element in the wavelengths, which are arbitrary, but fixed and known, and

$$u = \sin(\theta)\cos(\varphi), \; v = \sin(\theta)\sin(\varphi). \tag{2}$$

Here, we denote (θ_0, φ_0) as the desired direction, which yields $u_0 = \sin(\theta_0)\cos(\varphi_0)$ and $v_0 = \sin(\theta_0)\sin(\varphi_0)$. We also define an undesired region S, which includes all undesired directions. Furthermore, we refer to $|f(\theta, \varphi)|^2$ as the array response level to the direction of (θ, φ).

Our objective is to maximize the response level to the desired direction under the undesired-region response level constraint. We consider that the absolute squared sum of the complex weights is $\leq P_T$ for the OPC (ie, $\sum_{m=1}^{M} |w_m|^2 \leq P_T$) and the absolute square of each weight is less than or equal to P_I for the IPC (meaning $|w_m|^2 \leq P_I$ for all m). Then, the optimization problem can be written as

$$\begin{aligned} &\max_{\mathbf{w}} |\mathbf{a}(\theta_0, \varphi_0)\mathbf{w}|^2 \\ &\text{s.t.} \max_{(\theta,\varphi)\in S} |\mathbf{a}(\theta, \varphi)\mathbf{w}|^2 \leq \rho \\ &\quad \begin{cases} \mathbf{w}^\dagger \mathbf{w} \leq P_T, & \text{for OPC}, \\ |w_m|^2 \leq P_I, \forall m, & \text{for IPC}, \end{cases} \end{aligned} \tag{3}$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_M]^T$ is the complex excitation vector, the mth entry of the $1 \times M$ steering vector $\mathbf{a}(\theta, \varphi)$ is $g_m(\theta, \varphi)e^{j2\pi(ux_m + vy_m)}$, and ρ is the maximum allowable undesired-region response level.

It is noteworthy that a similar problem that does not consider array excitation power constraints can be found in [18]. However, [18] assumed a uniformly spaced antenna array, meaning linear programming could be exploited to solve the problem. As mentioned above, we considered that antenna elements are arbitrarily spaced in this study. The proposed scheme derived in the next section has no limitations regarding antenna spacing and can be used for both cases of uniform spacing and arbitrary spacing.

3 | ARRAY PATTERN SYNTHESIS

In this section, we solve (3) using semidefinite relaxation [15] and the bisection technique [16]. First, we rewrite (3) as

$$\begin{aligned} &\max_{\mathbf{W}} \text{tr}\,(\mathbf{A}(\theta_0,\varphi_0)\mathbf{W}) \\ &\text{s.t.}\ \text{tr}(\mathbf{A}(\theta,\varphi)\mathbf{W}) \leq \rho, \quad \forall(\theta,\varphi)\in S, \\ &\quad \text{rank}(\mathbf{W}) = 1, \quad \mathbf{W} \geq 0, \\ &\quad \begin{cases} \text{tr}(\mathbf{W}) \leq P_T, & \text{for OPC}, \\ \mathbf{W}_{mm} \leq P_I, \forall m, & \text{for IPC}, \end{cases} \end{aligned} \tag{4}$$

where $\mathbf{W} = \mathbf{w}\mathbf{w}^\dagger$, $\mathbf{A}(\theta, \varphi) = \mathbf{a}(\theta, \varphi)^\dagger \mathbf{a}(\theta, \varphi)$, tr(.) denotes trace operations, $(.)^\dagger$ denotes a conjugated transpose, $\mathbf{W} \succcurlyeq 0$ indicates that $\mathbf{W}$ must be a Hermitian positive semidefinite matrix, and $\mathbf{W}_{mm}$ denotes the mth diagonal entry of $\mathbf{W}$. To solve (4), we exploit semidefinite relaxation to eliminate the rank constraint [15]. Then, we have

$$\begin{aligned} &\max_{\mathbf{W}} \text{tr}(\mathbf{A}(\theta_0,\varphi_0)\mathbf{W}) \\ &\text{s.t.}\ \text{tr}(\mathbf{A}(\theta,\varphi)\mathbf{W}) \leq \rho, \quad \forall(\theta,\varphi)\in S, \mathbf{W} \geq 0, \\ &\quad \begin{cases} \text{tr}(\mathbf{W}) \leq P_T, & \text{for OPC}, \\ \mathbf{W}_{mm} \leq P_I, \forall m, & \text{for IPC}, \end{cases} \end{aligned} \tag{5}$$

which can be solved using the SeDuMi [19] and Yalmip [20] software. If (5) is feasible, we can obtain a solution $\mathbf{W}^*$. Because the rank constraint was eliminated in the original problem, we must check the rank of $\mathbf{W}^*$. When $\mathbf{W}^*$ is of rank one, we can use $\sqrt{\lambda^*_{\max}}\mathbf{w}^*_{\max}$ as an excitation vector, where $\lambda^*_{\max}$ and $\mathbf{w}^*_{\max}$ are the maximum eigenvalue and corresponding eigenvector of $\mathbf{W}^*$, respectively. It is noteworthy that in [13] and [14], the excitation vector was defined as $\sqrt{\lambda^*_{\max}}\mathbf{w}^*_{\max}$, even when $\mathbf{W}^*$ was not of rank one, meaning there is no guarantee of optimality.

To handle the case where the rank of $\mathbf{W}^*$ is higher than one, we propose the following process to find a rank-one solution. First, we set $\tau^* = \text{tr}(\mathbf{A}(\theta_0, \varphi_0)\mathbf{W}^*)$. Then, the problem can be expressed as

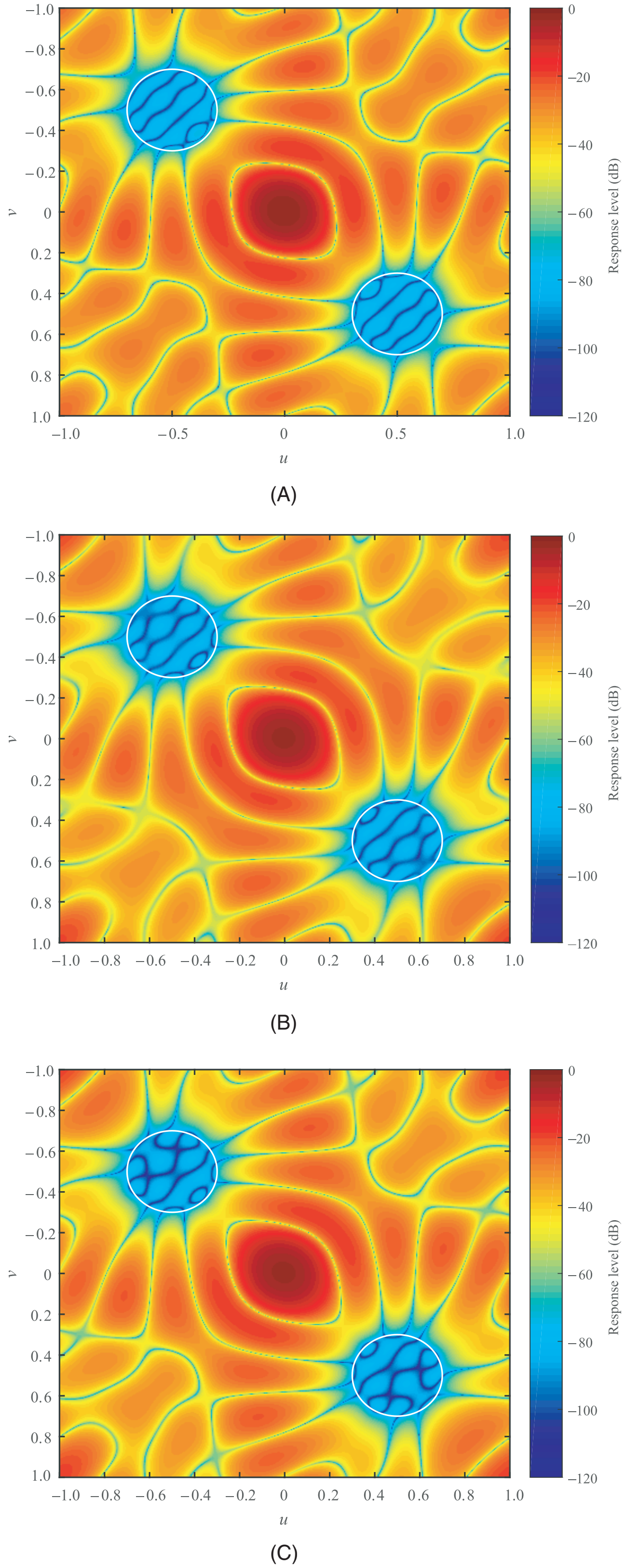

FIGURE 1 Comparison of array response levels for antenna pattern nulling with an IPC when $\tilde{M} = 10$ and $\rho = -80$ dB. (A) Response level of the conventional scheme. (B) Response level of the fmincon scheme. (C) Response level of the proposed scheme

$$\begin{aligned} &\text{find } \mathbf{W} \\ &\text{s.t. } \operatorname{tr}(\mathbf{A}(\theta_0,\varphi_0)\mathbf{W}) = \tau^*, \\ &\quad \operatorname{tr}(\mathbf{A}(\theta,\varphi)\mathbf{W}) \le \rho, \quad \forall(\theta,\varphi) \in S, \\ &\quad \operatorname{rank}(\mathbf{W}) = 1, \quad \mathbf{W} \succeq 0, \\ &\quad \begin{cases} \operatorname{tr}(\mathbf{W}) \le P_T, & \text{for OPC}, \\ \mathbf{W}_{mm} \le P_I, \forall m, & \text{for IPC}. \end{cases} \end{aligned} \tag{6}$$

In this work, we employ the PFM from [17] to solve (6). We set $\mathbf{W}'^{(0)} = \mathbf{W}^*$ and perform the initialization step of the PFM to obtain $\mathbf{W}^{(0)}$ with rank $(\mathbf{W}^{(0)}) \approx 1$. Using $\mathbf{W}^{(0)}$ as a starting point, we then perform the optimization step of the PFM. Both the initialization and optimization steps are iterative processes, where the following SDP problem is solved by using SeDuMi [19] and Yalmip [20] at the jth iteration:

$$\begin{aligned} \mathbf{W}^{(j+1)} = &\operatorname*{argmin}_{\tilde{\mathbf{W}}} \operatorname{tr}(\tilde{\mathbf{W}}) - \lambda_{\max}^{(j)} - \operatorname{tr}\left(\mathbf{w}_{\max}^{(j)}(\mathbf{w}_{\max}^{(j)})^{\dagger}(\tilde{\mathbf{W}} - \mathbf{W}^{(j)})\right) \\ &\text{s.t. } \operatorname{tr}(\mathbf{A}(\theta_0,\varphi_0)\tilde{\mathbf{W}}) = \tau^*, \quad \tilde{\mathbf{W}} \succeq 0, \\ &\operatorname{tr}(\mathbf{A}(\theta,\varphi)\tilde{\mathbf{W}}) \le \rho, \quad \forall(\theta,\varphi) \in S, \\ &\begin{cases} \operatorname{tr}(\mathbf{W}) \le P_T, & \text{for OPC}, \\ \mathbf{W}_{mm} \le P_I, \forall m, & \text{for IPC}, \end{cases} \end{aligned} \tag{7}$$

where $\lambda_{\max}^{(j)}$ and $\mathbf{w}_{\max}^{(j)}$ are the maximal eigenvalue and corresponding eigenvector of $\mathbf{W}^{(j)}$, respectively. If the final solution of the PFM is of rank one, we can use its maximum eigenvalue and corresponding eigenvector to form the excitation vector as discussed above.

The PFM utilizes the fact that a rank-one matrix $\mathbf{W}$ should satisfy $\operatorname{tr}(\mathbf{W}) = \lambda_{\max}$, where $\lambda_{\max}$ denotes the maximal eigenvalue of $\mathbf{W}$. Because $\operatorname{tr}(\mathbf{W}) > \lambda_{\max}$ when $\mathbf{W}$ is not of rank one, the PFM attempts to force $\operatorname{tr}(\mathbf{W}) - \lambda_{\max}$ to approach zero using the iterative method in (7). In [17], it was proven that the updated $\mathbf{W}^{(j+1)}$ in (7) always satisfies $\operatorname{tr}(\mathbf{W}^{(j+1)}) - \lambda_{\max}^{(j+1)} \le \operatorname{tr}(\mathbf{W}^{(j)}) - \lambda_{\max}^{(j)}$, which implies that the iteration in (7) always converges. An optimization problem with rank constraints was also investigated in [21]. Although the PFM in [17] is suitable only for a rank-one constraint, the iterative rank minimization approach in [21] can be used for a rank-r constraint with $r \ge 1$. When the approach in [21] was exploited for a rank-one constraint, the authors aimed to minimize the second largest eigenvalue of the matrix $\mathbf{W}$ through iteration, whereas the iteration of the PFM attempts to minimize $\operatorname{tr}(\mathbf{W}) - \lambda_{\max}$. Note that the authors of [21] introduced a penalty term requiring predetermined parameters to the objective function, whereas the PFM does not require any penalty terms or predetermined parameters. We expect that the iterative rank minimization algorithm can also be used in the proposed scheme instead of the PFM.

It is not always guaranteed that the converged solution of the PFM is of rank one. If the converged solution has a rank higher than one, we determine that a rank-one solution is not feasible for τ^* and perform the bisection technique [16,22]. For a given interval $[l, u]$, we set $\tau = \frac{l+u}{2}$ as the midpoint of the interval and solve the following convex feasibility problem using SeDuMi [19] and Yalmip [20]:

$$\begin{aligned} &\text{find } \mathbf{W} \\ &\text{s.t. } \operatorname{tr}(\mathbf{A}(\theta_0,\varphi_0)\mathbf{W}) = \tau, \quad \mathbf{W} \succeq 0, \\ &\quad \operatorname{tr}(\mathbf{A}(\theta,\varphi)\mathbf{W}) \le \rho, \quad \forall(\theta,\varphi) \in S, \\ &\quad \begin{cases} \operatorname{tr}(\mathbf{W}) \le P_T, & \text{for OPC}, \\ \mathbf{W}_{mm} \le P_I, \forall m, & \text{for IPC}. \end{cases} \end{aligned} \tag{8}$$

If (8) is infeasible, we update $u = \tau$. If (8) is feasible and the solution is of rank one, we update $l = \tau$. If the rank of the solution is higher than one, we perform the PFM method described above to determine if a rank-one solution is available or not. If the PFM provides a rank-one solution, we update $l = \tau$. Otherwise, $u = \tau$ is chosen. For the updated interval, we repeat the above process until the interval is sufficiently small (meaning $u - l \le \epsilon_b$, where ϵ_b is the desired accuracy). When the above bisection process is terminated, we obtain a rank-one solution for $\mathbf{W}$, and the maximum eigenvalue and corresponding eigenvector of the solution are used for our excitation vector. Note that the bisection technique requires an initial interval, which was set to $[0, \tau^*]$ in this study. Because the interval is halved at each iteration, the bisection technique converges after $\lceil \log_2 (\tau^*/\epsilon_b) \rceil$ iterations [23].

4 | NUMERICAL EVALUATION

In this section, we present numerical applications of the proposed scheme. Consider a planar array consisting of $\tilde{M} \times \tilde{M}$ isotropic elements ($M = \tilde{M}^2$). Although the proposed scheme can be used for arbitrary antenna spacing, we assume that each element is uniformly spaced at half-wavelength intervals in a two-dimensional grid for simplicity. We also assume that $g_n(\theta, \varphi) = 1/\sqrt{M}$ for all θ and φ [8]. For numerical implementation, we approximate the constraint $\operatorname{tr}(\mathbf{A}(\theta, \varphi)\mathbf{W}) \le \rho$ for all $(\theta, \varphi) \in S$ in (4) as

$$\begin{aligned} &\operatorname{tr}\left(\mathbf{A}(\theta_p,\varphi_q)\mathbf{W}\right) \le \rho, \\ &\text{for } p \in \{1, 2, \ldots, N_\theta\} \text{ and } q \in \{1, 2, \ldots, N_\varphi\}, \end{aligned} \tag{9}$$

where (θ_p, φ_q) denotes one of $N_\theta N_\varphi$ sample points in the undesired region S [9]. For the following analysis, we fixed the desired direction as $(u_0, v_0) = (0, 0)$.

To validate the proposed scheme, we present numerical results for antenna pattern nulling with an IPC. Consider the null region to be a circle in the $u - v$ domain defined by $S_i = \{(\theta, \varphi); (u - u_i)^2 + (v - v_i)^2 < R_i^2\}$, where (u_i, v_i) and R_i are the center and radius of the circle for the ith null region, respectively. We first consider two null regions with $(u_1, v_1) = (0.5, 0.5)$, $(u_2, v_2) = (-0.5, -0.5)$, and $R_1 = R_2 = 0.2$. Next, the undesired region S in the proposed scheme is set to $S_1 \cup S_2$. Furthermore, we set $\tilde{M} = 10$, $\rho = -80$ dB, and $P_I = 0.01$. Under the above settings, we evaluated the response levels of the proposed scheme and the conventional scheme from [13]. For comparison, we also evaluated the response levels obtained by the MATLAB subroutine "fmincon" [24,25].

Figures 1A, 1B and 1C presents the response levels of the conventional scheme, fmincon scheme, and proposed scheme, respectively. Note that the interiors of the white circles in the figures denote the null regions. It is worth noting that when the solution of (5) was not of rank one, the authors of [13] simply used the maximum eigenvalue and corresponding eigenvector of the solution of (5). Therefore, it cannot be guaranteed that the generated excitation vector is optimal and satisfies the undesired-region response level constraint [17]. From Figure 1A, we found that the response level to the desired direction achieved by the conventional scheme is −2.05 dB, whereas the maximum response level within S is −73.18 dB, meaning the undesired-region response level constraint of −80 dB was not achieved. Note that the proposed scheme exploits the bisection technique with the PFM to handle the additional rank one constraint. From Figure 1C, we found that the response level to the desired direction obtained by the proposed scheme is −2.93 dB, whereas the response level within S is suppressed to less than −80.00 dB. It is clear that the proposed scheme using the bisection technique and PFM is effective for handling the additional rank one constraint. From Figure 1B, we also found that the fmincon scheme suppresses the response level in the undesired region to less than −80 dB and provides a −2.97-dB response level to the desired direction. These results are comparable to those of the proposed scheme.

The above observations are confirmed by Figure 2, which presents magnified views of Figure 1A-C at the null region with a center at (0.5, 0.5). In Figure 2A, response levels over −80 dB are observed inside the white circle for the conventional scheme, whereas the fmincon scheme and proposed scheme suppress the response levels in the null region to less than −80 dB, as shown in Figures 2B and 2C. Another important performance metric is the ratio between the response level to the desired direction and the maximum response level in the undesired region. This ratio for the conventional scheme is 71.13 dB, whereas the proposed scheme achieves a ratio of 77.07 dB. This observation confirms the superiority of the proposed scheme compared to the conventional scheme. It is noteworthy that the fmincon scheme achieves a ratio of 77.03 dB. Overall, the fmincon scheme was found to provide comparable performance to the proposed scheme for array pattern nulling problems with an IPC. However, the next set of results demonstrate that the proposed scheme is superior to the fmincon scheme.

We now focus on the sidelobe suppression problem with an OPC to maximize the response level to the desired direction and suppress the response levels in the sidelobe region to less than a given threshold. The sidelobe region is defined by the exterior of a circle centered at (u_0, v_0) with a radius R ($S = \{(\theta, \varphi); (u - u_0)^2 + (v - v_0)^2 > R^2\}$), which was set as the undesired region for the proposed scheme. Figure 3 compares the response levels achieved by the conventional scheme,

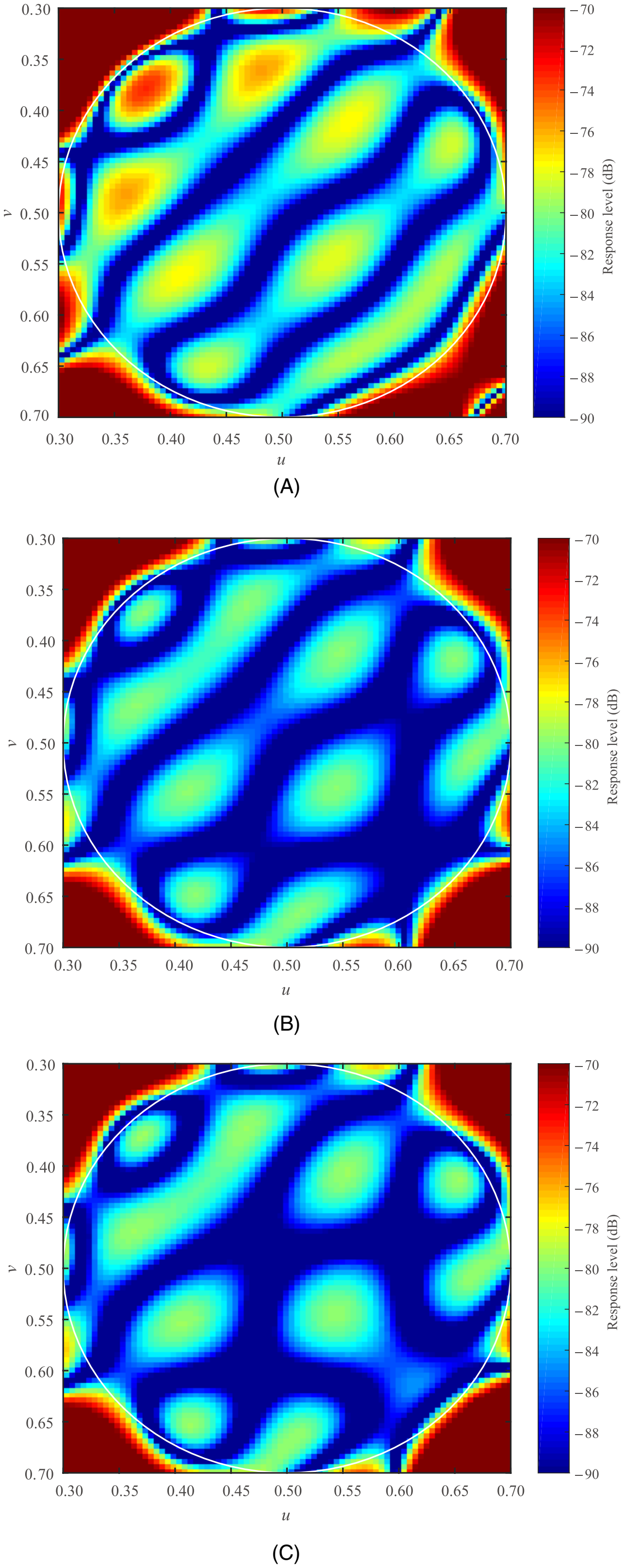

FIGURE 2 Magnified versions of Figure 1 at the null region with a center at (0.5, 0.5). (A) Response level of the conventional scheme. (B) Response level of the fmincon scheme. (C) Response level of the proposed scheme

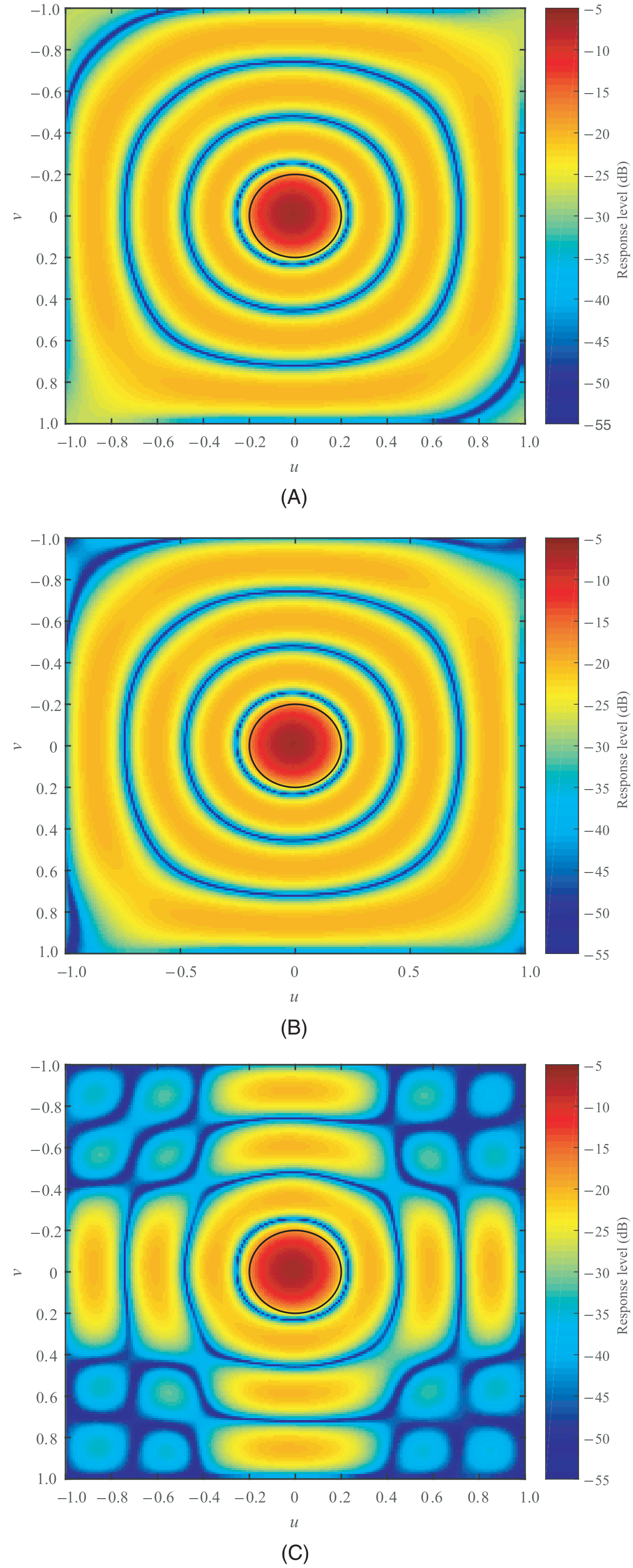

FIGURE 3 Array response levels for sidelobe suppression with an OPC when $\tilde{M} = 8$ and $\rho = -20$ dB. (A) Response level of the conventional scheme. (B) Response level of the fmincon scheme. (C) Response level of the proposed scheme

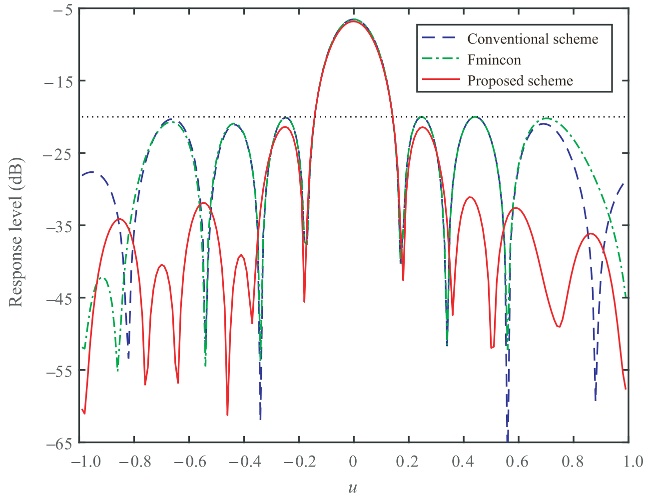

FIGURE 4 Comparison of array response levels as a function of u with $v = u$ for sidelobe suppression with an OPC when $\tilde{M} = 8$ and $\rho = -20$ dB

fmincon scheme, and the proposed scheme. Here, we set $\tilde{M} = 8$, $\rho = -20$ dB, $R = 0.2$, and $P_T = 1$. The exterior of the black circle denotes the sidelobe region. Although it was found that the maximum response level in the sidelobe region was −20 dB for all schemes, we observed that the pattern of the response levels in the $u - v$ domain achieved by the proposed scheme is significantly different from those obtained by the other schemes. To clarify this difference, we present the response levels as a function of u when $v = u$ in Figure 4. One can see that while the response levels of the proposed scheme and other schemes to the desired direction are very similar, the response levels of the proposed scheme in the sidelobe region are significantly lower than those of the other schemes. This observation confirms that the proposed scheme is superior to the conventional and fmincon schemes.

5 | CONCLUSION

In this paper, an array pattern synthesis method using SDP was considered under power consumption constraints, namely the OPC and IPC, for exciting an antenna array. Because the rank-one constraint must be resolved when an array pattern synthesis problem is formulated as an SDP problem, we proposed using the bisection technique and PFM to overcome this issue. We presented numerical applications to array pattern nulling and sidelobe suppression problems to verify that the proposed scheme outperforms conventional schemes.

REFERENCES

1. Y.-H. Nam et al., *Full-dimensional MIMO (FD-MIMO) for next generation cellular technology*, IEEE Commun. Mag. **51** (2013), no. 6, 172–179.
2. S. M. Razavizadeh, M. Ahn, and I. Lee, *Three-dimensional beamforming: a new enabling technology for 5G wireless networks*, IEEE Signal Process. Mag. **31** (2014), no. 6, 94–101.
3. C. L. Dolph, *A current distribution for broadside arrays which optimizes the relationship between beam width and sidelobe level*, Proc. IRE. **34** (1946), 335–348.
4. O. M. Bucci et al., *Antenna pattern synthesis: a general approach*, Proc. IEEE. **82** (1994), no. 3, 358–371.
5. P. Y. Zhou, M. A. Ingram, and P. D. Anderson, *Synthesis of minimax sidelobes for arbitrary arrays*, IEEE Trans. Antennas Propag. **46** (1998), no. 11, 1759–1760.
6. P. Y. Zhou and M. A. Ingram, *Pattern synthesis for arbitrary arrays using an adaptive array method*, IEEE Trans. Antennas Propag. **47** (1999), no. 5, 862–869.
7. H. Lebret and S. Boyd, *Antenna array pattern synthesis via convex optimization*, IEEE Trans. Signal Process. **45** (1997), no. 3, 526–532.
8. B. Fuchs and J. J. Fuchs, *Optimal narrow beam low sidelobe synthesis for arbitrary arrays*, IEEE Trans. Antennas Propag. **58** (2010), no. 6, 2130–2135.
9. B. Fuchs, A. Skrivervik, and J. R. Mosig, *Synthesis of uniform amplitude focused beam arrays*, IEEE Antennas Wireless Propag. Lett. **11** (2012), 1178–1181.
10. P. Rocca, R. J. Mailloux, and G. Toso, *GA-based optimization of irregular subarray layouts for wideband phased arrays design*, IEEE Antennas Wireless Propag. Lett. **14** (2015), 131–134.
11. S. Todnatee and C. Phongcharoenpanich, *Iterative GA optimization scheme for synthesis of radiation pattern of linear array antenna*, Int. J. Antennas. Prop. **2016** (2016), 7087298.
12. F. Wang et al., *Optimal array pattern synthesis using semidefinite programming*, IEEE Trans. Signal Process. **45** (1997), no. 3, 526–532.
13. P. J. Kajenski, *Phase only antenna pattern notching via a semidefinite programming relaxation*, IEEE Trans Antennas Propag. **60** (2012), no. 5, 2562–2565.
14. B. Fuchs, *Application of convex relaxation to array synthesis problems*, IEEE Trans Antennas Propag. **62** (2014), no. 2, 634–640.
15. Z. Luo et al., *Semidefinite relaxation of quadratic optimization problems*, IEEE Signal Process. Mag. **27** (2010), no. 3, 20–34.
16. S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge, Cambridge University Press, 2004.
17. H. M. Wang et al., *Hybrid cooperative beamforming and jamming for physical-layer security of two-way relay networks*, IEEE Trans. Inf. Forensics Security. **8** (2013), no. 12, 2007–2020.
18. O. M. Bucci, L. Caccavale, and T. Isernia, *Optimal far-field focusing of uniformly spaced arrays subject to arbitrary upper bounds in nontarget directions*, IEEE Trans. Antennas Propag. **50** (2002), no. 11, 1539–1554.
19. J. F. Sturm, *Using SeDuMi 1.02, a Matlab toolbox for optimization over symmetric cones*, Optim. Methods Softw. **11-12** (1999), 625–653.
20. J. Lofberg, YALMIP: A toolbox for modeling and optimization in MATLAB, *Proc. the CACSD Conf.*, New Orleans, LA, USA, 2004.
21. C. Sun and R. Dai, *Rank-constrained optimization and its applications*, Automatica. **82** (2017), 128–136.
22. J.-H. Lee, *Confidential multicasting assisted by multi-hop multi-antenna DF relays in the presence of multiple eavesdroppers*, IEEE Trans. Commun. **64** (2016), no. 10, 4295–4304.
23. E. Karipidis, N. D. Sidiropoulos, and Z.-Q. Luo, *Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups*, IEEE Trans. Signal Process. **56** (2008), no. 3, 1268–1279.

24. T. Isernia et al., *A hybrid approach of the optimal synthesis of pencil beams through array antennas*, IEEE Trans. Antennas Propag. **52** (2004), no. 11, 2912–2918.
25. P. Rocca, N. Anselmi, and A. Massa, *Optimal synthesis of robust beamformer weights exploiting interval analysis and convex optimization*, IEEE Trans. Antennas Propag. **62** (2014), no. 7, 3603–2612.

AUTHOR BIOGRAPHIES

Jong-Ho Lee received his BS degree in Electrical Engineering, and his MS degree and PhD in Electrical Engineering and Computer Science from Seoul National University, Seoul, Rep. of Korea in 1999, 2001, and 2006, respectively. From 2006 to 2008, he was a Senior Engineer at Samsung Electronics, Suwon, Rep. of Korea. From 2008 to 2009, he was a Postdoctoral Researcher at the Georgia Institute of Technology, Atlanta, GA, USA. From 2009 to 2012, he was an Assistant Professor at the Division of Electrical, Electronic, and Control Engineering, Kongju National University, Cheonan, Rep. of Korea. From 2012 to 2018, he was an Associate Professor at the Department of Electronic Engineering, Gachon University, Seongnam, Rep. of Korea. Since 2018, he has been a faculty member at the School of Electronic Engineering, Soongsil University, Seoul, Rep. of Korea. His research interests include wireless communication systems and signal processing for communication, with an emphasis on multiple-antenna techniques, multi-hop relay networks, physical layer security, and full-duplex wireless communications.

Jeongsik Choi received his BS degree in Electrical Engineering from Pohang University of Science and Technology, Pohang, Rep. of Korea, in 2010, and his MS degree and PhD in Electrical Engineering from Seoul National University (SNU), Seoul, Rep. of Korea in 2012 and 2016, respectively. From 2016 to 2017, he was a Senior Researcher at the Institute of New Media and Communications at SNU. Since 2017, he has been a Research Scientist at Intel Labs, Intel Corporation, Santa Clara, CA, USA. His research interests include wireless propagation channel measurement and modeling, wireless communication systems, and positioning algorithms.

Woong-Hee Lee received his BS degree in Electrical Engineering from the Korea Advanced Institute of Science and Technology, Daejeon, Rep. of Korea in 2009 and his PhD in Electrical Engineering from Seoul National University, Seoul, Rep. of Korea in 2017. From 2017 to 2019, he worked at the Advanced Standard Research and Development Laboratories, LG Electronics, Seoul, Rep. of Korea. He is a Postdoctoral Researcher with the Department of Communication Systems, KTH Royal Institute of Technology, Stockholm, Sweden. His research interests include signal processing, machine learning and game theory in wireless communications.

Jiho Song received his BS and MS degrees in Electrical Engineering from Seoul National University, Seoul, Rep. of Korea in 2009 and 2011, respectively, and his PhD in Electrical and Computer Engineering from Purdue University, West Lafayette, IN, USA in 2017. He is currently an Assistant Professor at the School of Electrical Engineering, University of Ulsan, Ulsan, Rep. of Korea. Before joining the University of Ulsan, he was a Senior Researcher at Motorola Mobility, Chicago, IL, USA. His research interests include the design and analysis of millimeter-wave communication, multiuser MIMO communication, and limited feedback strategies for massive MIMO systems. He was the recipient of the Bronze Prize in the Samsung Electronics 23rd Humantech Paper Contest in 2017.