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Partial Inverse Traveling Salesman Problems on the Line

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[Abstract]

The partial inverse optimization problem is an interesting variant of the inverse optimization problem in which the given instance of an optimization problem need to be modified so that a prescribed partial solution can constitute a part of an optimal solution in the modified instance. In this paper, we consider the traveling salesman problem defined on the line (TSP on the line) which has many applications such as item delivery systems, the collection of objects from storage shelves, and so on. It is worth studying the partial inverse TSP on the line, defined as follows. We are given n requests on the line, and a sequence of k requests that need to be served consecutively. Each request has a specific position on the real line and should be served by the server traveling on the line. The task is to modify as little as possible the position vector associated with n requests so that the prescribed sequence can constitute a part of the optimal solution (minimum Hamiltonian cycle) of TSP on the line. In this paper, we show that the partial inverse TSP on the line and its variant can be solved in polynomial time when the sever is equiped with a specific internal algorithm ***Forward Trip*** or with a general optimal algorithm.

▸ **Key words**: Inverse Optimization, Partial Inverse Problem, Traveling Salesman Problem, Poly-time Algorithm, TSP on the Line

[요 약]

부분역최적화는 역최적화의 흥미로운 변형으로, 주어진 최적화문제와 그 문제의 부분해가 주어지면 이 부분해가 최적해에 포함되도록 문제를 최소한으로 수정하는 문제이다. 이 논문은 라인 위에서 정의되는 순환외판원문제(TSP)를 다루는데, 이는 배달시스템, 창고 선반에서 물건을 수집하는 것, 등의 많은 응용을 가진다. 라인 위에서 위치하는 n개의 일이 주어지고 이 중 연속적으로 처리해야하는 일 k개가 부분적으로 주어진다. 각각의 일은 라인 위의 특정 장소에 위치하고 라인을 움직이는 서버에 의해 처리되어야 한다. 우리의 임무는 k개의 일이 최적해에서 연속적으로 처리되도록 n개의 일의 위치를 라인 위에서 최소한으로 조정하는 것이다. 이 논문에서 이 문제와 이 문제의 다양한 변종을 다항시간 내에 푸는 알고리즘을 개발한다. 구체적으로, 서버가 특정한 ***Forward Trip***이라는 특정한 내부 알고리즘을 사용하는 경우와 일반적인 최적 알고리즘을 사용하는 경우에 대한 부분역최적화를 다룬다.

▸ **주제어**: 역최적화, 부분역최적화, 순환외판원문제, 다항알고리즘, 라인 위에서의 TSP

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I. Introduction

Given a problem, we usually aim to find the optimal solution. However, it is not trivial to find the best solution, especially in the case where the given problem is intrinsically hard to solve. The traveling salesman problem(TSP for short) is the one of such problems.

On the other hand, it is relatively easy to get a feasible solution, even for the hard optimization problems. The inverse optimization problem is defined with the goal of making the prescribed feasible solution optimal in the new instance rather than finding an optimal solution[1-3]. The inverse optimization has been studied for many combinatorial optimization problem such as the shortest path problem, the minimum spanning tree problem, the maximum independent set problem, the minimum graph coloring problem, TSP, and so on[4-13].

The partial inverse optimization problem is an interesting variant of the inverse optimization problem in which a partial solution is given instead of a whole feasible solution. The objective is to modify the least possible the instance of the problem so that the prescribed partial solution becomes a part of the optimal solution in the modified instance. The partial inverse problem has been studied for the assignment problem, minimum cut problem, the shortest path problem, TSP, and so on[14-16].

For TSP, the inverse optimization and the partial inverse optimization problems have been studied by [8, 17]. In [8], the authors show that the inverse TSP against the closest neighbor algorithm can be solved in polynomial time, and the problem against *2-opt* is NP-hard when the edge length are restricted to 1 or 2. In [17], the partial inverse TSP is shown to be NP-hard against both Closest Neighbor, and *2-opt.*

In this paper, we study the partial inverse traveling salesman problem defined on a special metric space, the real line. While TSP is the one of the most well-known NP-hard problem[18], TSP on the line becomes trivial. For TSP on the line space, the inverse problem is studied by [8] and it is shown that the inverse traveling salesman problem can be solved in $O(n \log n)$ time. However, the partial inverse problem for TSP on the line has not been studied yet.

In practice, TSP on the line ($TSPL$, for short) has many applications such as robotic welding, item delivery systems, and the collection of objects from storage shelves, and so on. In practice, we may have some partial solution for $TSPL$. It can be a list of objects that need to be collected in order, or a list of items that need to be delivered consecutively. It is worth studying its partial inverse version where the given partial solution can become a part of an optimal solution for $TSPL$.

The partial inverse traveling salesman problem on the line is defined as follows and we denote it by $PI(TSPL)$.

We deal with two versions of partial inverse traveling salesman problem on the line. The one is defined with a partial solution of $TSPL$ which is a subsequence of requests that need to be served consecutively. The other is defined with a subset of requests with a specific prescribed order(e.g., precedence conditions). We denote the former by $PI(TSPL)$ and the latter $PI(TSPLP)$.

Definition 1. $PI(TSPL)$.

Given n requests on the horizontal line, and a sequence of k requests, $PI(TSPL)$ is to modify as little as possible the positions of some requests on the line so that the given sequence of k requests can be can be a part of the optimal $TSPL$ -solution.

Definition 2. $PI(TSPLP)$.

Given n requests on the horizontal line, and k requests with precedence conditions in the prescribed order, $PI(TSPL)$ is to modify as little as possible the positions of some requests on the line so that the given k requests can be served in the prescribed order by the server in the optimal $TSPL$-solution, while satisfying the precedence conditions between requests.

This paper is organized as follows: In Section II, we introduce the partial inverse traveling salesman problem on the line against a specific algorithm. In Section III, we discuss the polynomial time solvability of the partial inverse traveling salesman problem on the line against any optimal algorithm. We consider in Section IV the partial inverse traveling salesman problem on the line with precedence conditions, and discuss the relation with the minimum cost flow problem. We then conclude the paper.

II. Partial Inverse Traveling Salesman Problems on the Line against a Specific Algorithm

Let us consider the partial inverse traveling salesman problem on the line against a specific algorithm A for TSP, denoted by $PI(TSPL)_A$. In [4], the authors introduced the inverse problem against a specific algorithm. The objective is to determine a new instance at a minimum cost (with minimum modification) such that the given feasible solution can be selected as a best solution when applying the algorithm.

In $PI(TSPL)_A$, we are given n requests dispersed along a horizontal line space, and k requests that need to be served consecutively by the server. The server decides the order to serve the requests by using its specific internal algorithm A in order to minimize the makespan, i.e., the time until all requests have been served. It may often happen that the given k requests cannot be visited in the prescribed sequential order in the optimal tour returned by the algorithm. The objective of $PI(TSPL)_A$ is to modify as little as possible the positions of some requests on the line in such a way that the given sequence k requests can constitute a part of the best solution returned by A.

As a specific algorithm for solving TSP on the line space, we consider the greedy algorithm, called ***Forward Trip*** (FT, for short). FT starts the trip from the leftmost request on the line and satisfies all the requests in its forward trip; no request can be served on the way back. It is clear that FT is an optimal algorithm for TSP on the line ($TSPL$ for short).

Note that an instance of $TSPL$ can be described as a position vector $x=(x_1,\cdots,x_n)$ of n requests on the real line space. Then $PI(TSPL)_{FT}$ can be defined as follows.

Definition. Given a position vector $x=(x_1,\cdots,x_n)$ of n requests on the real line and a sequence $r_1-r_2-\cdots-r_k$ of k requests, $PI(TSPL)_{FT}$ is to find a new position vector $y=(y_1,\cdots,y_n)$ such that the given sequence can be a part of the optimal tour returned by FT in the new instance, and the total deviation $\| x-y \|_1$ between the original and new position vectors becomes minimum under the L_1-norm.

Focusing on the given k requests that need to be served consecutively (without preemption), we consider the following sets of indices. We denote by $L=\{i\in\{k+1,\cdots,n\} \mid x_i \leq x_1\}$ the indices of the requests located on the left-hand side with respect to the request r_1, and denote by

$R=\{i\in\{k+1,\cdots,n\}\mid x_i \geq x_k\}$ the indices of the requests originally located on the right-hand side with respect to the request r_k.

Then, one can rewrite $PI(TSPL)_{FT}$ as follows.

$$(P_1) \qquad \min \sum_{i=1}^{n}|x_i - y_i|$$

subject to

$$\max_{i\in L}x_i \leq y_1 \leq y_2 \leq \cdots \leq y_k \leq \min_{j\in R}x_j$$

Proposition 1. $PI(TSPL)_{FT}$ has an optimal solution such that $y_i \leq y_j$ if $x_i \leq x_j$ for any $i,j\in\{k+1,\cdots,n\}$.

Proof. Let $y=(y_1,\cdots,y_n)$ be an optimal solution of $PI(TSPL)_{FT}$. Assume that for some $i,j\in\{k+1,\cdots,n\}$, we have $x_i \leq x_j$ and $y_i \geq y_j$. Now we consider another feasible solution y' such that $y'_i = y_j$, $y'_j = y_i$ and $y'_k = y_k$ for any $k \neq i,j$. It is easy to see that y' is at least as good as the given optimal solution y. By repeating this, we can obtain an optimal solution for $PI(TSPL)_{FT}$ such that $y_i \leq y_j$ if $x_i \leq x_j$ for any $i,j>k$.

■

Due to Proposition 1, one can assume without loss of generality that $x_{k+1} \leq x_{k+2} \leq \cdots \leq x_n$. Then, we can rewrite $PI(TSPL)_{FT}$ as the problem of solving the following $n-k+1$ subproblems:

$$(P_2) \qquad \min \sum_{i=1}^{n}|x_i - y_i|$$

subject to

$$y_1 \leq \cdots \leq y_k \leq y_{k+1} \leq \cdots \leq y_n$$

or

$$y_{k+1} \leq y_1 \leq \cdots \leq y_k \leq y_{k+2} \leq \cdots \leq y_n$$

or

$$y_{k+1} \leq y_{k+2} \leq y_1 \leq \cdots \leq y_k \leq y_{k+3} \leq \cdots \leq y_n$$

or

$$\cdots$$

or

$$y_{k+1} \leq y_{k+2} \leq \cdots \leq y_n \leq y_1 \leq \cdots \leq y_k$$

It is straightforward to see that each of these subproblems is a special case of the isotonic median regression problem[19-21] In [20], the PAV-algorithm (Pool Adjacent Violaters) is proposed for solving the isotonic median regression problem, and this algorithm solves each subproblem of $PI(TSPL)_{FT}$ in time $O(n\log n)$ in the worst case. This leads to the following result.

Proposition 2. $PI(TSPL)_{FT}$ can be solved in polynomial time.

Proof. One can solve $PI(TSPL)_{FT}$ by applying the PAV algorithm to the $n-k+1$ problems, and the new position vector (the optimal solution) which has the minimum deviation value.

■

III. Partial Inverse Traveling Salesman Problems on the Line against Any Optimal Algorithm

In this section, we consider the partial inverse traveling salesman problem on the line against any general optimal algorithm rather than a specific algorithm for $TSPL$. We denote this problem by $PI(TSPL)_{OPT}$.

In $PI(TSPL)_{OPT}$, the requests on the line can be served during both forward and backward trips, while in $PI(TSPL)_{FT}$, the requests can be served only on the forward trip. It is easy to see that the result of Proposition 1 still holds for $PI(TSPL)_{OPT}$.

We assume that the server starts the trip by visiting the request r_1 and finishes the trip by returning to that request. Let r_s and r_t be the leftmost and rightmost request on the line with $1 \le s, t \le n$.

Then, one can observe that any instance of $PI(TSPL)_{OPT}$ admits an optimal Hamiltonian cycle $r_1 - r_2 - \cdots - r_n - r_1$ if the associated position vector $x = (x_1, \cdots, x_n)$ satisfies the following conditions (a) and (b):

(a) if $1 \le s < t \le n$,

$x_s \le x_{s+1} \le \cdots \le x_t$ and
$x_s \le x_{s-1} \le \cdots \le x_2 \le x_1$
$x_1 \le x_n \le x_{n-1} \le \cdots \le x_{t+1} \le x_t$

(b) if $1 \le t < s \le n$,

$x_s \le x_{s-1} \le \cdots \le x_t$ and
$x_s \le x_{s+1} \le x_{s+2} \le \cdots \le x_n$
$x_n \le x_1 \le x_2 \le x_{t-1} \le x_t$

Based on these results, we devise an optimal algorithm for $PI(TSPL)_{OPT}$, using the PAV-algorithm for solving the special case of the isotonic median regression.

When an original position vector $x = (x_1, \cdots, x_n)$ and a sequence $r_1 - r_2 - \cdots - r_k$ of k requests in the increasing order are given, our algorithm computes a new vector $y_{(i_0, j_0)} = \left(y_1, \cdots, y_{i_0}, \cdots, y_{j_0}, \cdots, y_k, \cdots, y_n\right)$ for each (i_0, j_0) with $1 \le i_0, j_0 \le k$ that minimizes the quantity $\sum_{i=1}^{n} |x_i - y_i|$, while satisfying the following:

(i) $\forall i \in \{k+1, \cdots, n\}$, $y_i = x_i$,

(ii) By using the PAV-algorithm, determine y_i $\forall i \in \{1, \cdots, k\}$ by solving the following problems.

In the case $1 \le i_0 < j_0 \le k$,

$$(P_3) \qquad \min \sum_{i=1}^{k} |x_i - y_i|$$

subject to

$y_{i_0} \le y_{i_0+1} \le \cdots \le y_{j_0}$,
$y_{i_0} \le y_{i_0-1} \le \cdots \le y_2 \le y_1 \le y_k$, and
$y_k \le y_{k-1} \le \cdots \le y_{j_0+1} \le y_{j_0}$

In the case $1 \le j_0 < i_0 \le k$,

$$(P_4) \qquad \min \sum_{i=1}^{k} |x_i - y_i|$$

subject to

$y_{i_0} \le y_{i_0-1} \le \cdots \le y_{j_0}$,
$y_{i_0} \le y_{i_0+1} \le \cdots \le y_k$, and
$y_k \le y_1 \le y_2 \le \cdots \le y_{j_0-1} \le y_{j_0}$

For each (i_0, j_0) with $1 \le i_0, j_0 \le k$, the PAV-algorithm [16] solve this problem in time $O(k \log k)$. Hence, our algorithm computes the best solution for $PI(TSPL)_{OPT}$ with the minimum deviation with respect to the L_1-norm in time $O(k^3 \log k)$.

Proposition 3. The overall computational complexity for $PI(TSPL)_{OPT}$ is $O(k^3 \log k)$.

IV. Partial Inverse Traveling Salesman Problems on the Line with Precedence Constraints

In the previous sections, we have considered the partial inverse traveling salesman problem on the line, defined with the prescribed partial solution. In this section, we consider an interesting variant of $PI(TSPL)_A$ where instead of a sequence of requests, the precedence conditions $r_i \prec r_j$ are imposed on some requests. We mean by $r_i \prec r_j$ that the request r_i needs to be served before r_j, however the other requests can be served between r_i and r_j by the server. We assume that the precedence conditions are imposed on k requests, i.e., $r_1 \prec r_2 \prec \cdots \prec r_k$. We call this problem the partial inverse traveling salesman problem on the line with the precedence conditions, and denote it by $PI(TSPLP)_A$.

Let us consider $PI(TSPLP)_A$ against FT. Let $\Omega = \{(i,j) | r_i < r_j\}$ be the set of the pairs of requests with precedence conditions. Then, the problem $PI(TSPLP)_{FT}$ can be formulated as follows:

$$(P_5) \qquad \min \sum_{i=1}^{n} |x_i - y_i|$$
$$\text{subject to}$$
$$-y_i + y_j \geq 0, \ \forall (i,j) \in \Omega$$

By linearization, we obtain the following model.

$$(P_6) \qquad \min \sum_{i=1}^{n} (d_i^+ - d_i^-)$$
$$\text{subject to}$$
$$y_i + d_i^+ - d_i^- = x_i, \ \forall i = 1, \cdots, n$$
$$-y_i + y_j \geq 0, \ \forall (i,j) \in \Omega$$
$$d_i^+, d_i^- \geq 0, \forall i = 1, \cdots, n$$

Let μ_i and $\lambda_{i,j}$ be the dual variables corresponding to the first and second constraints of the problem (P_2), respectively. Then, the dual of the problem (P_2) can be written as follows.

$$(P_7) \qquad \max \sum_{i=1}^{n} x_i \mu_i$$
$$\text{subject to}$$
$$\sum_{j|(i,j) \in \Omega} \lambda_{i,j} - \sum_{j|(j,i) \in \Omega} \lambda_{j,i} = \mu_i, \forall i = 1, \cdots, n$$
$$-1 \leq \mu_i \leq 1, \ \forall i = 1, \cdots, n$$
$$\lambda_{i,j} \geq 0, \quad \forall (i,j) \in \Omega$$

or equivalently,

$$(P_8) \qquad \min \ -\sum_{i=1}^{n} x_i \mu_i$$
$$\text{subject to}$$
$$\sum_{j|(i,j) \in \Omega} \lambda_{i,j} - \sum_{j|(j,i) \in \Omega} \lambda_{j,i} = \mu_i, \ \forall i = 1, \cdots, n$$
$$-1 \leq \mu_i \leq 1, \ \forall i = 1, \cdots, n$$
$$\lambda_{i,j} \geq 0, \quad \forall (i,j) \in \Omega$$

Using this formulation, we will show that $PI(TSPLP)_{FT}$ can be reduced to the minimum cost flow problem (MCF, for short). We construct an instance of MCF as follows:

- Graph structure:
  $G = (V, E)$, with $V = \{1, \cdots, n\} \cup \{s, t\}$ and $E = \Omega \cup \{(s,i), (i,t) | i = 1, \cdots, n\} \cup \{(s,t)\}$.

- Arc capacity:
  $u_{i,j} = \infty$, if $\forall (i,j) \in \Omega$,
  $u_{s,i} = u_{i,t} = 1$, $\forall i = 1, \cdots, n$
  and $u_{s,t} = n$

- Arc cost:
  $c_{i,j} = 0$, if $\forall (i,j) \in \Omega$,
  $c_{s,i} = -x_i$, $c_{i,t} = x_i$, $\forall i = 1, \cdots, n$
  and $c_{s,t} = 0$

- Node balance:
  $b(s) = n, b(t) = -n$
  and $b(i) = 0$, $\forall i = 1, \cdots, n$

- Let be $f_{i,j}$ the flow on each arc $(i,j) \in \Omega$.

Proposition 4. $PI(TSPLP)_{FT}$ can be reduced to the minimum cost flow problem and thus can be solved in polynomial time.

Proof. Given an instance (P_4) of $PI(TSPLP)_{FT}$, we have just constructed an instance of MCF. It remains to us to show that a feasible solution of $PI(TSPLP)_{FT}$ is feasible for the constructed instance of MCF, with the same objective value, and vice versa.

Given a feasible solution $(\lambda_{i,j},\mu_i)$ of (P_4), we can find a feasible solution for MCF by setting $f_{i,j}=\lambda_{i,j},\ \forall\,(i,j)\in\Omega$, $f_{s,i}=\mu_i, f_{i,t}=0$, if $\mu_i\geq 0$, and $f_{s,i}=0, f_{i,t}=-\mu_i$, if $\mu_i\leq 0$, and by setting $f_{s,t}=n-\sum_{i=1}^{n}\max\{0,\mu_i\}$. Clearly, this is a feasible solution for MCF.

We can also find a feasible solution $(\lambda_{i,j},\mu_i)$ for $PI(TSPLP)_{FT}$ using a feasible solution $f_{i,j}$ for MCF by setting $\lambda_{i,j}=f_{i,j},\ \forall\,(i,j)\in\Omega$ and $\mu_i=f_{s,i}-f_{i,t}$ for any $i=1,\cdots,n$.

Hence, $PI(TSPLP)_{FT}$ can be reduced to the minimum cost flow problem and can be solved in polynomial time.

■

V. Conclusions

In this paper, we have studied the partial inverse traveling salesman problem on the line against any optimal algorithm as well as against a specific algorithm for $TSPL$. When a sequence of some requests is given as a partial solution, $PI(TSPL)_{FT}$ and $PI(TSPL)_{OPT}$ turn out to be polynomially solvable by using the PAV-algorithm devised for the isotonic median regression problem. When precedence conditions are given instead of a sequence of requests, we show that $PI(TSPLP)_{FT}$ is reducible to the minimum cost flow problem, one of the most well-known network optimization problem. The obtained results are summarized in the following table.

Table 1. Complexity Status of Problems

Problems	Complexity Status
$PI(TSPL)_{FT}$	poly-time solvable
$PI(TSPL)_{OPT}$	poly-time solvable
$PI(TSPLP)_{FT}$	poly-time solvable
$PI(TSPLP)_{OPT}$	unknown

Our paper contributes to the inverse optimization literature by investigating the computational complexity of the partial inverse traveling salesman problem on the line, which has not yet been studied to the best of our knowledge.

This study has also some limitations. First, the complexity status for $PI(TSPLP)_{OPT}$ has not been investigated yet. Second, even though the algorithms we proposed for solving $PI(TSPL)_{FT}$ and $PI(TSPL)_{OPT}$ are theoretically correct, they have not yet been used for solving the real examples. The complexity status for $PI(TSPLP)_{OPT}$ and the computation part remain as a future work. In addition, it would be an interesting future research to consider the partial inverse traveling salesman problem on some other metric space such as trees and lattices.

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