

A Study on the Quantitative Determination of Failure Effect Probability for Criticality Analysis on System

Myeong-seok Lee*, Seong-Dae Choi*, Jang-wook Hur*[#]

*Department of Mechanical System Engineering, Kumoh National Institute of Technology

시스템의 치명도 분석을 위한 고장영향확률 정량화 방안 연구

이명석*, 최성대*, 허장욱*[#]

*금오공과대학교 기계시스템공학과

(Received 10 June 2019; received in revised form 22 June 2019; accepted 23 July 2019)

ABSTRACT

The inter-development of FMECA is very important to assess the effect of potential failures during system operation on mission, safety and performance. Among these, criticality analysis is a core task that identifies items with high risk and selects the analyzed objects as the key management targets and reflects their effects to the design optimization. In this paper, we analyze the theory related to criticality analysis following US military standard, and propose a method to quantify the failure effect probability for objective criticality analysis. The criticality analysis according to the US military standard depends on the subjective judgment of the failure probability. The methodology for quantifying the failure effect probability is presented by using the reliability theory and the Bayes theorem. The failure rate is calculated by applying the method to quantify failure effect probability.

Keywords : Criticality Analysis(치명도 분석), Failure Effect Probability(고장영향확률), Quantification(정량화), Bayes's Theory(베이즈 이론)

1. Introduction

In order to evaluate the effect of potential failures of system operations on mission, safety, and performance, FMECA(Failure Mode, Effects and Criticality Analysis) is performed among the developments. The FMECA

result is used to identify the items with high risk by evaluating the expected failure effects of the items constituting the system.

Besides choosing the analyzed objects as the key management targets, FMECA also allows us to reflect them in design, develop exclusion procedure, prepare technical manuals and analyze complete reliability centered maintenance^[1-4].

Usually, in order to establish reasonable preventive

Corresponding Author : hhjw88@kumoh.ac.kr

Tel: +82-53-478-7399, Fax: +82-53-478-7399

maintenance between life cycle of systems, it is necessary to select preventive maintenance items and preventive maintenance cycles through RCM analysis. However, in the analysis of the criticality using US military specifications, many procedures are applied in different aspects according to the developer's capability and requirements. For an ease in accessibility, it is required to establish an objective standard procedure of criticality analysis^[5-6].

Therefore, this paper summarizes the reliability and Bayes theory related to the criticality analysis, examines the problems of the US military standard for criticality analysis and suggests a method for quantifying the probability of failure for objective criticality analysis.

2. Theoretical background

2.1 Reliability theory^[7]

The instantaneous failure rate $\{\lambda(t)\}$ is the rate at which an item that has been operating up to a certain point causes a failure within a certain unit time. It is used as a scale to indicate the frequency of the occurrence of failure, and can be derived using equation (1).

$$\lambda(t) = \frac{\Pr[t < T \leq t + \Delta t | T > t]}{\Delta t} = \frac{f(t)}{1 - F(t)} \quad (1)$$

$F(t)$ is the unreliability function, that represents the probability of failure occurring within a certain point of time, and $f(t)$ is the failure density function, which means the frequency of failure occurrence over time. In general, the failure rate can be defined as the number of failures occurring during the unit time. In many applications, the average failure rate $\{\bar{\lambda}(t_1, t_2)\}$ is also used. It can also be expressed as equation (2) using the reliability function $\{R(t)\}$.

$$\bar{\lambda}(t_2, t_1) = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1}, t_2 > t_1 \quad (2)$$

$R(t)$ means the probability of satisfactorily operation during the intended use period under given operating conditions. And, $\lambda(t)$ and $R(t)$ used in equation (4) are derived from the relation of (3).

$$\int_0^t \lambda(u) du = - \int_0^t \frac{R'(u)}{R(u)} du = - \ln R(t) \quad (3)$$

$$R(t) = \exp[- \int_0^t \lambda(u) du] \quad (4)$$

Assuming that only one failure occurs at a certain point in the above equation (4), and assuming that the life of the part follows the exponential distribution, $\lambda(t)$ becomes a constant. In that case, $F(t)$ can be expressed as equation (5).

$$F(t) = 1 - R(t) = 1 - \exp(-\lambda t) \quad (5)$$

Taylor's theorem is an approximation of the analytic function by the polynomial function. The approximation of the analytic function by the n^{th} polynomial function is called the Taylor approximation, and the Taylor approximation of $F(t)$ following the exponential distribution is expressed as (6). The approximate value of the second order Taylor function is given by equation (7)^[8].

$$F(t) = 1 - e^{-\lambda t} = 1 - \sum_{n=0}^{\infty} \frac{(-\lambda t)^n}{n!} \quad (6)$$

$$F(t) \approx \lambda t \quad (7)$$

As a result, when the failure rate and time are very small, the probability of a failure occurring within a period can be expressed as a product of failure rate and time.

2.2 Bayes Theory

Bayes's theorem describes the process of inferring the probability of an event based on previous experience and current evidence. If event A is any

event of sample space S , The conditional probability that event B occurs under the condition that A has occurred is expressed as (8)^[9].

$$P(B|A_i) = \frac{P(B) \cdot P(A_i|B)}{\sum_{i=1}^n P(B)P(A_i|B)} \quad (8)$$

$P(A_i)$ is the prior probability of A , $P(B)$ is the prior probability of B , $P(A_i|B)$ is the conditional probability of A when event B is given, $P(B|A_i)$ is the posterior probability for the evidence of event A .

In the criticality analysis, the failure effect probability (β_j) is defined as the probability that the failure effect will belong to a specific severity classification, under the condition that a particular failure mode has occurred. This implies that conditional probability $P(B_j|A_i)$ can be expressed as equation (9) by applying Bayes's theorem. Where A_i is defined as a specific failure mode occurrence event and B_j as a failure effect occurrence event according to a specific failure mode.

$$\beta_j = P(B_j|A_i) = \frac{P(B_j) \cdot P(A_i|B_j)}{P(A_i)} \quad (9)$$

$P(A_i|B_j)$ represents the conditional probability of A_i when the event B_j is given, $P(A_i)$ represents the probability of occurrence of a specific failure mode, and $P(B_j)$ represents the probability of a specific failure effect occurring.

2.3 Relationship between failure mode and failure effect

The terms failure cause, failure effect, and failure mode are used interchangeably in practical applications. According to the definition, a failure mode means a symptom of a failure due to a failure effect^[10], and a failure effect indicates a result of a failure mode occurring in terms of the operation, function, or state of the system^[11]. As a result, the failure mode can be defined as a prior probability event in which the causal

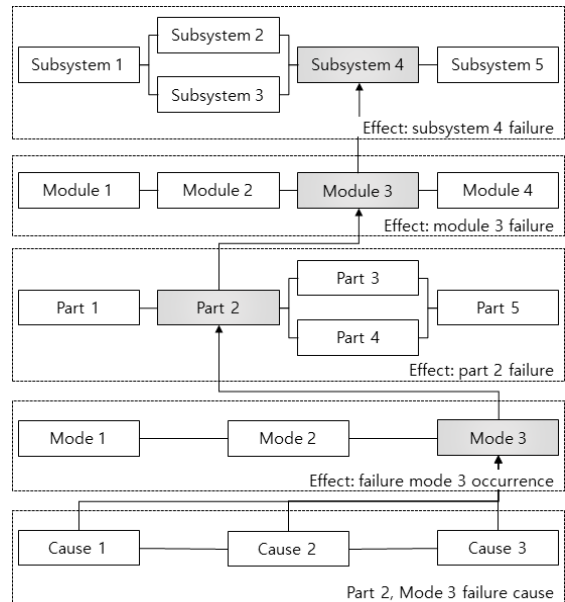


Fig. 1 Relationship between failure modes and failure effects

relation of the failure effect is established, and the failure mode and the failure effect are in the dependency relation as the causal relation is established.

When the criticality analysis is performed, the failure effect should be extended not only to the analysis target level but also to the system and subsystem level as shown in Fig. 1. From the relationship between the failure mode and the failure effect, there is a feature that the upper failure of the analysis target is described as the failure effect.

3. Application of military standards

The criticality analysis between system development is based on Task 101 and Task 102 of MIL-STD-1629A. The critical value(C_m) is expressed as the product of the failure mode ratio(α), the failure effect probability(β), the failure rate(λ) and the annual operating time(t)^[11].

$$C_m = \alpha \times \beta \times \lambda \times t \quad (10)$$

Table 1 Criteria of failure effect probability

Failure effect	β value
Actual loss	1.00
Probable loss	> 0.10 to < 1.00
Possible loss	> 0.00 to = 0.10
No effect	0.00

Table 2 Criteria of failure effect probability

1	Based on the characteristics of the lowest-level components to be analyzed, values between 0 and 1 are assigned, and the failure effect probability of the upper fault type is calculated by the bottom-up method.
2	Granted by mode of failure(Table 2 applies)
3	Giving batch 1

C_m denotes the degree of criticality for a specific failure mode of the item to be analyzed, and the failure mode ratio means a rate at which failure occurs due to the mode of the identified failure.

In equation (10), most of the parameters are applied quantitative specifications, but the failure effect probability is generally determined by the developer's qualitative indicators such as Table 1, and the decision method as shown in Table 2 is applied. In addition, there is no guideline for cases where several failure effects occur in one failure mode, so there is a possibility of error in judgment, and the validity of the analysis result depends greatly on the developer's capability. Therefore, it is necessary to study the methodology that can quantitatively judge the failure effect probability to reduce the workload of the developer and to make a more accurate and rational analysis^[13].

4. Failure effect probability quantification

The approximation of the probability of failure by Taylor theorem is applied with Bayes' theorem to

quantify the failure effect probability. According to Bayes' theorem (9), the failure effect probability can be calculated using three parameters $P(A_i)$, $P(A_i|B_j)$ and $P(B_j)$. For the analysis, MTBF(Mean Time between Failure), failure mode ratio derived from reliability analysis and FTA (Fault Tree Analysis) technique were used.

First, $P(A_i)$ means the probability of occurrence of the failure mode, and it can be derived by expressing the top event failure event of the FTA as the failure mode event^[14].

And, $P(A_i|B_j)$ means the rate of occurrence of the failure mode(A_i) under the failure effect occurrence condition(B_j). It can be expressed as equation (11)^[15], from the ratio of the failure effect occurrence frequency($O_{fe,i}$) to the failure mode occurrence frequency($O_{fm,i}$).

$$P(A_i|B_j) = \frac{O_{fm,i}}{O_{fe,j}} \quad (11)$$

If there is failure data of similar system in the above equation, it can be applied immediately. However, when there is no failure data, the frequency of occurrence of the failure mode is calculated from the failure rate and the failure mode ratio of the analysis target as shown in equation (12). Considering that the failure effect due to the failure mode has progressed according to a certain failure mechanism, the frequency of occurrence of the failure effect can be calculated as in equation (13)^[16].

$$O_{fm,i} = \alpha_i \times \lambda_t \quad (12)$$

$$O_{fe,j} = \sum_{i \in m} (\alpha_i \times \lambda_t) \quad (13)$$

In this case, the occurrence frequency of the failure refers to the sum of the failure effects having the same mission function as well as the severity classification for the occurrence of the failure mode of the analysis object. And, m is the group having the same mission function and severity classification.

In addition, $P(B_j)$ means the probability of

occurrence of the failure effect, and the probability approximation can be performed using the failure effect event's failure rate($\lambda_{fe,j}$) according to the Taylor theorem (7). Here, the failure effect event's failure rate can be calculated using the frequency of occurrence($O_{fe,total}$) of the total failure, the frequency of occurrence of the failure, and the system failure rate(λ_s) as shown in equation (15)^[4].

$$P(B_j) \approx \lambda_{fe,j} \times t \quad (14)$$

$$\lambda_{fe,j} = \frac{O_{fe,j}}{O_{fe,total}} \times \lambda_s \quad (15)$$

5. Failure effect probability quantification

The aircraft's fuel system must be capable of continuously supplying and maintaining fuel to the engine, while minimizing the pilot's attention during flight. In this fuel system, the failure effect probability is quantified by applying equation (15) to the fuel transfer pump.

The fuel transfer pump is mounted outside the fuel tank to enable fuel transfer between the tanks. Through the FMEA, the Fuel transport degradation, the Low

discharge pressure, and the failure mode of the external leakage are identified as shown in Table 3.

$P(A_i)$ is calculated by applying the FTA to the components of the fault cause derived by FMEA as shown in Fig. 2. In this case, equation (7), MTBF and FMD data of the components are applied to the probability of failure of the basic event^[17].

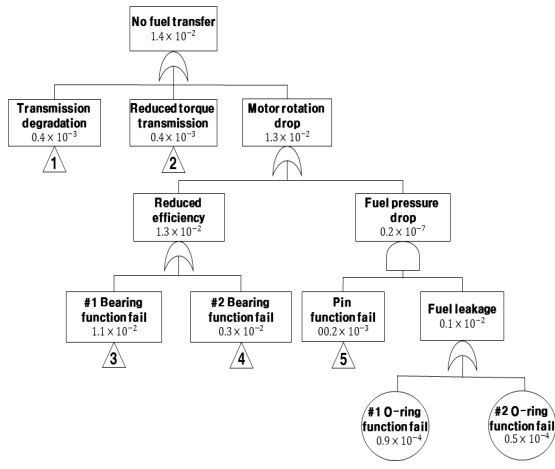
$P(A_i|B_j)$ is shown in Table 4, and frequency of occurrence of failure mode is calculated using equation (12) based on FMEA data. The failure effect is classified based on the mission function shown in Fig. 3 and the severity classification shown in Table 3, and the frequency of occurrence of the failure effect is calculated by applying equation (13).

Finally, $P(B_j)$ is calculated as shown in Table 5 by applying equation (14) and (15) through the system failure rate and the frequency of occurrence of the failure effect.

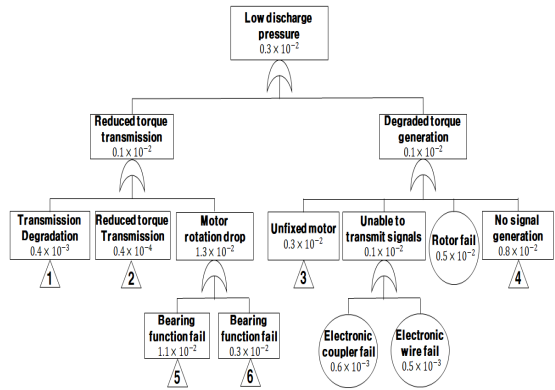
Analyzed parameters of $P(A_i)$, $P(A_i|B_j)$ and $P(B_j)$ are applied to the Bayes equation (9) to calculate the failure effect probability of the fuel transfer pump. The critical value is calculated by applying the failure rate, the operating time (234H) and the failure mode ratio to the equation (10), and the critical value(C_m) using the quantification method for the fuel transfer pump is shown in Table 6.

Table 3 FMEA of fuel transfer pump

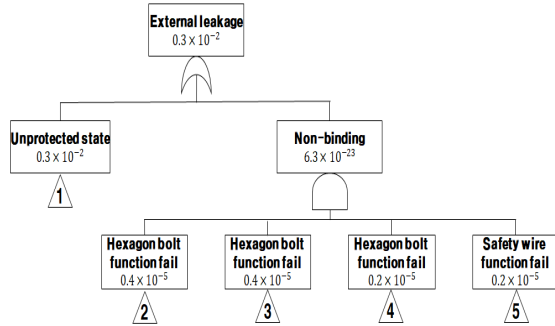
Failure mode	Failure effect			Severity classification	Failure ratio	Failure rate	t(H)
	Part	The next level	System				
1.Fuel transport degradation	The function of adjusting the flow rate to pressure ratio is lost, and fuel between the fuel tanks can not be transferred to each other.	It is impossible to control the fuel quantity imbalance of two tank groups.	The fuel transfer function between the main fuel tanks deteriorated.	3	0.255	202.18	234
2.Low discharge pressure	It is impossible to transfer fuel between the fuel tanks in both directions due to the impossibility of transmitting torque.	It is impossible to control the fuel quantity imbalance of two tank groups.	Degradation of fuel transfer between main fuel tanks	3	0.710	202.18	234
3.External leakage	Can not generate flow and hydraulic pressure due to housing damage	Loss of fuel transfer function to main fuel tank	No fuel transfer function of ground and in flight	2	0.035	202.18	234



(a) Failure mode 1(no fuel transfer)



(b) Failure mode 2(low discharge pressure)



(c) Failure mode 3(external leakage)

Fig. 2 FTA of fuel transfer pump

Table 4 P(A_i|B_j) of fuel transfer pump

Failure mode	α	λ_r	$O_{fm,i}$	$O_{fe,j}$	$P(A_i B_j)$
No fuel transfer	0.255	202.18	51.56	195.11	0.264
Low discharge pressure	0.710	202.18	143.56	195.11	0.736
External leakage	0.035	202.18	7.08	7.08	1

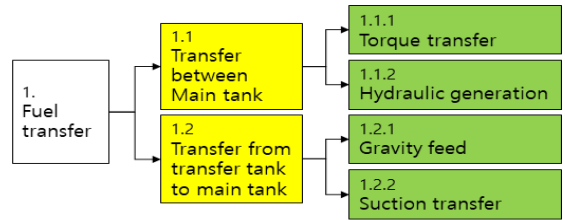


Fig. 3 Segmentation of mission tree

Table 5 P(B_j) of fuel transfer pump(t=234H)

Failure effect	$O_{fe,j}$	$\lambda_{fe,j}$	$P(B_j)$
Loss of boosting capacity	7.08	7.08	0.002
Degraded fuel transport function (between main tank)	195.11	195.11	0.046
Gravity feed Not possible	23.52	23.52	0.011
Degraded fuel transport function (Transfer tank to Main tank)	95.10	95.10	0.003
Total	320.81	320.81	0.06

Table 6 Criticality of fuel transfer pump

Failure mode	β	λ_r	α	C_m
No fuel transfer	0.62	202.18	0.255	7531.66
Low discharge pressure	0.79	202.18	0.710	26655.60
External leakage	0.55	202.18	0.035	906.85

6. Conclusion

In this study, theories related to criticality analysis and US military standards were analyzed, and a method for quantifying the failure effect probability

was proposed for objective criticality analysis. Based on our findings, the following conclusions can be drawn:

1. Criticality analysis according to US military standards depends on the subjective judgment of the expert or the indicator of the failure effect probability.
2. A methodology for quantifying the probability of failure is proposed using the reliability theory and Bayes' theorem.
3. The calculation of criticality was demonstrated by applying the method of quantification of failure effect probability to the fuel transfer pump for aircraft.

Acknowledgment

This article is based on the results of a study carried out in support of Agency for Defense Development(RAM specialized laboratory, UD180018AD).

References

1. MIL-STD-1629A, "Procedures for performing a failure mode, effects and criticality analysis", pp. 18-33, 1999.
2. Park, C. K., "The Study on Optimum Design of Main Control Valve", J. Korean Soc. Mech. Technol, Vol. 20, No. 2, pp.238-245, 2018.
3. Kim, K. M., Park, J. C., Yin, J. J., "Optimization of Sheet Metal Forming Process Using Designer's Desirability Functionl", J. Korean Soc. Mech. Technol, Vol. 20, No. 5, pp.556-561, 2018.
4. Kang, D. H., Ranjit, S., Chung, Y. J., Kim, W. T., "A Precedent Research on Personal Armor and Anti-stab. Wearing System Development", J. Korean Soc. Mech. Technol, Vol. 20, No. 6, pp.879-885, 2018.
5. DAPA, "Comprehensive Logistics Support Development Practical Guide", Human Culture Arirang, pp. 169-186, 2015.
6. Cho, S. H., "A Study on Performance Calculation Option for M&V of Power Demand Management Investment Project", J. Korean Soc. Mech. Technol, Vol. 20, No. 5, pp.697~701, 2018.
7. Seo, S. K., Kim, H. K., Kwon, H. M., Cha, M. S., Yun, W. Y., "Reliability engineering", Kyobobook, pp. 97-151, 2015.
8. Kim G. W., "On the Elementary Complex Functions and Convergence of power Series", University of Kangnung, pp. 26-30, 2008.
9. MIL-HDBK-338B, "Electronic Reliability Design Handbook", pp. 119-136, 1988.
10. BS EN 60812, "Analysis techniques for system reliability Procedure for Failure Mode and Effects Analysis", pp. 9-26, 2016.
11. IEC 60812, "Analysis techniques for system reliability Procedure for failure mode and effects analysis", pp. 19-40, 2006.
12. Jeong, J., Jang, J. H., Seon, C. H., Jeong, J. W., "A comparison of the FMECA result from the different decision ways of β value", Korean Institute Of Industrial Engineers, pp. 3083-3084, 2014.
13. Cha, J. H., "Study on the Improvement of Preventive Maintenance on Pneumatic Braking System of KTX-II Using FMECA", School of Rail way Seoul National University of Technology, pp. 15-20, 2010.
14. Ruijters, E., Stoelinga, M., "Fault Tree Analysis: A survey of the state-of-the-art in modeling, analysis and tools", Computer Science Review, pp. 29-62, 2015.
15. Song, K. T., Kim, M. H., Baek, Y. G., Lee, K. S., Kim, S. M., "A Study for FMEA and Optimization of Failure Diagnosis Sequence Using Probability of Failure Cause", The Korean society for railway, pp. 745-753, 2007.
16. Lee, J. M., Lee, J. S., Lee, S. R., Park, K. D., "A study on FMECA(Criticality & Cost) based on failure rate and cost incurred by failure", The Korean Institute of Industrial Engineers, pp.843-845, 2010.
17. FMD 2016, "Failure mode mechanism distributions", Reliability databook series, failure data for a wide variety of component types, 2016.