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R, Fuzzy R, and Set-Theoretic Kripke-Style Semantics*

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【Abstract】 This paper deals with set-theoretic Kripke-style semantics for **FR**, a fuzzy version of **R** of Relevance. For this, first, we introduce the system **FR** and its corresponding Kripke-style semantics. Next, we provide set-theoretic completeness results for it.

【Key Words】 Set-theoretic Kripke-style semantics, Algebraic Kripke-style semantics, Fuzzy logic, **R**, **FR**.

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1. Introduction

As Yang said in Yang (2012), it is well known that many relevance logicians have had difficulties in providing binary relational Kripke-style semantics, i.e., semantics with binary accessibility relations, for relevance logics (see e.g. Dunn (1986)). In that paper, he showed that such semantics can be provided for a fuzzy version of the system **R** of Relevance, although not for **R** itself.

Although Yang provided Kripke-style semantics for **FR**, a fuzzy version of **R**, in Yang (2012), his investigation was still *algebraic* in the sense that his completeness proof for **FR** was provided using the fact that the Kripke-style semantics for **FR** is equivalent to the algebraic semantics for **FR** and **FR** is algebraically complete. Because of this, he called his semantics *algebraic* Kripke-style semantics.

However, having a set-theoretic model is sometimes on a wish list for some people working on the semantics of a formal system. The main reason would be the familiarity of people with sets. Using sets, one could get a more intuition about the structure of the formal system. The other reason would be the simplicity of the given set-theoretic semantics compared to any other sorts.[1]

Note that Yang has provided not only algebraic but also

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[1] An anonymous referee mentioned this paragraph. The present author thinks that it includes a very important motivation to introduce set-theoretic semantics and thus restates his comments.

set-theoretic Kripke-style semantics for several fuzzy logics (Yang (2016a; 2016b; 2017; 2018a; 2018b; 2019)). This gives rise to the following natural question:

● Can we introduce set-theoretical Kripke-style semantics for **FR**?

The answer to the question is positive in the sense that we can provide such Kripke-style semantics for **FR**. This answer is important (to some readers) when one considers the above reasons that to provide a set-theoretic model for a logic is a wish list (for them). For this, first, in Section 2 we introduce the fuzzy logic system **FR** and its corresponding Kripke-style semantics. In Section 3, using set-theoretic method, we provide soundness and completeness results for it.

For convenience, we shall adopt the notations and terminology similar to those in Metcalfe & Montagna (2007), Montagna & Sacchetti (2003; 2004), and Yang (2012; 2016a; 2016b; 2017; 2018a; 2018b; 2019), and we assume reader familiarity with them (along with results found therein).

2. FR and Kripke-style semantics

We base **FR** on a countable propositional language with formulas *Fm* built inductively as usual from a set of propositional variables *VAR* and connectives →, ∧, ∨, and constants **f**, **t**, with defined connectives:[2)]

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2) Note that while ∧ is the extensional conjunction connective, & is the

df1. $\sim\phi := \phi \rightarrow \mathbf{f}$
df2. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$
df3. $\phi \mathbin{\&} \psi := \sim(\phi \rightarrow \sim\psi)$.

We moreover define $\phi_{\mathbf{t}} := \phi \wedge \mathbf{t}$. For the remainder we shall follow the customary notation and terminology. We use the axiom systems to provide a consequence relation.

We start with the following axiomatization of **FR**.

Definition 2.1 **FR** consists of the following axiom schemes and rules:[3)]

A1. $\phi \rightarrow \phi$ (self-implication, SI)
A2. $(\phi \wedge \psi) \rightarrow \phi$, $(\phi \wedge \psi) \rightarrow \psi$ ($\wedge$-elimination, $\wedge$-E)
A3. $((\phi\rightarrow\psi)\wedge(\phi\rightarrow\chi)) \rightarrow (\phi\rightarrow(\psi\wedge\chi))$ ($\wedge$-introduction, $\wedge$-I)
A4. $\phi \rightarrow (\phi \vee \psi)$, $\psi \rightarrow (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)
A5. $((\phi\rightarrow\chi)\wedge(\psi\rightarrow\chi)) \rightarrow ((\phi\vee\psi)\rightarrow\chi)$ ($\vee$-elimination, $\vee$-E)
A6. $(\phi\wedge(\psi\vee\chi))\rightarrow((\phi\wedge\psi)\vee(\phi\wedge\chi))$ ($\wedge\vee$-distributivity, $\wedge\vee$-D)
A7. $(\phi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\phi \rightarrow \chi))$ (suffixing, SF)
A8. $(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\psi \rightarrow (\phi \rightarrow \chi)$ (permutation, PM)
A9. $(\phi \rightarrow (\phi \rightarrow \psi)) \rightarrow (\phi \rightarrow \psi)$ (contraction, CR)
A10. $\sim\sim\phi \rightarrow \phi$ (double negation elimination, DNE)
A11. $(\phi \rightarrow \psi)_{\mathbf{t}} \vee (\psi \rightarrow \phi)_{\mathbf{t}}$ (**t**-prelinearity, $PL_{\mathbf{t}}$)

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intensional conjunction one.

3) Here we introduce the axiomatization of **FR** without & connective. For the axiomatization of **FR** with &, see Yang (2012). A6, indeed, is redundant in **FR**. But we introduce this in order to show that **R** is the **FR** omitting A11. Note that the system omitting both A6 and A11 is not **R** (cf see Anderson & Belnap (1975), Anderson, Belnap, and Dunn (1992), Dunn (1986)).

$\phi \to \psi, \phi \vdash \psi$ (modus ponens, mp)

$\phi, \psi \vdash \phi \wedge \psi$ (adjunction, adj).

A11 is the axiom scheme for linearity, and logics being complete w.r.t. linearly ordered (corresponding) algebras are said to be fuzzy logics (see e.g. Cintula (2006)).

Note that, as Yang mentioned in Yang (2012), the system **R** is the **FR** omitting A11. Note also that in **R** (and so **FR**), $\phi \to \psi$ can be defined as $\sim(\phi \,\&\, \sim\psi)$ (df4).

Proposition 2.2 **FR** proves:

(1) $((\phi \to (\psi \vee \chi)) \wedge (\psi \to \chi)) \to (\phi \to \chi)$.

(2) $(\phi \,\&\, \psi) \to (\psi \,\&\, \phi)$.

(3) $(\phi \,\&\, \mathbf{t}) \leftrightarrow \phi$.

(4) $(\phi \to (\psi \to \chi)) \leftrightarrow ((\phi \,\&\, \psi) \to \chi)$

(5) $\phi \to (\phi \,\&\, \phi)$

(6) $(\phi \,\&\, (\phi \to \psi)) \to \psi$

(7) $\phi \to ((\phi \to \psi) \to \psi)$.

Proof: For (1), see Proposition 2.2 in Yang (2012). For the other cases, we prove (2) as an example.

For (2), first note that we have $(\psi \to \sim\phi) \to (\phi \to \sim\psi)$ using (df1) and A8. Thus, we further have $\sim(\phi \to \sim\psi) \to \sim(\psi \to \sim\phi)$ using (df1) and A7. Therefore, we obtain $(\phi \,\&\, \psi) \to (\psi \,\&\, \phi)$ by (df3).

The proof for the other cases is left to the interested reader. □

Note that **R** does not prove (1) in Proposition 2.2 (see Dunn (1986)).

In **FR**, **f** can be defined as $\sim$**t**. A *theory* over **FR** is a set T of formulas. A *proof* in a theory T over **FR** is a sequence of formulas whose each member is either an axiom of **FR** or a member of T or follows from some preceding members of the sequence using the two rules in Definition 2.1. T $\vdash \phi$, more exactly T $\vdash_{\mathbf{FR}} \phi$, means that ϕ is *provable* in T w.r.t. **FR**, i.e., there is a **FR**-proof of ϕ in T. The relevant deduction theorem ($\mathrm{RDT}_{\mathbf{t}}$) for **FR** is as follows:

Proposition 2.3 (Meyer, Dunn, & Leblanc (1976)) Let T be a theory, and ϕ, ψ formulas.

($\mathrm{RDT}_{\mathbf{t}}$) $T \cup \{\phi\} \vdash \psi$ iff $T \vdash \phi_{\mathbf{t}} \to \psi$.

For convenience, "$\sim$", "$\wedge$", "$\vee$", and "$\to$" are used ambiguously as propositional connectives and as algebraic operators, but context should clarify their meaning.

Here we consider Kripke-style semantics for **FR**.

Definition 2.4 (Kripke frame, Yang (2012)) A *Kripke frame* is a structure $\mathbf{X} = (X, t, f, \leq, *, \to)$ such that $(X, t, f, \leq, *, \to)$ is a linearly ordered residuated pointed commutative monoid, i.e., it satisfies the following:

(I) $(A, \wedge, \vee)$ is a distributive lattice.

(II) $(A, *, t)$ is a commutative monoid.

(III) $y \leq x \to z$ iff $x * y \leq z$, for all $x, y, z \in A$

(residuation).

(IV) $t \leq (x \rightarrow y)_t \vee (y \rightarrow x)_t$ (pl_t).

The elements of **X** are called *nodes*.[4)]

Definition 2.5 (FR frame, Yang (2012)) A *FR frame* is a Kripke frame, where $x = (x \rightarrow f) \rightarrow f$ (double negation), $x \leq x * x$ (contraction), and $*$ is left-continuous (i.e., whenever $\sup\{x_i : i \in I\}$ exists, $x * \sup\{x_i : i \in I\} = \sup\{x * x_i : i \in I\}$), and so its residuum $\rightarrow$ is defined as $x \rightarrow y := \sup\{z: x * z \leq y\}$ for all $x, y \in X$.

An *evaluation* or *forcing* on a Kripke frame is a relation $\Vdash$ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(AHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$; and

for arbitrary formulas,

(t) $x \Vdash \mathbf{t}$ iff $x \leq t$;

(f) $x \Vdash \mathbf{f}$ iff $x \leq f$;

($\wedge$) $x \Vdash \phi \wedge \psi$ = iff $x \Vdash \phi$ and $x \Vdash \psi$;

($\vee$) $x \Vdash \phi \vee \psi$ iff $x \Vdash \phi$ or $x \Vdash \psi$;

($\rightarrow$) $x \Vdash \phi \rightarrow \psi$ iff for all $y \in X$, if $y \Vdash \phi$, then $x * y \Vdash \psi$.

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4) Yang (2012) introduced this as algebraic Kripke frame. But we introduce it as Kripke frame because we can consider both algebraic and set-theoretic completeness for **FR** using this frame. Similarly, we will introduce Kripke model.

Definition 2.6 (Yang (2012)) (i) (Kripke model) A *Kripke model* is a pair (**X**, ⊩), where **X** is a Kripke frame and ⊩ is a forcing on **X**.

(ii) (FR model) A *FR model* is a pair (**X**, ⊩), where **X** is a FR frame and ⊩ is a forcing on **X**.

Definition 2.7 Given a Kripke model (**X**, ⊩), a node x of **X** and a formula ϕ, we say that *x forces ϕ* to express x ⊩ ϕ. We say that ϕ is *true* in (**X**, ⊩) if t ⊩ ϕ, and that ϕ is *valid* in the frame **X** (expressed by **X** models ϕ) if ϕ is true in (**X**, ⊩) for every forcing ⊩ on **X**.

3. Soundness and completeness for FR

We first introduce two lemmas, which can be easily proved:

Lemma 3.1 (Cf, Yang (2016b)) (Hereditary Lemma, HL) Let **X** be a Kripke frame. For any sentence ϕ and for all nodes x, y $\in$ **X**, if x ⊩ ϕ and y $\leq$ x, then y ⊩ ϕ.

Lemma 3.2 t ⊩ $\phi \rightarrow \psi$ iff for all x $\in$ **X**, if x ⊩ ϕ, then x ⊩ ψ.

We then provide soundness and completeness results for **FR**.

Proposition 3.3 (Soundness) If $\vdash_{\mathbf{FR}} \phi$, then ϕ is valid in every FR frame.

Proof: We prove the validity of A9 and A10 as examples.

For A9, by Lemma 3.2, it suffices to assume $x \Vdash \phi \to (\phi \to \psi)$ and to show $x \Vdash \phi \to \psi$. To show this last, we further assume $y \Vdash \phi$ and show $x * y \Vdash \psi$. By the suppositions and ($\to$), we have $x * y \Vdash \phi \to \psi$ and thus $(x * y) * y \Vdash \psi$. Then, since $*$ is associative and contractive, we have $x * y \leq (x * y) * y$. Therefore, using Lemma 3.1, we obtain $x * y \Vdash \psi$, as required.

For A10, by Lemma 3.2, it suffices to show that if $x \Vdash {\sim}{\sim}\phi$, then $x \Vdash \phi$. Assume $x \Vdash {\sim}{\sim}\phi$. Then, by (df1), we have $x \Vdash {\sim}(\phi \to \mathbf{f})$. Then, since $\mathbf{f} \leftrightarrow {\sim}\mathbf{t}$, we also have $x \Vdash {\sim}(\phi \to {\sim}\mathbf{t})$, and thus $x \Vdash \phi \,\&\, \mathbf{t}$ by (df3). Therefore, using (3) in Proposition 2.2, we obtain $x \Vdash \phi$, as required.

The proof for the other cases is left to the interested reader. □

Now we provide set-theoretic completeness results for **FR** using Kripke-style semantics. A theory T is said to be *linear* if, for each pair ϕ, ψ of formulas, we have $T \vdash \phi \to \psi$ or $T \vdash \psi \to \phi$. By a **FR**-theory, we mean a theory T closed under rules of **FR**. As in relevance logic, by a regular **FR**-theory, we mean a **FR**-theory containing all of the theorems of **FR**. Since we have no use of irregular theories, by a **FR**-theory, we henceforth mean a **FR**-theory containing all of the theorems of **FR**.

Moreover, where T is a linear **FR**-theory, we define the *canonical FR frame* determined by T to be a structure $\mathbf{X} = (X_{can}, t_{can}, f_{can}, \leq_{can}, *_{can}, \to_{can})$, where $t_{can} = T$, $f_{can} = \{\phi : T \vdash_{\mathbf{FR}} \mathbf{f}$

$\to \phi\}$, X_{can} is the set of linear L-theories extending t_{can}, $\leq_{can}$ is $\supseteq$ restricted to X_{can}, i.e, x $\leq_{can}$ y iff $\{\phi : x \vdash_{\mathbf{FR}} \phi\} \supseteq \{\phi : y \vdash_{\mathbf{FR}} \phi\}$, $\to_{can}$ is defined as $x \to_{can} y := \{Z : T \vdash_{\mathbf{FR}} \phi \to (Z \to \psi$ for $\phi \in x$ and $\psi \in y\}$, and $*_{can}$ is defined as $x *_{can} y := \{\phi \,\&\, \psi :$ for some $\phi \in x, \psi \in y\}$ satisfying groupoid properties corresponding to FR frames on $(X_{can}, t_{can}, \leq_{can})$. Note that the base t_{can} is constructed as the linear **FR**-theory that excludes nontheorems of **FR**, i.e., excludes ϕ such that $\nvdash_{\mathbf{FR}} \phi$. The partial orderedness and the linear orderedness of the canonical FR frame depend on $\leq_{can}$ restricted on X_{can}. Then, first, the following is obvious.

Proposition 3.4 A canonical FR frame is linearly ordered.

Proof: Since $\leq_{can}$ is an order reversed subset relation, it is obvious that a canonical FR frame is partially ordered. For linearly orderedness, suppose for contradiction that neither x $\leq_{can}$ y nor y $\leq_{can}$ x. Then, there exist ϕ, ψ such that $\phi \in y$, $\phi \notin x$, $\psi \in x$, and $\psi \notin y$. Since t_{can} is a linear theory, we have that $\phi \to \psi \in t_{can}$ or $\psi \to \phi \in t_{can}$. Let $\phi \to \psi \in t_{can}$. Then, since $\phi \to \psi \in y$, by (mp), we have $\psi \in y$, a contradiction. The case, where $\psi \to \phi \in t_{can}$, is analogous. □

Next, we define a canonical evaluation as follows:

(a) $x \Vdash_{can} \phi$ iff $\phi \in x$.

We then consider the following two lemmas.

Lemma 3.5 $t_{can} \Vdash_{can} \phi \rightarrow \psi$ iff for all $x \in X_{can}$, if $x \Vdash_{can} \phi$, then $x \Vdash_{can} \psi$.

Proof: By (a), we instead show that $\phi \rightarrow \psi \in t_{can}$ iff for all $x \in X_{can}$, if $\phi \in x$, then $\psi \in x$. For the left-to-right direction, we assume $\phi \rightarrow \psi \in t_{can}$ and $\phi \in x$, and show $\psi \in x$. By the suppositions and the definition of $*_{can}$, we have that $\phi \,\&\, (\phi \rightarrow \psi) \in x *_{can} t_{can} = x$. Then, since $(\phi \,\&\, (\phi \rightarrow \psi)) \rightarrow \psi \in t_{can}$ by (5) in Proposition 2.2 and thus $(\phi \,\&\, (\phi \rightarrow \psi)) \rightarrow \psi \in x$, we also obtain that $\psi \in x$ by (mp). For the right-to-left direction, suppose contrapositively that $\phi \rightarrow \psi \notin t_{can}$. Set $x_0 = \{Z : \text{there exists } X \in t_{can} \text{ and } t_{can} \vdash X \rightarrow (\phi \rightarrow Z)\}$. Clearly, $x_0 \supseteq t_{can}$, $\phi \in x_0$, and $\psi \notin x_0$. (Otherwise, $t_{can} \vdash X \rightarrow (\phi \rightarrow \psi)$; therefore, $t_{can} \vdash \phi \rightarrow \psi$, a contradiction, by (mp), since $t_{can} \vdash X$.)

Then, by the Linear Extension Property of Theorem 12.9 in Cintula, Horčík, & Noguera (2015), we have a linear theory $x \supseteq x_0$ with $\psi \notin x$; therefore $\phi \in x$ but $\psi \notin x$. □

Lemma 3.6 (Canonical Evaluation Lemma) $\Vdash_{can}$ is an evaluation.

Proof: First, consider the conditions for propositional variables.

For (AHC), we need to show that: for every propositional variable p,

$$\text{if } x \Vdash_{can} p \text{ and } y \leq_{can} x, \text{ then } y \Vdash_{can} p.$$

Assume that $x \Vdash_{can} p$ and $y \leq_{can} x$. By (a), we have that $p \in x$ and $x \subseteq y$, and thus $p \in y$; therefore, $y \Vdash_{can} p$.

We next consider the conditions for propositional constants **t** and **f**.

For (t), we need to show that:

$$x \Vdash_{can} \mathbf{t} \text{ iff } x \leq_{can} t_{can}.$$

Since $t_{can} = T$ and x is a theory extending T, we can ensure that $\mathbf{t} \in x$ iff $x \supseteq t_{can}$; therefore, $x \Vdash_{can} \mathbf{t}$ iff $x \leq_{can} t_{can}$ by (a).

For (f), we need to show that:

$$x \Vdash_{can} \mathbf{f} \text{ iff } x \leq_{can} f_{can}.$$

Note first that $f_{can} = \{\phi : T \vdash_{\mathbf{FR}} \mathbf{f} \rightarrow \phi\}$. We take x as a theory extending T and including **f**. Then, we can ensure that $\mathbf{f} \in x$ iff $x \supseteq f_{can}$; therefore, $x \Vdash_{can} \mathbf{f}$ iff $x \leq_{can} f_{can}$ by (a).

Now we consider the conditions for arbitrary formulas.

For ($\wedge$), we need to show

$$x \Vdash_{can} \phi \wedge \psi \text{ iff } x \Vdash_{can} \phi \text{ and } x \Vdash_{can} \psi.$$

By (a), we instead show that $\phi \wedge \psi \in x$ iff $\phi \in x$ and $\psi \in x$. The left-to-right direction follows from ($\wedge$-E) and (mp).

The right-to-left direction follows from (adj).

For ($\vee$), we must show

$$x \Vdash_{can} \phi \vee \psi \text{ iff } x \Vdash_{can} \phi \text{ or } x \Vdash_{can} \psi.$$

By (a), we instead show that $\phi \vee \psi \in x$ iff $\phi \in x$ or $\psi \in x$. The left-to-right direction follows from the fact that linear theories are also prime theories in **FR** (see Cintula & Noguera (2011)). The right-to-left direction follows from ($\vee$-I) and (mp).

For (&), we need to show

$$x \Vdash_{can} \phi \,\&\, \psi \text{ iff there are } y, z \in X \text{ such that } y \Vdash_{can} \phi, z \Vdash_{can} \psi, \text{ and } x \leq_{can} y *_{can} z.$$

The definition of $*_{can}$ ensures that $\phi \,\&\, \psi \in x$ iff there are $y, z \in X$ such that $\phi \in y$, $\psi \in z$, and $x \leq_{can} y *_{can} z$. Then, by (a), we obtain the claim.

For ($\rightarrow$), we need to show

$$x \Vdash_{can} \phi \rightarrow \psi \text{ iff for all } y \in X, \text{ if } y \Vdash_{can} \phi, \text{ then } x *_{can} y \Vdash_{can} \psi.$$

By (a), we instead show that $\phi \rightarrow \psi \in x$ iff for all $y \in X$, if $\phi \in y$, then $\psi \in x *_{can} y$. For the left-to-right direction, we assume $\phi \rightarrow \psi \in x$ and $\phi \in y$, and show $\psi \in x *_{can} y$. The definition of $*_{can}$ ensures $(\phi \rightarrow \psi) \,\&\, \phi \in x *_{can} y$. Since $\phi \rightarrow ((\phi \rightarrow \psi) \rightarrow \psi) \in t_{can}$ by (7) in Proposition 2.2 and thus

$((\phi \to \psi) \,\&\, \phi) \to \psi \in t_{can}$ and so $((\phi \to \psi) \,\&\, \phi) \to \psi \in x *_{can} y$, by (mp), we have that $\psi \in x *_{can} y$. For the right-to-left direction, suppose contrapositively that $\phi \to \psi \notin x$. As in Lemma 3.5, we can construct a linear theory y such that $\phi \in y$ and $\psi \notin x *_{can} y$. □

Let us call a model **M**, = (**X**, $\Vdash_{can}$) (i.e., $(X_{can}, t_{can}, f_{can}, \leq_{can}, *_{can}, \to_{can}, \Vdash_{can})$), for **FR**, a FR model. Then, by Lemma 3.6, the canonically defined (**X**, $\Vdash_{can}$) is a FR model. Thus, since, by construction, t_{can} excludes our chosen nontheorem ϕ, and the canonical definition of models agrees with membership, we can state that, for each nontheorem ϕ of **FR**, there is a FR model in which ϕ is not t_{can} models ϕ. It gives us the weak completeness of **FR** as follows.

Theorem 3.7 (Weak completeness) If $\vDash_{\mathbf{FR}} \phi$, then $\vdash_{\mathbf{FR}} \phi$.

Furthermore, using Lemma 3.6 and the Linear Extension Property, we can show the strong completeness of **FR** as follows.

Theorem 3.8 (Strong completeness) **FR** is strongly complete w.r.t. the class of all FR-frames.

4. Concluding remark

We investigated set-theoretic Kripke-style semantics for **FR**, a fuzzy version of **R**. We proved soundness and completeness theorems. This fact shows that following two. (1) We may have the same Kripke-style semantics for **FR** and then give algebraic and set-theretic Kripke-style completeness for it. (2) The Kripke-style semantics for **FR** is based on its corresponding algebraic structures.

(2) makes us to consider the following two future works. First, it shows that algebraic semantics and Kripke-style semantics can be considered on the same algebraic structures. This is a subject related to the *duality* property between *n*-ary algebras and their corresponding *n+1*-ary relations. Second, (2) means that we have not yet provided set-theoretic Kripke-style semantics for **FR**, *not based on* its corresponding algebraic structures. To provide this kind of set-theoretic Kripke-style semantics for **FR** is an another subject left in this paper.

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양 은 석

이 글에서 우리는 연관 논리 R을 퍼지화한 체계 FR을 위한 집합 이론적인 크립키형 의미론을 다룬다. 이를 위하여 먼저 FR 체계와 그에 상응하는 크립키형 의미론을 소개한다. 다음으로 FR을 위한 집합 이론적 완전성 결과를 제공한다.

주요어: 집합 이론적 크립키형 의미론, 대수적 크립키형 의미론, 퍼지 논리, R, FR.