

# Prediction of Tensile Strength of a Large Single Anchor Considering the Size Effect

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## Abstract

An anchorage system is essential for most reinforced concrete structures to connect building components. Therefore, the prediction of strength of the anchor is very important issue for safety of the structures themselves as well as structural components. The prediction models in existing design codes are, however, not applicable for large anchors because they are based on the small size anchors with diameters under 50 mm. In this paper, new prediction models for strength of a single anchor, especially the tensile strength of a single anchor, is developed from the experimental results with consideration of size effect. Size effect in the existing models such as ACI or CCD method is based on the linear fracture mechanics which is very conservative way to consider the size effect. Therefore, new models are developed based on the nonlinear fracture mechanics rather than the linear fracture mechanics for more reasonable prediction. New models are proposed by the regression analysis of the experimental results and it can predict the tensile strength of both small and large anchors.

*Keywords: Large Anchor, Tensile Strength, CCD Method, 45 Degree Cone Method, Size Effect*

## I. INTRODUCTION

Anchorage systems in concrete structures resist tensile and shear loading either alone or together. There are several failure modes of the anchor under tensile loading, i.e., steel failure, pullout failure, concrete cone failure and so on. Among these failure modes, concrete cone failure is exhibited by the majority of mechanical and cast-in-place anchor systems [11]. Prediction of resistance of a single anchor by tensile loading, especially failed by concrete cone failure is the main subject of this research.

Variety of experiments and analyses on the tensile anchorage system have conducted and many prediction models are also suggested [10][13][14][20][21]. The existing design codes for reinforced concrete structures such as ACI 318 (2015) or ACI 349 (2015) give the prediction equations for the strength of anchorage systems. However, these prediction equations are not suitable for the large size anchors with deep embedment depth which are frequently

used in reinforced concrete structures such as containment buildings of nuclear power plants.

Large anchors have bigger diameters and deeper embedment depths than small anchors. These parameters, the diameter and the embedment depth, are related to the size effect of concrete. The diameter of anchor head is related to the initial crack size, and the embedment depth is related to the characteristic length which is very important in the size effect. The size effect in the existing codes (ACI 318 2015, ACI 349 2015) follows the linear fracture mechanics and therefore the nominal strength of anchors can be underestimated. In other words, predicted strengths of anchors from the model equation are smaller than the actual strengths and the error increases as the size of the anchor deviates from the norm. Therefore, the size effect should be considered based on the nonlinear fracture mechanics rather than the linear fracture mechanics.

The purpose of this paper is to suggest new models for the prediction of the tensile strength of a single anchor

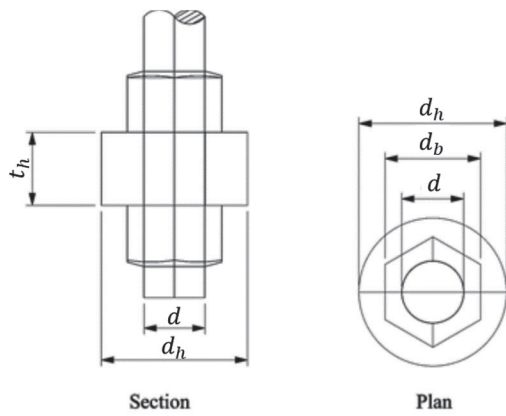
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Anchor diameter $d$ (mm)	Plate diameter $d_h$ (mm)	Plate thickness $t_h$ (mm)	Nut width $d_b$ (mm)
69.9	152.4	76.2	98.4
95.3	215.9	101.6	136.5
108.0	254.0	127.0	174.6

Fig. 1. Details of anchor head.

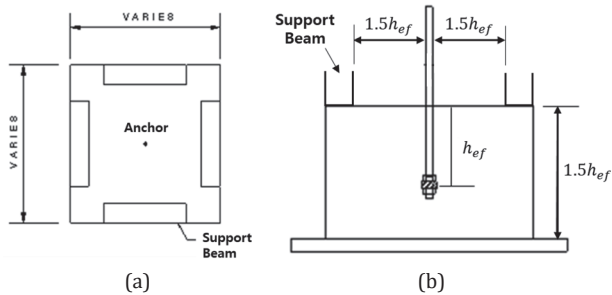


Fig. 2. Details of tensile test setup. (a) Plan. (b) Specimen size.

including large size in non-cracked concrete failed by concrete cone failure. Experiments on a large single anchor under tensile loading are conducted. Prediction models are suggested based on the regression analysis of the experimental results with the reasonable consideration of the size effect.

## II. EXPERIMENT FOR LARGE ANCHOR

### A. Test Variables

Test variables for the research are the diameter and the embedment depth of the anchor as shown in Table 1. The embedment depths of anchors are set as 635 mm (25 in.), 889 mm (35 in.), and 1143 mm (45 in.) and the diameters are set as 70 mm (2.75 in.), 95 mm (3.75 in.), and 108 mm (4.25 in.), respectively.

The anchor head consisted of a round thick plate which was fixed to the bolt by clamping nuts. Details of the anchor head is as shown in Fig. 1 [19].

Table 1  
Test Variables

Specimen No.	$f'_c$ (MPa)	$h_{ef}$ (mm)	$d$ (mm)	$d_h$ (mm)	$\frac{d_h}{d}$	$\frac{h_{ef}}{d}$	$\frac{h_{ef}}{d_h}$
T1	44.7	635	70	152	2.18	9.08	4.17
T2	44.7	889	95	216	2.27	9.33	4.12
T3	44.7	1143	108	254	2.35	10.6	4.50

Table 2  
Mix Proportion of Concrete

$w/b$ (%)	$S/a$ (%)	Unit weight (kg/m <sup>3</sup> )					
		W	C	FA	S	G	Ad.
36.7	40.7	184	426	75	658	959	3.01

Table 3  
Mechanical Properties of Anchor Bolts

Standard	$f_y$ (MPa)	$f_u$ (MPa)	$E_s$ (MPa)
ASTM A540 B23 Class2	980	1085	$2.058 \times 10^5$

### B. Materials

Ready-mixed concrete is used for the test and mix proportion of concrete is provided in Table 2. Compressive strength of concrete is tested according to the standards, KS F 2405(2010) corresponding to ASTM C39 (2016), with the cylindrical specimens of  $\phi 100 \times 200$  made according to the standards, KS F 2403 (2014) corresponding to ASTM C192 (2016). The average strength is 44.7 MPa.

All anchors were made of ASTM A540 (2015) B23 Class 2 steel which is equivalent to ASME SA 549 B23 Class 2 steel used in Korean nuclear power plants. Properties of the anchor bolts are as shown in Table 3.

### C. Test Setup

Test is basically following the standard, ASTM E 488 (1998). This standard specifies the test methods for static, seismic, fatigue, shock, tensile, shear strength of post-installed or cast-in-place anchors in concrete. The support for the tension test equipment shall be of sufficient size to prevent failure of the surrounding test member. The loading rod shall be of sufficient diameter to develop the anticipated ultimate strength of the anchorage hardware with an elastic elongation not exceeding 10 percent of the anticipated elastic elongation of the anchor, and shall be attached to the anchorage system by a connector that will minimize the direct transfer of bending stress to the anchor. The displacement measuring device(s) shall be positioned to measure the movement of the anchors with respect to points on the test member so that the device is not influenced during the test by deflection or failure of the anchor or test member.

In this research, all these test conditions are satisfied and Fig. 2 shows the test setup used for this research. Four steel beams are supporting because the failure mode of tensile anchor can be flexural failure depending on the boundary conditions. Therefore, four beam supports are designed to prevent the splitting failure due to flexural behavior of concrete block.

Table 4  
Test Results of Tensile Strength for Anchors

Specimen No.	$f'_c$ (MPa)	$d$ (mm)	$h_{ef}$ (mm)	Test result (kN)	Mean (kN)	COV %	
T1	44.7	70	635	T1-A	2097.2	2286	5.76
				T1-B	2234.4		
				T1-C	2440.2		
				T1-D	2371.6		
T2	44.7	95	889	T2-A	3253.6	3222	1.71
				T2-B	3253.6		
				T2-C	3253.6		
				T2-D	3126.2		
T3	44.7	108	1143	T3-A	5390	5446	4.88
				T3-B	5635		
				T3-C	5723.2		
				T3-D	5037.2		

Fig. 3 shows the actual tensile test setup. As shown in the Fig. 3, four support beams are placed to apply tensile load to the anchor.

### C. Test Results

Test variables and the cone failure loads obtained from each specimen are as shown in Table 4. At least four specimens for each type of specimen are tested for reliability.

## III. MODEL EQUATION FOR TENSILE STRENGTH OF ANCHOR

### A. Area of Failure Surface

The resisting capacity of a concrete section is dependent on the strength of concrete and area of failure surface. It means that the failure load of an anchor can be different depending on the area of failure surface even if the compressive strength of concrete is constant.

If the diameter of anchor head is very large so that it is hard to neglect the area of the cone tip portion as shown in Fig. 4, area should be calculated by Eq. (1a). If the diameter of anchor head is small or  $d_h/h_{ef}$  is constant, then the area can be calculated by Eq. (1b) or Eq. (1c).

$$A = \frac{\pi h_{ef}^2}{\sin^2 \theta} \left( \frac{d_h \sin \theta}{h_{ef}} + \cos \theta \right) \quad (1a)$$

$$A = \frac{\pi h_{ef}^2}{\sin^2 \theta} \cos \theta \quad (1b)$$

$$A = \frac{\pi h_{ef}^2}{\sin^2 \theta} (\alpha \sin \theta + \cos \theta) \quad (1c)$$

where  $A$  is area of failure surface,  $h_{ef}$  is embedment depth,  $d_h$  is the diameter of anchor head, and  $\theta$  is angle of failure surface from the horizontal line.

Failure area is a function of not only the embedment



Fig. 3. Photograph of actual tensile test setup.

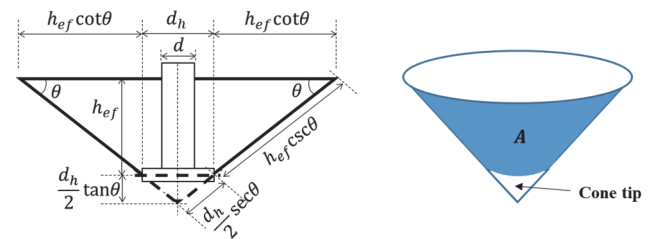


Fig. 4. Failure surface area of a single anchor with large diameter.

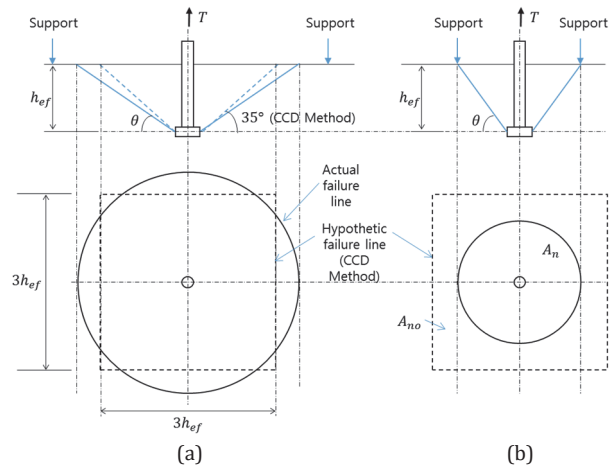


Fig. 5. Failure surface area of a single anchor under tensile loading. (a) Distant supports. (b) Close supports.

depth  $h_{ef}$  and the angle  $\theta$  but also the diameter of anchor head  $d_h$  as shown in Eq. (1a). However, the effect of diameter of anchor head can be neglected as shown in Eq. (1b) or Eq. (1c) in most cases, even for large anchors used in this research, because the term  $d_h \sin \theta / h_{ef}$  is relatively small compared to the term  $\cos \theta$ , and the diameter of anchor head  $d_h$  is normally proportional to the embedment depth  $h_{ef}$ .

In case of small  $d_h$  or the constant value of  $d_h/h_{ef}$ , the area of failure surface is proportional to the square of the embedment depth  $h_{ef}^2$ . However, the area can be also different depending on the position of the support as shown in Fig. 5. If the supports are located far enough from the

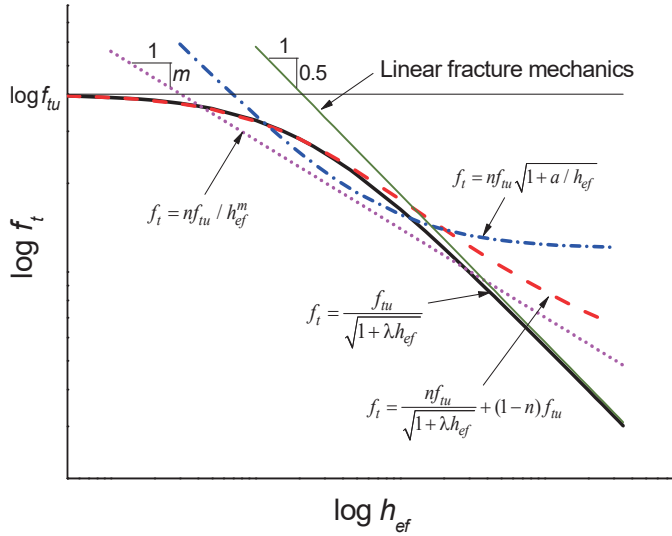


Fig. 6. Illustration of size effect according to various model equations.

anchor, failure occurs along with the certain line of which slope is around  $35^\circ$  as shown in Fig. 5(a) [12]. However, if the supports are located closely to the anchor, failure occurs along with the line from the head of anchor and the support as shown in Fig. 5(b). The angle  $\theta$  in Fig. 5(b) increases as the distance between the anchor and support decreases.

Compressive strength of concrete is an important factor because tensile strength and diagonal tensile strength of concrete are dependent on the compressive strength. If the supports are located far enough from the anchor as shown in Fig. 5(a), failure is related with the diagonal tensile strength of concrete, but if the supports are located closely to the anchor as shown in Fig. 5(b), the diagonal tensile strength of concrete would be larger. Therefore, the resisting capacity per unit area of concrete can be different depending on the location of the supports because the tensile strength in Fig. 5(b) is greater than the tensile strength of concrete in Fig. 5(a). It can be written as Eq. (2).

$$f_{tu} = \alpha f_{cu}^\beta \quad (2)$$

where  $f_{tu}$  is the resisting capacity per unit area of concrete,  $f_{cu}$  is compressive strength of concrete, and  $\alpha$  and  $\beta$  which dependent on compressive strength of concrete and supports' distances in Fig. 5 are the experimental constants.

### B. Size Effect

Concrete is always fractured by crack. Therefore, size effect appears and the characteristic length for the tensile anchor is embedment depth  $h_{ef}$ . Size effect can be expressed by many researchers as shown in Eq. (3). Eq. (3) is proposed by Bazant [8] and Kim [15] based on nonlinear fracture mechanics. Eq. (3c) and Eq. (3d) are proposed by Carpinteri [9] based on fractal theory and Weibull [22] based on weakest-link-theory by statistics, respectively.

$$f_t = \frac{f_{tu}}{\sqrt{1 + \lambda h_{ef}}} \quad (3a)$$

$$f_t = \frac{n f_{tu}}{\sqrt{1 + \lambda h_{ef}}} + (1 - n) f_{tu} \quad (3b)$$

$$f_t = n f_{tu} \sqrt{1 + \frac{a}{h_{ef}}} \quad (3c)$$

$$f_t = \frac{n f_{tu}}{h_{ef}^m} \quad (3d)$$

where  $f_t$  is the tensile strength of concrete considering the size effect,  $f_{tu}$  is the strength of small concrete specimen whose size effect can be ignored,  $h_{ef}$  is embedment depth, and  $n$  and  $m$  are the experimental constants.

Fig. 6 shows the size effect according to the various equations in Eq. (3). The plate diameter  $d_h$  in Fig. 4 can be regarded as the size of initial crack because of the weak interfacial bond between the plate and concrete. If the diameter of anchor head which represents the initial crack size is proportional to the size of specimen, i.e. the embedded length of anchor, Eq. (3a) is reasonable in this research. If the diameter is not proportional to the specimen size which is called dissimilar crack, then Eq. (3b) is more reasonable.

Among the equations, Eq. (3d) is currently used for the existing model equations with the exponent  $m=0.5$ . As shown in Fig. 6, if  $m=0.5$  in Eq. (3d), Eq. (3d) represents the linear fracture mechanics. It means that the existing model follows the linear fracture mechanics. From the Fig. 6, it is able to notify that the size effect by linear fracture mechanics is too strong, i.e., the strength of concrete by this equation is underestimated and the difference is bigger for anchors deviated from the norm. Therefore, new model equations are necessary.

### C. Suggestion of Model Equations

Among various versions of Eq. (3), Eq. (3a) which is based on the nonlinear fracture mechanics and reasonable for this experiment because the ratio of anchor diameter to the embedment depth is almost constant and Eq. (3d) which is the simplest form and used for the existing standards in Europe and Japan are considered to suggest new model equations. The tensile cone failure load  $T_n$  of anchor can be expressed as shown in Eq. (4) from Eq. (1), Eq. (2), and Eq. (3a) and (3d).

$$T_n = A_n f_t = \pi h_{ef}^2 \frac{\cos \theta}{\sin^2 \theta} \times \frac{\alpha f_{cu}^\beta}{\sqrt{1 + \lambda h_{ef}}} = \frac{n_1 f_{cu}^\beta h_{ef}^2 \cos \theta}{\sqrt{1 + \lambda h_{ef}} \sin^2 \theta} \quad (4a)$$

$$T_n = A_n f_t = \pi h_{ef}^2 \frac{\cos \theta}{\sin^2 \theta} \times \frac{n \alpha f_{cu}^\beta}{h_{ef}^m} = n_1 h_{ef}^{2-m} f_{cu}^\beta \frac{\cos \theta}{\sin^2 \theta} \quad (4b)$$

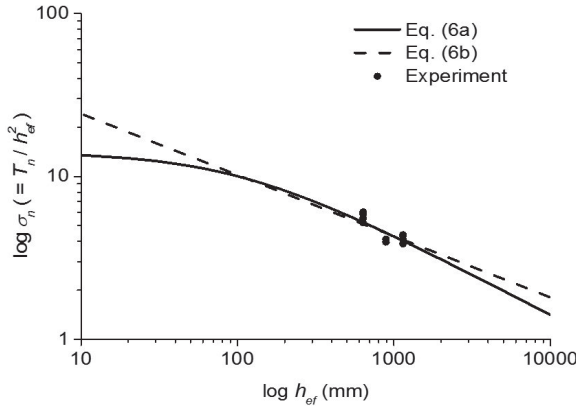


Fig. 7. Size effect of large anchors.

Also, if the supports are located far enough from the anchor, the angle  $\theta$  becomes constant as  $30\sim 35^\circ$  and Eq. (4) can be simplified as Eq. (5).

$$T_n = \frac{\bar{n} f_{cu}^\beta h_{ef}^2}{\sqrt{1 + \lambda h_{ef}}} \quad (5a)$$

$$T_n = \bar{n} f_{cu}^\beta h_{ef}^{2-m} = \bar{n} f_{cu}^\beta h_{ef}^{\bar{m}} \quad (5b)$$

where  $T_n$  is the tensile cone failure load of anchor,  $f_{cu}$  is the compressive strength of concrete,  $h_{ef}$  is embedment depth, and  $\bar{n}, \beta, \bar{m}$  are the experimental constants.

The experimental constant  $\bar{m}$  in Eq. (5b) should be  $\bar{m} \geq 1.5$  based on fracture mechanics. In other words,  $\bar{m} = 1.5$  when the largest possible size effect predicted by linear fracture mechanics is assumed.

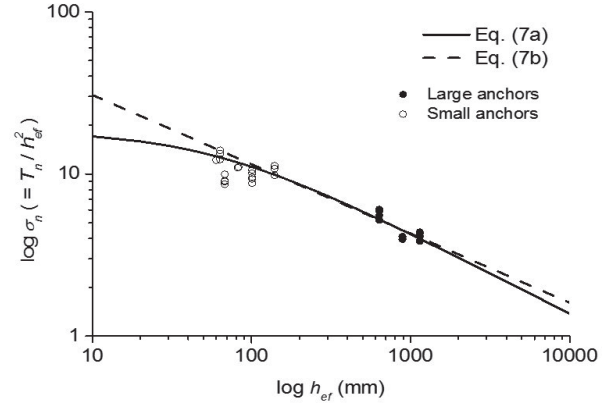
First of all, the experimental results of large anchors are analyzed whether the size effect is observed or not. Test results of three kinds of anchors in Table 4 are used for the regression analysis to find the experimental constants in Eq. (5a) and (5b). It is assumed that tensile strength of concrete is proportional to the square root of compressive strength as provided in the existing models because the effect of compressive strength on tensile strength is not tested herein. In other words, the regression analysis is conducted with fixed  $\beta$  as  $\beta=0.5$  in Eq. (5) and the optimum values of the other constants are found as shown in Eq. (6).

$$T_n = \frac{2.12 \sqrt{f_{cu}} h_{ef}^2}{\sqrt{1 + 0.01 h_{ef}}} \quad (6a)$$

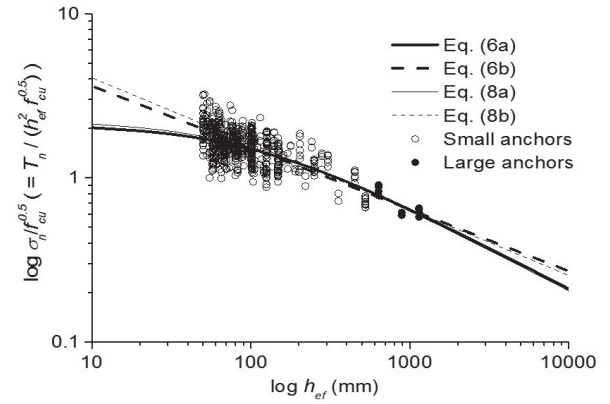
$$T_n = 8.63 \sqrt{f_{cu}} h_{ef}^{1.62} \quad (6b)$$

Experimental result of large anchors and Eq. (6) are presented in Fig. 7. Size effect appears as shown in Fig. 7.

Secondly, experimental results of anchors from the literature [16] as well as the large anchors tested herein are



(a)



(b)

 Fig. 8. Size effect of all anchors. (a) Unfixed constant  $\beta$ . (b) Fixed constant  $\beta=0.5$ 

used altogether for regression analysis to find the constants in Eq. (5) and the constants are as shown as Eq. (7).

$$T_n = \frac{1.73 f_{cu}^{0.62} h_{ef}^2}{\sqrt{1 + 0.018 h_{ef}}} \quad (7a)$$

$$T_n = 8.36 f_{cu}^{0.60} h_{ef}^{1.57} \quad (7b)$$

If  $\beta$  in Eq. (5) is fixed as  $\beta=0.5$  like Eq. (6), the regression result is changed into Eq. (8).

$$T_n = \frac{2.26 \sqrt{f_{cu}} h_{ef}^2}{\sqrt{1 + 0.012 h_{ef}}} \quad (8a)$$

$$T_n = 10.23 \sqrt{f_{cu}} h_{ef}^{1.60} \quad (8b)$$

Fig. 8(a) shows Eq. (7) with the results from the experiment and literatures [16]. Small anchors from the literatures in Fig. 8(a) only includes data of which the compressive strength is similar to the large anchor because the exponent of compressive strength in Eq. (7a) and Eq. (7b)

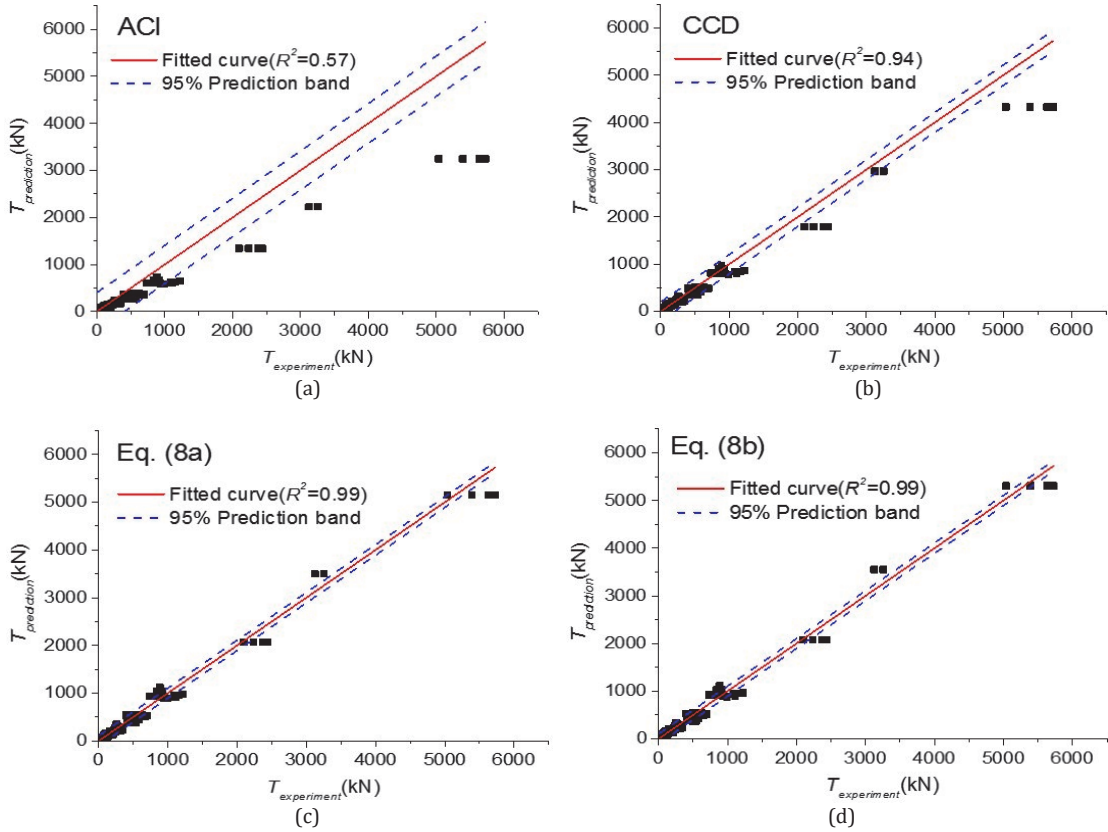


Fig. 9. Size effect of all anchors. (a) ACI. (b) CCD. (c) Eq. (8a). (d) Eq. (8b).

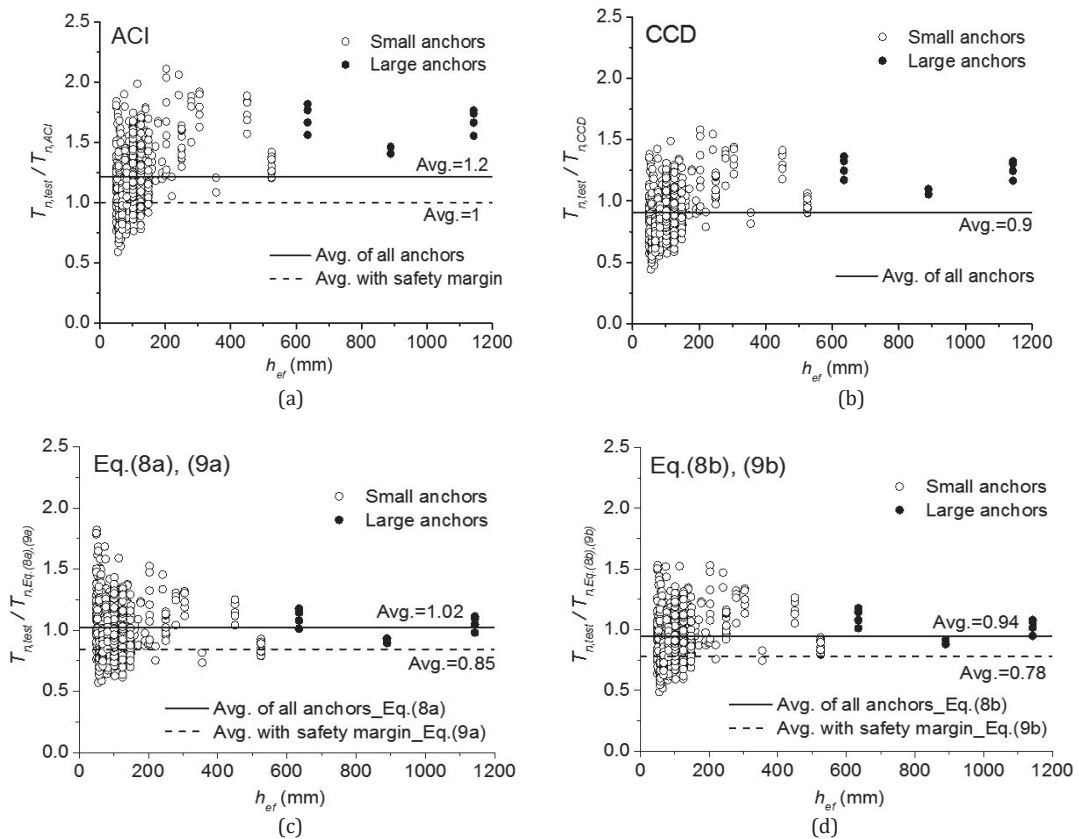


Fig. 10. Ratio of the measured strength to predicted strength. (a) ACI. (b) CCD. (c) Eq. (8a) and Eq. (9a). (d) Eq. (8b) and Eq. (9b).

are not same. Fig. 8(b) compares Eq. (6) and Eq. (8). All data are included in Fig. 8(b) with y-axis expressed as nominal stress divided by square root of compressive strength. Size effect can be noticed from both small and large anchors although there exists small difference as shown in Fig. 8(b) depending on the data considered.

Comparison between the predicted tensile strength of anchors and calculated strength by the existing prediction method of ACI and concrete capacity design(CCD) method [12] and the suggested Eq. (8) is represented in Fig. 9.

In this paper, Eq. (9) is finally suggested as an equation for prediction of tensile strength of anchors with similar safety margin as equation by ACI.

$$T_n = \frac{1.88\sqrt{f_{cu}}h_{ef}^2}{\sqrt{1 + 0.012h_{ef}}} \quad (9a)$$

$$T_n = 8.5\sqrt{f_{cu}}h_{ef}^{1.60} \quad (9b)$$

Fig. 10 shows the ratio of the measured cone failure load to the calculated load by each prediction method depending on the embedment depth. New model equations predict the experimental results more accurately as shown in Fig. 10, especially for large size anchors.

#### IV. CONCLUSION

Through this experimental and analytical research for large anchors to suggest new prediction equations, it is able to draw the following conclusions.

1) The size effect is clearly shown in experimental results for large size anchors with diameters of over 50 mm.

2) New model equations for prediction of concrete cone failure load are suggested by regression analysis of experimental data with the reasonable consideration of the size effect of anchors.

3) Two types of model equations are finally suggested. The suggested equations reduce the error and increase reliability of tensile strength of anchors.

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