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A Comparison of Admission Controls of Reservation Requests with Callable Products

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임의상환가능 상품 도입하의 예약 요청 승인 방법 비교

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Abstract A callable product is one of service derivatives using options to generate demand and reduce risk. This paper compares two booking admission controls for callable products, the online and the batch admission controls. To this end, the paper computes the optimal booking policy by using the backward dynamic programming and the stochastic optimization method. Intuitively, the provider should outperform under the batch control by utilizing demand information. The contribution of the paper is to show that the two controls are equivalent in terms of the booking strategy and the expected profit, which enables the provider to keep its current control method. The paper develops the closed-form solutions for the three fare classes. The future work is to extend the result to the model with complicated fare structures.

Key Words : Callable products, Revenue management, Option, Admission control, Booking strategy

요 약 임의상환가능 상품은 옵션을 이용하여 수요창출과 리스크를 줄이는 서비스 파생상품의 일종이다. 본 논문은 임의상환가능 상품이 도입된 후, 예약 요청의 온라인승인 방법과 일괄승인 방법 둘의 성능을 비교한다. 최적의 예약관리법을 계산하기 위하여, 역행 동적계획법(Backward Dynamic Programming)과 확률적 최적화(Stochastic Optimization) 방법을 이용한다. 직관적으로는 공급자는 수요정보를 이용하여 일괄승인 방법으로 더 높은 수익을 올릴 것으로 예상되지만, 본 논문은 두 예약 요청 승인 방법의 예약관리전략과 수익은 동일하다는 것을 증명함으로써, 현행 예약 요청 승인 방법의 변화 없이 임의상환가능 상품을 빠르게 도입할 수 있다는 실무적인 기여도가 있다. 본 논문은 세 종류의 서비스 요금 클래스가 있을 경우 최적의 해를 정확히 구하였다. 향후 연구는 다양한 요금 모델에서 최적의 해를 구하는 것이다.

주제어 : 임의상환가능 상품, 수익관리, 옵션, 예약 승인, 예약 전략

1. Introduction

The revenue management model assumes that low-fare customers arrive to book capacity before high-fare customers. For example, in the airline industry, price-sensitive leisure passengers tend to book early, while business passengers are willing to pay a premium to book late. To protect capacity for high-fare customers, service providers set booking limits for low-fare

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customers. Products are spoiled when high-fare demand turns out to be low, while cannibalization occurs when high-fare bookings are displaced by low-fare bookings[1].

To hedge against the uncertain future high-fare demand, Gallego et al.[2] propose callable products. With callable products, low-fare customers may grant call options to the service provider at the time of purchase. The service provider then can recall capacity from those low-fare customers at a pre-specified price. This allows the service provider to sell more units at a low price while reducing spoilage. At the same time, the service provider can reduce demand cannibalization by eliciting enough call options to satisfy most of the high-fare demand.

Gallego et al.[2] assume that options are exercised at any time prior to the service delivery time. Gallego and Lee[3] make callable products more operational by adding a pre-departure exercise date to give displaced customers an opportunity to make alternative plans. So the service provider must exercise call options at a certain time before service delivery time (e.g., flight departure). To make the callable products operational, the early exercise constraint is also considered in our model.

The service providers employ different booking admission controls for their booking systems[4]. The widely used ones are the online admission control and the batch admission control. Under the online admission control, service providers decide whether to accept or reject each booking request in real time based on the first-come-first served rule. So it is guaranteed that customers are booked in the order in which they arrive. In contrast, under the batch admission control, service providers hold booking requests and later make accept-or-reject decisions simultaneously. Gallego and Lee[3] study the optimal booking and option exercise policy under the online admission control but not for the batch admission control. This paper studies the optimal booking and option exercise policy under the batch admission control.

Intuitively, service providers seem to outperform under the batch admission control than under the online admission control when callable products are employed. This is because service providers can make call option decisions after observing the high-fare demand. However. this paper shows that the option operations are the same under the two different admission controls. So the two admission controls are equivalent in terms of the expected profit for service providers. The practical implication is that the service providers can keep their current control methods when callable products are implemented because there is no benefit to changing them.

In summary, the contribution of the paper is twofold. First, the paper develops the optimal booking policy under the new admission control policy (i.e., the batch admission control) when callable products are introduced. Second, the paper shows that the two admission controls, the online and the batch controls, are equivalent in terms of the expected profit.

We organize the paper as follows. Section 2 describes the revenue management model with callable products under the two admission controls. Section 3 introduces the optimal policy developed by Gallego and Lee[3] under the online admission control. Section 4 develops the optimal policy under the batch admission control. We conclude in Section 5.

2. Literature Review

To hedge against the uncertain future high-fare demand, Biyalogorsky et al.[5] propose opportunistic cancellations. This idea is extended by Biyalogorsky and Gerstner[6] by using contingent pricing. Biyalogorsky and Gerstner[6] consider two booking periods and a single product for sale. During the initial period, the seller makes an arrangement to sell a product at a low price if he/she fails in obtaining a higher price in the second period. Gallego et al.[2] further extend the analysis to multiple products for sale by introducing callable products. Gallego and Lee[3] make callable products more operational by adding a pre-departure exercise date. The benefit of using callable products are empirically shown in Lardeux et al.[7].

Callable products can be considered in supply chain settings. Some customers are willing to pay a significant premium for shorter order-fulfillment lead times. The heterogeneous customer valuation for different fulfillment lead times gives rise to advance demand information. Chen[8], Gallego and Özer[9], and Özer[10] show that advance demand information improves the performance of inventory and distribution systems. Özer and Wei[11] and Boyaci and Özer[12] discuss more supply chain strategies including pricing, capacity planning.

Gallego and Stefanescu[13] claim that callable products may be sold to customers with predictable demands who operate with low margins. Low margins make a recall price attractive. Then, the service provider can accommodate customers who are willing to pay a significant premium for short lead times. The callable service can also be applied to supply chain contract designs as in Lee et al.[14] and to the car sharing service[15].

Callable products can also be applied to sporting events. The options enable customers to reserve tickets for the final game. When the teams for the final game is identified, option holders pay the tickets if they decide to attend and otherwise cancel the tickets. Sainam et al.[16] show that call options can generate extra revenue. Their work is further extended by Balseiro et al.[17] to include pricing analysis of call options.

3. Model Description

The total capacity that the service provider initially has is denoted by C. We assume low-fare customers book before high-fare customers. The service provider announces the recall price d. Low-fare customers may grant the option to the service provider at the time of purchase if the recall price is attractive to them. The service provider can recall capacity at the recall price to resell to the high-fare customers if capacity is exhausted.

Low-fare customers must be notified a few days before the service delivery time (e.g., the flight departure). To deal with this, the high-fare booking period is divided into two sub-periods. During the last sub-period, no options are allowed to be exercised. Hence, we consider three fare classes: low-fare price f_3, high-fare price in the first sub-period f_2, and high-fare price in the last sub-period f_1. So we have $f_3 < d < f_2 \leq f_1$.

We let the probability that a low-fare customer will grant a call option be q. Because he/she tends to more willingly grant an option at a higher recall price, the probability q is increasing in d. The binomial random variable with parameters n and q is denoted by $B(n)$.

The random integer-valued demand of fare-class i is denoted by D_i, for i=1, 2, 3. We assume those demands are independent of each other. Based on the anticipated demands, the provider determines the booking limits, b_3 and b_2 for fare-class 3 and fare-class 2 demands respectively. In addition, the provider needs to determine the number of call options, denoted by n_2, to exercise during the fare class 2 booking period and the number of options , denoted by n_1, to exercise before the fare class 1 booking period starts.

Under the online admission control studied by

Gallego and Lee[8], the provider needs to determine n_2 before observing the full demand D_2. This is because the service provider needs to determine whether to accept or reject each booking request in real time. In contrast, the provider determines n_2 after fully observing the full demand D_2 and just before fare−class 1 booking period starts.

4. Admission Control

4.1 Online Admission Control

We introduce the main results developed by Gallego and Lee[8]. Readers may refer to Gallego and Lee[8] for detailed results and proofs. The problem is formulated using a backward dynamic programming. Hence, the optimal policy is developed for the fare−class 1 booking period, then for fare−class 2, and finally for fare−class 3 booking periods.

Assume that the provider is at the beginning of the fare−class 1 booking period with a state (s, x), where s represents the units of residual capacity and x represents the number of options available. Because it is the final booking period to sell, the provider makes all the residual capacity available. So, the only decision to make is to determine the number of options exercised. We define the target capacity, denoted by $\hat{c}_1$ as follows.

$\hat{c}_1 = \min\{m \in Z_+ : f_1 \Pr(D_1 > n) < d\}$.

Then the optimal number of options to exercise is determined by

$n_1^* = \min\{(\hat{c}_1 - s)^+, x\}$.

The optimal policy means that options are exercised only if the target capacity exceeds the residual capacity if options are available. If the provider recalls n_1^* units, with the cost of $d \cdot n_1^*$, the number of sales is $\min\{D_1, s+n_1^*\}$. Therefore, the value function is

$$V_1(s,x) = f_1 E\min\{D_1, s+n_1^*\} - d \cdot n_1^*. \quad (1)$$

Now, we go back to the beginning of fare−class 2 booking period. Assume that the provider starts with a state (s, x). It needs to determine the booking limit b_2 and the number of options allowed n_2. We define the protection level for fare−class 1 demand as follows.

$p_1 = \max\{p \in Z_+ : f_1 \Pr(D_1 \geq p) > f_2\}$

Note that $p_1 \leq \hat{c}_1$. Then the optimal booking limit is

$b_2^* = \min\{(s+x-p_1)^+, s\}$

and the optimal number of options allowed is

$n_2^* = (x-p_1)^+$.

The optimal policy means that the provider protects call options for fare−class 1 demand before protecting any inventory. Then initial sale is $\min\{D_2, b_2^*\}$, and surplus demand is covered by exercising options but not more than n_2^*. So the number of options exercised is $\min\{(D_2-b_2)^+, n_2\}$. Then the value function under the online admission control is

$$V_2(s,x) = f_2 E\min\{D_2, b_2^*\}$$
$$+ (f_2-d)E\min\{(D_2-b_2^*)^+, n_2^*\}$$
$$+ EV_1(s-\min\{D_2, b_2^*\}, x-\min\{(D_2-b_2^*)^+, n_2^*\}. (2)$$

Finally, assume that the provider just starts the booking process with a state $(C, 0)$ and needs to determine the booking limit b_3. The sale is $\min\{D_3, b_3\}$, and each booking has the probability q of granting a call option. So the number of options granted is a binomial random variable denoted by $B(\min\{D_3, b_3\})$. The optimal booking limit is

$b_3^* = \min\{b \in Z_+ : H_2(b) - H_2(b+1) > f_3\}$,

where $H_2(b) = EV_2(C-b, B(b))$. So the optimal total expected revenue corresponds to

$$V_3(C,0) = f_3 E\min\{D_3, b_3^*\} + H_2(\{D_3, b_3^*\}).$$

4.2 Batch Admission Control

Under the batch admission control, the

provider is able to hold demand D_i at the end of fare-class $i$booking period for $i=1,2,3$. Note that the value function for fare-class 1 booking period does not change, so the value function $V_1(s,x)$ can still be used.

However, for fare-class 2 booking period, the provider determines n_2 at the end of its booking period. This enables the provider to make the decision n_2 after observing the demand D_2. To distinguish with the value function under the online control in Section 3, the value function under the batch control is denoted by $V_2^b(s,x)$. Intuitively, we may think that it is greater than $V_2(s,x)$ because the provider can utilize the demand information. However, we will show that the two value functions are exactly the same.

At the end of fare-class 2 booking period, the expected profit is

$$\pi_2(n_2|b_2)=(f_2-d)\min\{(D_2-b_2)^+,n_2\}$$
$$+V_1(s-\min\{D_2,b_2\},x-\min\{(D_2-b_2)^+,n_2\}).$$

The problem to solve at the beginning of the booking period is to determine b_2 maximizing

$$\Pi_2(b_2;s,x)=E[f_2\min\{D_2,b_2\}+\max_{n_2\le x}\pi_2(n_2|b_2)].$$

Then our value function is given by

$$V_2^b(s,x)=\max_{0\le b_2\le s}\Pi_2(b_2;s,x).$$

The following lemma determines the optimal number of options exercised at the end of booking period, denoted by $n_2^b(b_2)$, when the booking limit was determined by b_2 at the start of the booking period.

Lemma 4.1 *The optimal number of options allowed for the surplus demand $(D_2-b_2)^+$ is*

$$n_2^b(b_2)=\min\{(s-b_2+x-p_1)^+,x\}.$$

Proof : We consider the two cases separately.

Case 1: There is surplus demand (i.e., $D_2>b_2$). There is no need to allow options more than the surplus demand. So if $n_2<D_2-b_2$, we have

$$\pi_2(n_2+1|b_2)-\pi_2(n_2|b_2)$$
$$=f_2-d+V_1(s-b_2,x-n_2-1)-V_1(s-b_2,x-n_2).$$

If $\hat{c}_1-s+b_2\le 0$, no options are exercised for fare-class 1 demand. So we have

$\pi_2(n_2+1|b_2)-\pi_2(n_2|b_2)=f_2-d>0$. This means that $\pi_2(n_2|b_2)$ is increasing in n_2. So it is maximized at $n_2=\min\{D_2-b_2,x\}$. Next, consider $\hat{c}_1-s+b_2>0$. If $n_2<x-\hat{c}_1+s-b_2$, we have $\pi_2(n_2+1|b_2)-\pi_2(n_2|b_2)=f_2-d>0$. Otherwise, $\pi_2(n_2+1)-\pi_2(n_2)=f_2-f_1\Pr(D_1\ge s-b_2+x-n_2)$. Altogether, the function is maximized at

$$n_2=\{(s-b_2+x-p_1)^+,x,D_2-b_2\}.$$

Case 2: There is no surplus demand (i.e., $D_2\le b_2$). In this case, we have $n_2=0$.

For cases 1 and 2, the function is maximized at

$n_2=\{(s-b_2+x-p_1)^+,x,(D_2-b_2)^+\}$. We can simplify the policy, because $n_2^b(b_2)$ results in the same value of $\pi_2(n_2|b_2)$. □

According to Lemma 4.1, the optimal option policy for a given booking limit b_2 does not depend on the demand D_2.

Lemma 4.2 *If options are to be exercised (i.e., $b_2<s+x-p_1$), $\Pi_2(b_2;s,x)$ is increasing in b_2.*

Proof : If $b_2\ge s-p_1$, we have

$\Pi_2(b_2+1;s,x)-\Pi_2(b_2;s,x)=0$. If $b_2<s-p_1$, we have $\Pi_2(b_2+1;s,x)-\Pi_2(b_2;s,x)$

$$=[f_2-f_1\Pr(D_1\ge s-b_2)]\Pr(D_2>b_2)\ge 0.$$

Hence, we can say that $\Pi_2(b_2;s,x)$ is increasing in $b_2$if $b_2<s+x-p_1$.

Lemma 4.2 tells that the booking limit needs to be increased as long as the number of options exercised is positive. Now, we have the main result. The following theorem develops the optimal booking limit and the optimal number of options allowed under the batch admission control, denoted by b_2^b and n_2^b.

Theorem 4.1 *When batch admission policy is applied, for a given state (s,x), the optimal booking limit is*

$$b_2^h=\min\{(s+x-p_1)^+,s\}$$

and the optimal number of options allowed is $n_2^h = (x - p_1)^+$.

In turn, we have $V_2^b(s,x) = V_2(s,x)$.

Proof : If $x - p_1 > 0$, we have $s - b_2 + x - p_1 > 0$. By Lemma 4.2, $\Pi_2(b_2;s,x)$ is increasing in b_2. Therefore, the value of b_2 maximizing $\Pi_2(b_2;s,x)$ is s.

Next, consider $x - p_1 \le 0$. In this case, we first consider $b_2 \ge s + x - p_1$. Then,

$$\Pi_2(b_2+1;s,x) - \Pi(b_2;s,x)$$
$$= [f_2 - f_1 \Pr(D_1 \ge s - b_2 + x)]\Pr(D_2 > b_2).$$

Because $s - b_2 + x \le p_1$, we have $\Pi_2(b_2+1;s,x) - \Pi(b_2;s,x) \le 0$. So it is decreasing. In contrast, if $b_2 < s + x - p_1$, if is increasing by Lemma 4.2.

Altogether, we have $b_2^h = \min\{(s + x - p_1)^+, s\}$.

By Lemma 4.1, $n_2^h = \min\{(s - b_2^h + x - p_1)^+, x\}$.

If $x > p_1$, $b_2^h = s$. So, $n_2^h = x - p_1$. If $x \le p_1$, $b_2^h = (s + x - p_1)^+$. Then, $n_2^h = x$. Therefore, we can simply have $n_2^h = (x - p_1)^+$.

4.3 Comparison

Intuitively, the provider may outperform under the batch control by utilizing demand information. Hence, the provider may be tempted to change the current method when callable products are implemented if the booking system is based on the online admission control.

However, Theorem 4.1 shows that the optimal booking limit and the optimal number of options allowed under the batch admission control, denoted by b_2^b and n_2^b, are exactly the same as the optimal booking limit and the optimal number of options allowed under the online admission control. This means that we have $b_2^b = b_2^*$ and $n_2^b = n_2^*$. Furthermore, the two value functions are the same, which means $V_2^b(s,x) = V_2(s,x)$. Therefore, this results shows that there is no benefit to changing the existing control method when callable products are implemented.

5. Conclusions

This paper compares the online admission control and the batch admission control when callable products are implemented. The optimal policy was studied by Gallego and Lee[8] for the online admission control, but not for the batch control. This paper develops the optimal policy for the batch admission control. The optimal option policy is based on the threshold values that do not depend on the state of the dynamic programming and the demand distributions. For fare-class 1 demand, there is the target capacity, and options are exercised only when the target capacity exceeds the residual capacity. For fare-class 2 demand, options are protected for the future demand up the protection level and the rest are exercised.

Surprisingly, the two admission controls result in the same booking and option policies, and in turn the same expected revenue for the service provider, even though there is a strong information advantage of observing the full demands under the batch admission control.

The equivalence of the two admission control policy means that the provider does not need to change its current online admission method. The provider may be tempted to change the existing admission method to earn the extra money by utilizing the information advantage. However, this paper shows that there is no benefit to changing the current admission method.

We present several future research directions. First, our analysis is tractable for the revenue manage model with three fare classes. We believe that it is difficulty to derive closed-form solutions for the model with more than three fare classes. However, it is certain that insights found in this paper still hold for the model with complicated fare structures. Second, we assume that the demands are stochastically independent, but the analysis should be extended to the dependent demand for a practical application of callable products. Finally, the recall price can be optimized along with the booking limit and option exercise policy.

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