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# An Alternative Optimization Procedure for Parameter Design

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## Abstract

Taguchi has used the signal-to-noise ratio (SN) to achieve the appropriate set of operating conditions where variability around target is low in the Taguchi parameter design. Taguchi has dealt with having constraints on both the mean and variability of a characteristic (the dual response problem) by combining information on both mean and variability into an SN. Many Statisticians criticize the Taguchi techniques of analysis, particularly those based on the SN. In this paper we propose a substantially simpler optimization procedure for parameter design to solve the dual response problems without resorting to SN.

**Keywords** : Parameter Design, Optimization Procedure

## 1. Introduction

Taguchi Quality Engineering has contributed greatly to the statistical field widely used to improve the quality throughout product design stage[1].

In the Taguchi parameter design, the product array using the orthogonal array table was subjected to data analysis using the SN by performing experimental setup considering all the interaction effects of the control factor and the noise factor. In the product array, the noise factor plays a role in reducing quality variation of quality characteristics, which makes it possible to design a parameter which can find the optimum condition of control factor approaching the target value while the average of quality characteristics is insensitive to variation. Products and their manufacturing processes are influenced both by control factors that can be controlled by designers and by noise factors that are difficult or expensive to control such as environmental conditions. The basic idea of parameter design is to identify, through exploiting interactions between control factors and noise factors, appropriate settings of control factors that make the system's performance robust to changes in the noise factors. Parameter design is a quality improvement technique proposed by the Japanese quality expert Taguchi[2], which was described by Kackar[3] and others.

Although Taguchi quality engineering has made a great contribution to improving quality, many problems have been pointed out in the use of SN in analyzing data, and alternatives have been studied by various scholars. In this regard, the analysts such as Box[4] proposed an analytical method through data transformation. Vining and Myers[5] used regression analysis from repeated measurement data instead of using SN, For the first time as an alternative method to obtain the optimum condition of quality characteristics. They analyzed the experimental data using the optimization technique for the dual response function of Myers and Carter[6]. Since then, scholars such as Copeland and Nelson[7] have proposed different optimization methods for three different characteristics.

The purpose of this paper is to propose a reasonable optimization formula for data analysis in parameter design and to find an optimization method. In addition, the new optimization formula proposed in this paper can be applied to the alternative method that improves the problem in the Taguchi method mentioned above. This paper is to use the alternative method to find the optimum condition of the quality characteristic by separating the estimated mean model and the variance model without using SN, Chapter 4 presents the advantages of this paper based on the contents of Chapters 2

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and 3. An example is illustrated to show the proposed method.

## 2. An Optimization Procedure for Parameter Design

In analyzing the data in the parameter design, problems have arisen due to the use of SN, a performance measure that combines mean and variation(or variance). Suppose that the quality characteristics ($\underline{y}$) are determined by the control factors ($\underline{x}$) and the noise factors ($\underline{z}$). The product array is possible to obtain repeated quality characteristics ($\underline{y}$) by the noise factor under the different experimental condition of the control factors.

The sample mean ($\bar{y}$) and sample variance(or standard deviation ($v$)) can be obtained from these data. Therefore, instead of using the SN, we can consider an alternative method to obtain the optimal condition of the quality characteristic by separating the estimated mean model and the standard deviation model from the repeated measurement data. From the repeated measurement data, the estimated mean model $\hat{\mu}(\underline{x})$ by the least squares method for the sample mean of the quality characteristics. the estimated standard deviation model $\hat{\sigma}(\underline{x})$ by the least squares method for the sample standard deviation of the quality characteristics. The parameter design is an experimental design and analysis method to reduce the expected loss. To reduce the expected loss in a product or process design, the quality mean should be brought close to the target while reducing quality variation. First, we determine an average model that measures the average of products and a standard deviation model that measures product variability. Next, we find an optimal process that minimizes variance $\hat{\sigma}(\underline{x})$ while keeping the mean $\hat{\mu}(\underline{x})$ within the range possible.

Copeland and Nelson[7] proposed the optimization formula for the nominal-is-best case as follows.

$$\min_{\underline{x} \in R_x} \hat{\sigma}(\underline{x})$$

$$\text{such that } (\hat{\mu}(\underline{x}) - T)^2 \le \Delta^2$$

Here $R_x$ is the region of interest of the control factors. In case of the larger-is-better as follows. Here $\sigma_T$ is a constant.

$$\max_{\underline{x} \in R_x} \hat{\mu}(\underline{x})$$

$$\text{such that } \hat{\sigma}(\underline{x}) \le \sigma_T$$

On the other hand, in case of the smaller-is-better as follows.

$$\min_{\underline{x} \in R_x} \hat{\mu}(\underline{x})$$

$$\text{such that } \hat{\sigma}(\underline{x}) \le \sigma_T$$

Copeland and Nelson[7] proposed an optimization formula for the larger-is-better case and the smaller-is-better case, while limiting the standard deviation model and maximizing or minimizing the mean model. The optimization formula for the nominal-is-best case is to minimize the standard deviation model while limiting the mean model. The method proposed by them has a problem that the restriction conditions vary depending on the quality characteristics. This procedural problem is a wrong idea in that it is priority to reduce quality variation rather than quality mean in parameter design. Maintaining consistency of constraints in using optimization formulas is helpful in determining the optimal solution of the control factors. This is helpful when analyzing data because it can be applied consistently to all quality characteristics. They do not describe at all how to set the specificity constant $\sigma_T$ or $\sigma_T$ as the upper limit of the standard deviation model as a constraint in the case of the larger-is-better and the smaller-is-better case.

We propose a new formula to overcome the problems of the optimization formula proposed by Copeland and Nelson[7] as follows. All quality characteristics of the world have some specific target value. Therefore, we want to set the target values for the three quality characteristics in the mean model as follows.

(1) the nominal-is-best : $T_\mu$ = Specific constant

(2) the larger-is-better : $T_\mu = \max_{\underline{x} \in R_x} \hat{\mu}(\underline{x})$ (1)

(3) the smaller-is-better : $T_\mu = \min_{\underline{x} \in R_x} \hat{\mu}(\underline{x})$

From the above, all the quality characteristics can be regarded as the nominal-is-best. in that they have a certain target value $T_\mu$ in the region of interest of the control factors. Therefore, the method of finding the optimal process by data analysis can be equally applied to all the quality characteristics as the nominal-is-best. In addition, since the target value for all quality characteristics in the standard deviation model is smaller-is-better as follows.

all the quality characteristics :

$$T_\sigma = \min_{\underline{x} \in R_x} \hat{\sigma}(\underline{x}) \tag{2}$$

In parameter design, it is first of all to reduce quality variation rather than quality mean. Therefore, we propose the following optimization formula for all quality characteristics.

$$\min_{\underline{x} \in R_x} \left| \hat{\mu}(\underline{x}) - T_\mu \right|$$

$$\text{such that } \left| \hat{\sigma}(\underline{x}) - T_\sigma \right| \le \Delta \tag{3}$$

Here, the allowable range $\Delta$ is an arbitrary constant. The method of determining $\underline{x}$ is to use the confidence interval or various methods, but the basic idea is to place the quality fluctuation within the range of the target as much as possible. Therefore, looking at the behavior of the control factors in various ranges, It will be a way to save them. Furthermore, since the mean and standard deviation models are fitted by regression analysis, the estimated average model of quality characteristics and the allowable range of target values should be considered in finding the optimal process.

## 3. Numerical Example

In this chapter, we introduce the optimization procedure using the newly proposed optimization formula and optimization method in the parameter design.

Box and Draper[8] conducted an experiment on the printing process. Table 1 shows that the factorial experiment was repeated three times at different locations. This can be seen as a product array data from repeated measurements with three levels of noise factor, $z_1$, $z_2$ and $z_3$.

The mean model estimated by the least squares method for the sample mean ($\bar{y}$) of the quality characteristics from the experimental data on the printing process is as follows.

$$\hat{\mu}(\underline{x}) = 327.6 + 117.0x_1 + 109.4x_2 + 131.5x_3 + 32.0x_1^2 - 22.4x_2^2 - 29.1x_3^2 + 66.0x_1x_2 + 75.5x_1x_3 + 43.6x_2x_3 \tag{4}$$

**Table 1.** Printing process data

| | $x_1$ | $x_2$ | $x_3$ | $y_1$ | $y_2$ | $y_3$ | $\bar{y}$ | $v^2$ |
|---|---|---|---|---|---|---|---|---|
| 1 | -1 | -1 | -1 | 34 | 10 | 28 | 24.0 | 12.49 |
| 2 | 0 | -1 | -1 | 115 | 116 | 130 | 120.3 | 8.39 |
| 3 | 1 | -1 | -1 | 192 | 186 | 263 | 213.7 | 42.80 |
| 4 | -1 | 0 | -1 | 82 | 88 | 88 | 86.0 | 3.46 |
| 5 | 0 | 0 | -1 | 44 | 178 | 188 | 136.7 | 80.41 |
| 6 | 1 | 0 | -1 | 322 | 350 | 350 | 340.7 | 16.17 |
| 7 | -1 | 1 | -1 | 141 | 110 | 86 | 112.3 | 25.57 |
| 8 | 0 | 1 | -1 | 259 | 251 | 259 | 256.3 | 4.62 |
| 9 | 1 | 1 | -1 | 290 | 280 | 245 | 271.7 | 23.63 |
| 10 | -1 | -1 | 0 | 81 | 81 | 81 | 81.0 | 0.00 |
| 11 | 0 | -1 | 0 | 90 | 122 | 93 | 101.7 | 17.67 |
| 12 | 1 | -1 | 0 | 319 | 376 | 376 | 357.0 | 32.91 |
| 13 | -1 | 0 | 0 | 180 | 180 | 154 | 171.3 | 15.01 |
| 14 | 0 | 0 | 0 | 372 | 372 | 372 | 372.0 | 0.00 |
| 15 | 1 | 0 | 0 | 541 | 568 | 396 | 501.7 | 92.50 |
| 16 | -1 | 1 | 0 | 288 | 192 | 312 | 264.0 | 63.50 |
| 17 | 0 | 1 | 0 | 432 | 336 | 513 | 427.0 | 88.61 |
| 18 | 1 | 1 | 0 | 713 | 725 | 754 | 730.7 | 21.08 |
| 19 | -1 | -1 | 1 | 364 | 99 | 199 | 220.7 | 133.80 |
| 20 | 0 | -1 | 1 | 232 | 221 | 266 | 239.7 | 23.46 |
| 21 | 1 | -1 | 1 | 408 | 415 | 443 | 422.0 | 18.52 |
| 22 | -1 | 0 | 1 | 182 | 233 | 182 | 199.0 | 29.45 |
| 23 | 0 | 0 | 1 | 507 | 515 | 434 | 485.3 | 44.64 |
| 24 | 1 | 0 | 1 | 846 | 535 | 640 | 673.7 | 158.20 |
| 25 | -1 | 1 | 1 | 236 | 126 | 168 | 176.7 | 55.51 |
| 26 | 0 | 1 | 1 | 660 | 440 | 403 | 501.0 | 138.90 |
| 27 | 1 | 1 | 1 | 878 | 991 | 1161 | 1010.0 | 142.50 |

The estimated standard deviation model for the sample standard deviation ($v$) of the quality characteristics is as follows.

$$\hat{\sigma}(\underline{x}) = 34.9 + 11.5x_1 + 15.3x_2 + 29.2x_3 + 2x_1^2 - 1.3x_2^2 + 16.8x_3^2 + 7.7x_1x_2 + 5.1x_1x_3 + 14.1x_2x_3 \tag{5}$$

On the other hand, the interest region of the three control factors is $-1 \le x_1, x_2, x_3 \le 1$.

We use a grid search method with a lattice spacing of 0.01 within the interest range of the three control factors to find the optimal point of the control factor from the two fitted models. In this case, $201^3$ grid points are generated and optimized only at the generated grid points.

Table 2 shows the optimum point of the control factor minimizing $|\hat{\mu}(\underline{x}) - 500|$ in the constraint condition $|\hat{\sigma}(\underline{x}) - 12.50| \leq \Delta$ when the target value $T_\mu$ for the quality mean is 500 and the target value $T_\sigma$ for the quality variation is 12.50. In the optimization results, the optimum point of the control factor is $x_1 = -1.00$, $x_2 = 0.52$ and $x_3 = -0.43$ when the allowable range $\Delta$ is 1.00, and then $\hat{\mu}(\underline{x}) = 219.91$ and $\hat{\sigma}(\underline{x}) = 13.50$. In addition, no matter how large the allowable range $\Delta$ increases from 54.93, it can be seen that $x_1$ is 0.20, $x_2$ is 0.89, and $x_3$ is 0.32.

Table 3 shows the optimum point of the control factor minimizing $|\hat{\mu}(\underline{x}) - 851.01|$ in the constraint $|\hat{\sigma}(\underline{x}) - 12.50| \leq \Delta$, with the maximum value of $\hat{\mu}(\underline{x})$ being $T_\mu = 851.01$ for the case of the larger-is-better. As the allowable range $\Delta$ increases in the optimization result, $x_1$, $x_2$ and $x_3$ tend to gradually increase at -1.00, 0.53 and -0.4, respectively.

Table 4 shows the optimum value of the control factor minimizing $|\hat{\mu}(\underline{x}) - 74.11|$ in the constraint condition $|\hat{\sigma}(\underline{x}) - 12.50| \leq \Delta$, with the minimum value of $\hat{\mu}(\underline{x})$ being the target value of $T_\mu = 74.11$ for the case of the smaller-is-better. Table 4 shows the optimum value of the control factor minimizing $|\hat{\mu}(\underline{x}) - 74.11|$ in the constraint condition $|\hat{\sigma}(\underline{x}) - 12.50| \leq \Delta$, with the minimum value of $\hat{\mu}(\underline{x})$ being the target value of $T_\mu = 74.11$ for the case of the smaller-is-better. Even if the optimization result tolerance range $\Delta$ increases from 0.01 to 2.65, $x_1$, $x_2$ and $x_3$ do not change at -1.00, 1.00 and -1.00. In addition, it can be seen that no matter how large the allowable range $\Delta$ increases from 7.62, $x_1$ becomes 0.38, $x_2$ becomes -1.00 and $x_3$ becomes -1.00.

Therefore, it is possible to find a certain trend by observing the optimal solution of the control factors by optimizing the newly proposed optimization formula in various permissible ranges $\Delta$ through the grid search method. In addition, better parameter design can be achieved by identifying and analyzing the causes of such trends.

**Table 2.** Optimization of the nominal-is-best

| $\Delta$ | $\hat{\mu}$ | $\hat{\sigma}$ | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|---|---|
| 1.00 | 219.91 | 13.50 | -1.00 | 0.52 | -0.43 |
| 3.00 | 229.52 | 15.50 | -0.87 | 0.62 | -0.42 |
| 5.00 | 239.97 | 17.49 | -0.72 | 0.77 | -0.44 |
| 10.00 | 268.04 | 22.50 | -0.49 | 0.91 | -0.42 |
| 15.00 | 297.86 | 27.50 | -0.26 | 0.97 | -0.42 |
| 20.00 | 328.75 | 32.48 | -0.10 | 1.00 | -0.38 |
| 30.00 | 393.24 | 42.50 | 0.23 | 0.95 | -0.31 |
| 40.00 | 460.38 | 52.49 | 0.53 | 0.99 | -0.27 |
| 50.00 ~ 54.92 | 500.01 | 59.46 | 0.59 | 0.66 | 0.02 |
| 54.93 ~ ∞ | 500.00 | 67.43 | 0.20 | 0.89 | 0.32 |

**Table 3.** Optimization of the larger-is-better

| $\Delta$ | $\hat{\mu}$ | $\hat{\sigma}$ | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|---|---|
| 1.00 | 219.90 | 13.50 | -1.00 | 0.53 | -0.43 |
| 2.50 | 239.03 | 15.00 | -0.89 | 0.64 | -0.43 |
| 5.00 | 239.97 | 17.50 | -0.72 | 0.77 | -0.44 |
| 10.00 | 268.04 | 22.50 | -0.49 | 0.91 | -0.42 |
| 20.00 | 328.75 | 32.48 | -0.10 | 0.94 | -0.38 |
| 30.00 | 393.24 | 42.50 | 0.23 | 0.96 | -0.31 |
| 40.00 | 460.38 | 52.49 | 0.53 | 1.00 | -0.27 |
| 60.00 | 599.16 | 72.48 | 1.00 | 0.96 | -0.10 |
| 100.00 | 778.60 | 112.35 | 1.00 | 0.99 | 0.49 |
| 124.99 ~ ∞ | 851.01 | 137.50 | 1.00 | 1.00 | 1.00 |

**Table 4.** Optimization of the smaller-is-better

| $\Delta$ | $\hat{\mu}$ | $\hat{\sigma}$ | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|---|---|
| 0.01 ~ 2.65 | 134.90 | 12.50 | -1.00 | 1.00 | -1.00 |
| 2.66 ~ 2.70 | 134.62 | 15.15 | -0.11 | -1.00 | -0.58 |
| 2.80 | 132.33 | 15.26 | -0.09 | -1.00 | -0.60 |
| 2.90 | 130.00 | 15.37 | -0.07 | -1.00 | -0.62 |
| 3.00 | 128.78 | 15.44 | -0.05 | -1.00 | -0.63 |
| 3.50 | 119.98 | 15.97 | 0.03 | -1.00 | -0.70 |
| 4.00 | 113.26 | 16.46 | 0.09 | -1.00 | -0.75 |
| 5.00 | 100.29 | 17.49 | 0.15 | -1.00 | -0.84 |
| 6.00 | 89.33 | 18.50 | 0.23 | -1.00 | -0.91 |
| 7.00 | 79.37 | 19.49 | 0.30 | -1.00 | -0.97 |
| 7.62 ~ ∞ | 74.11 | 20.11 | 0.38 | -1.00 | -1.00 |

## 4. Conclusions

This paper proposes a new optimization method for parameter design and finds a way to apply it.. SN, which is a performance measure of the nominal-is-best used in data analysis in Taguchi parameter design, has been pointed out by many scholars. Therefore, in this paper, we propose an optimization formula for parameter design using the model which is separated from the mean model and the standard deviation (or variance) model from the experimental data. Accordingly, we can apply the primary goal of the Taguchi methodology which is to obtain a target condition on the mean while achieving the variance, or to minimize the variance while constraining the mean. In addition, the proposed optimization method finds the optimum solution of the control factors through the grid search method. The grid search method can be easily used by anyone because it is easy to program according to the user's purpose.

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