J. Appl. Math. & Informatics Vol. **37***(2019), No. 5 - 6, pp. 459 - 467*
https://doi.org/10.14317/jami.2019.459

ON ASYMPTOTICALLY f−ROUGH STATISTICAL EQUIVALENT OF TRIPLE SEQUENCES

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ABSTRACT. In this work, via Orlicz functions, we have obtained a generalization of rough statistical convergence of asymptotically equivalent triple sequences a new non-matrix convergence method, which is intermediate between the ordinary convergence and the rough statistical convergence. We also have examined some inclusion relations related to this concept. We obtain the results are non negative real numbers with respect to the partial order on the set of real numbers.

AMS Mathematics Subject Classification : 41A36, 40G15.
Key words and phrases : rough statistical convergence, strong Cesaro summability, triple sequence space, Orlicz function, asymptotically equivalent triple sequences.

1. Introduction

Let K be a subset of the set of positive integers $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$, and let us denote the set $\{(m,n,k) \in K : m \leq u, n \leq v, k \leq w\}$ by K_{uvw}. Then the natural density of K is given by $\delta(K) = lim_{u,v,w \to \infty} \frac{|K_{uvw}|}{uvw}$, if the limit exists, where $|K_{uvw}|$ denotes the number of elements in K_{uvw}. Clearly, a finite subset has natural density zero, and we have $\delta(K^c) = 1 - \delta(K)$ where $K^c = \mathbb{N} \backslash K$ is the complement of K. If $K_1 \subseteq K_2$, then $\delta(K_1) \leq \delta(K_2)$.

The triple sequence $x = (x_{mnk})$ is said to be rough statistically convergent to l if for every $\epsilon > 0$, the set $K_\epsilon = \left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - l| \geq \beta + \epsilon\right\}$ has natural density zero, for each $\epsilon > 0$,

$$lim_{r,s,t \to \infty} \frac{1}{rst} |\{(m,n,k) \leq (r,s,t) : |x_{mnk} - l| \geq \beta + \epsilon\}| = 0.$$

In this case, we write $l = st - limx$. Throughout the paper, $\mathbb{R}$ denotes the real of three dimensional space with metric (X,d). Consider a triple sequence $x = (x_{mnk})$ such that $x_{mnk} \in \mathbb{R}, m,n,k \in \mathbb{N}$.

Received June 20, 2019. Revised August 12, 2019. Accepted September 2, 2019.
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A triple sequence $x = (x_{mnk})$ is said to be statistically convergent to $0 \in \mathbb{R}$, written as $st - lim\ x = 0$, provided that the set

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk}, 0| \geq \epsilon\right\}$$

has natural density zero for any $\epsilon > 0$. In this case, 0 is called the statistical limit of the triple sequence x.

If a triple sequence is statistically convergent, then for every $\epsilon > 0$, infinitely many terms of the sequence may remain outside the $\epsilon-$ neighbourhood of the statistical limit, provided that the natural density of the set consisting of the indices of these terms is zero. This is an important property that distinguishes statistical convergence from ordinary convergence. Because the natural density of a finite set is zero, we can say that every ordinary convergent sequence is statistically convergent.

If a triple sequence $x = (x_{mnk})$ satisfies some property P for all m, n, k except a set of natural density zero, then we say that the triple sequence x satisfies P for almost all (m, n, k) and we abbreviate this by a.a. (m, n, k).

Let $\left(x_{m_i n_j k_\ell}\right)$ be a sub sequence of $x = (x_{mnk})$. If the natural density of the set $K = \left\{(m_i, n_j, k_\ell) \in \mathbb{N}^3 : (i, j, \ell) \in \mathbb{N}^3\right\}$ is different from zero, then $\left(x_{m_i n_j k_\ell}\right)$ is called a non thin sub sequence of a triple sequence x.

$c \in \mathbb{R}$ is called a statistical cluster point of a triple sequence $x = (x_{mnk})$ provided that the natural density of the set

$$\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk} - c| < \epsilon\right\}$$

is different from zero for every $\epsilon > 0$. We denote the set of all statistical cluster points of the sequence x by Γ_x.

A triple sequence $x = (x_{mnk})$ is said to be statistically analytic if there exists a positive number M such that

$$\delta\left(\left\{(m,n,k) \in \mathbb{N}^3 : |x_{mnk}|^{1/(m+n+k)} \geq M\right\}\right) = 0$$

The theory of statistical convergence has been discussed in trigonometric series, summability theory, measure theory, turnpike theory, approximation theory, fuzzy set theory and so on.

The idea of rough convergence was introduced by Phu [18], who also introduced the concepts of rough limit points and roughness degree. The idea of rough convergence occurs very naturally in numerical analysis and has interesting applications. Aytar [1] extended the idea of rough convergence into rough statistical convergence using the notion of natural density just as usual convergence was extended to statistical convergence. Pal et al. [17] extended the notion of rough convergence using the concept of ideals which automatically extends the earlier notions of rough convergence and rough statistical convergence.

Let (X, ρ) be a metric space. For any non empty closed subsets $A, A_{mnk} \subset X\,(m, n, k \in \mathbb{N})$, we say that the triple sequence (A_{mnk}) is Wijsman statistical convergent to A is the triple sequence $(d\,(x, A_{mnk}))$ is statistically convergent to $d\,(x, A)$, i.e., for $\epsilon > 0$ and for each $x \in X$

$$lim_{r,s,t} \frac{1}{rst} \left|\{m \leq r, n \leq s, k \leq t : |d\,(x, A_{mnk}) - d\,(x, A)| \geq \epsilon\}\right| = 0.$$

In this case, we write $St-lim_{m,n,k}A_{mnk}=A$ or $A_{mnk}\longrightarrow A\,(WS)$. The triple sequence (A_{mnk}) is bounded if $sup_{mnk}d\,(x,A_{mnk})<\infty$ for each $x\in X$.

A triple sequence (real or complex) can be defined as a function $x:\mathbb{N}\times\mathbb{N}\times\mathbb{N}\to\mathbb{R}\,(\mathbb{C})$, where $\mathbb{N},\mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by *Sahiner et al. [19,20]*, *Esi et al. [5-10]*, *Dutta et al. [3]*, *Subramanian et al. [22-27]*, *Debnath et al. [4]*, *Aiyub et al. [2]* and Zweier sequence was introduced and investigated at the initial by *Fadile Karababa et al. [11]*, *Sharma et al. [21]*, *Khan et al. [14]* , *Hazarika et al. [12]* , *Velmurugan et al. [28]* many others.

Throughout the paper let β be a nonnegative real number.
By using Orlicz functions, we have defined a generalization of rough statistical convergence of asymptotically equivalent of triple sequences and obtained some inclusion relations related to this concept.

2. Definition and Preliminaries

Definition 2.1. An Orlicz function ([see [13]) is a function $M:[0,\infty)\to[0,\infty)$ which is continuous, non-decreasing and convex with $M\,(0)=0$, $M\,(x)>0$, for $x>0$ and $M\,(x)\to\infty$ as $x\to\infty$. If convexity of Orlicz function M is replaced by $M\,(x+y)\le M\,(x)+M\,(y)$, then this function is called modulus function.

Lindenstrauss and Tzafriri ([15]) used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g=(g_{mnk})$ defined by

$$g_{mnk}\,(v)=sup\left\{|v|\,u-(f_{mnk})\,(u):u\ge0\right\},m,n,k=1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function f. For a given Musielak-Orlicz function f, [see [16]] the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f=\left\{x\in w^3:I_f\,(|x_{mnk}|)^{1/(m+n+k)}\to0\,as\,m,n,k\to\infty\right\},$$

where I_f is a convex modular defined by

$$I_f\,(x)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}f_{mnk}\,(|x_{mnk}|)^{1/(m+n+k)}\,,x=(x_{mnk})\in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d\,(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}f_{mnk}\left(\frac{|x_{mnk}|^{1/(m+n+k)}}{mnk}\right)$$

is an extended real number.

Definition 2.2. Let f be an unbounded Orlicz function. Two non negative triple sequences $x=(x_{mnk})$ and $y=(y_{mnk})$ are said to be asymptotically $f-$rough statistical equivalent of multiple l provided that for every $\beta,\epsilon>0$

$$d^f\left(\left\{(m,n,k)\le(r,s,t):\left|\tfrac{x_{mnk}}{y_{mnk}}-l\right|\ge\beta+\epsilon\right\}\right)=0,$$

that is,

$$lim_{r,s,t\to\infty}\frac{1}{f(rst)}f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\geq\beta+\epsilon\right\}\right|\right)=0,$$

it is denoted by $x\overset{rs_l^f}{\equiv}y$ and simply asymptotically $f-$ rough statistical equivalent if $l=1$. Further more, let rs_l^f denote the set of x and y such that $x\overset{rs_l^f}{\equiv}y$.

Definition 2.3. Two non negative triple sequences $x=(x_{mnk})$ and $y=(y_{mnk})$ are said to be strong Cesaro asymptotically equivalent of multiple l with respect to an Orlicz function f provided that

$$lim_{r,s,t\to\infty}\frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}f\left(\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\right)=0$$

it is denoted by $x\overset{w_l^f}{\equiv}y$ and simply strong Cesaro asymptotically equivalent if $l=1$. In addition, let w_l^f denote the set of x and y such that $x\overset{w_l^f}{\equiv}y$.

3. Main results

Theorem 3.1. *Let f be any unbounded Orlicz function for which $lim_{t\to\infty}\left(\frac{f(t)}{t}\right)>0$ and c be a positive constant such that $f(xy)\geq c\,f(x)\,f(y)$ for all $x\geq0,y\geq0$. If $x\overset{w_l^f}{\equiv}y$ then $x\overset{rs_l^f}{\equiv}y$.*

Proof. Let f be any unbounded Orlicz function for which $lim_{t\to\infty}\left(\frac{f(t)}{t}\right)>0$ and c be a positive constant such that $f(xy)\geq c\,f(x)\,f(y)$ for all $x\geq0,y\geq0$. For $x\overset{w_l^f}{\equiv}y$ and $\beta,\epsilon\in(0,\infty)$, we have

$$\frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}f\left(\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\right)\geq\frac{1}{rst}f\left(\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\right)$$

$$\geq\frac{1}{rst}f\left(\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1,\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\geq\beta+\epsilon}^{t}\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\right)$$

$$\geq\frac{1}{rst}f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\geq\beta+\epsilon\right\}\right|\cdot(\beta+\epsilon)\right)$$

$$\geq\frac{c}{rst}f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\geq\beta+\epsilon\right\}\right|\right)\cdot f(\beta+\epsilon)$$

$$=\frac{1}{f(rst)}f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\geq\beta+\epsilon\right\}\right|\right)\cdot\frac{f(rst)}{rst}\cdot c\cdot f(\beta+\epsilon)$$

$$\Longrightarrow x\overset{rs_l^f}{\equiv}y$$ □

Theorem 3.2. *If $x\overset{rs_l^f}{\equiv}y$, then $x\overset{rs_l}{\equiv}y$*

Proof. Suppose that $x\overset{rs_l^f}{\equiv}y$. Then by the definition of the limit and the fact that f being Orlicz function is subadditive, for every $p\in\mathbb{N}$, there exists $(r_0s_0t_0)\in\mathbb{N}$

such that for $(rst) \geq (r_0 s_0 t_0)$, we have
$f\left(\left|\left\{(m,n,k) \leq (r,s,t) : \left|\frac{x_{mnk}}{y_{mnk}} - l\right| \geq \beta + \epsilon\right\}\right|\right) \leq \frac{1}{p} f(rst) \leq \frac{1}{p} \cdot p \cdot f(rst) = f\left(\frac{rst}{p}\right)$ and since f is non-decreasing function, we have
$\frac{1}{rst} f\left(\left|\left\{(m,n,k) \leq (r,s,t) : \left|\frac{x_{mnk}}{y_{mnk}} - l\right| \geq \beta + \epsilon\right\}\right|\right) \leq \frac{1}{p}$.
Hence $x \overset{rs_l}{\equiv} y$. □

Corollary 3.3. *Let f be an unbounded Orlicz function such that $lim_{t\to\infty} \frac{f(t)}{t} > 0$ and c be a positive constant such that $f(xy) \geq c \cdot f(x) \cdot f(y)$ for all $x \geq 0, y \geq 0$. If $x \overset{w_l^f}{\equiv} y$, then $x \overset{rs_l}{\equiv} y$*

Proof. It is followed by result of Theorem (3.1) and Theorem (3.2). □

Theorem 3.4. *If $x \in \ell_\infty^3$ (the space of all bounded real valued triple sequences) and $x \overset{rs_l^f}{\equiv} y$, then $x \overset{w_l^f}{\equiv} y$ for any unbounded Orlicz function f.*

Proof. Suppose that $x = (x_{mnk}) \in \ell_\infty^3$ and $x \overset{rs_l^f}{\equiv} y$. Then we can assume that there exists $M > 0$ such that

$$\left|\frac{x_{mnk}}{y_{mnk}} - l\right| \leq M$$

for all m, n, k. Given $\beta, \epsilon > 0$

$$\begin{aligned}
&\frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t} f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right)\\
=&\frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1,\left|\frac{x_{mnk}}{y_{mnk}}-l\right|>\beta+\epsilon}^{t} f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right)\\
&\qquad + \frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1,\left|\frac{x_{mnk}}{y_{mnk}}-l\right|<\beta+\epsilon}^{t} f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right)\\
\leq&\frac{1}{rst} f\left(\left|\left\{(m,n,k) \leq (r,s,t) : \left|\frac{x_{mnk}}{y_{mnk}} - l\right| \geq \beta + \epsilon\right\}\right|\right) \cdot f(M)\\
&\qquad + \frac{1}{rst} \cdot rst \cdot f(\beta + \epsilon).
\end{aligned}$$

Taking limit on both sides as $(r,s,t) \to \infty$, we get
$lim_{(r,s,t)\to\infty} \frac{1}{rst} \sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t} f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right) = 0$, by Theorem (3.2) and the fact that f is non-decreasing function. □

Theorem 3.5. *Let f be an Orlicz function such that $lim_{t\to\infty}\frac{f(t)}{t} > 0$. If $x \overset{w_l^f}{\equiv} y$ then $x \overset{w_l}{\equiv} y$.*

Proof. We have $\eta = lim_{t\to\infty}\frac{f(t)}{t} = inf\left\{\frac{f(t)}{t} : t > 0\right\}$. By definition of η, we have $\frac{f(t)}{t} \geq \eta$ for all $t \geq 0$. Since $\eta > 0$, we have $\frac{t}{f(t)} \leq \eta^{-1}$. Hence

$$\frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}\left|\frac{x_{mnk}}{y_{mnk}} - l\right|$$

$$= \frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}\left|\frac{x_{mnk}}{y_{mnk}} - l\right| \cdot \frac{1}{f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right)} \cdot f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right)$$

$$\leq \eta^{-1} \cdot \frac{1}{rst}\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t} f\left(\left|\frac{x_{mnk}}{y_{mnk}} - l\right|\right)$$

$\Longrightarrow x \overset{w_l}{\equiv} y.$ □

Theorem 3.6. *For any Orlicz function f, if $x \overset{w_l}{\equiv} y$, then $x \overset{w_l^f}{\equiv} y$.*

Proof. It is similar to Theorem (3.2). □

Corollary 3.7. *Let f be any Orlicz function such that $lim_{t\to\infty}\frac{f(t)}{t} > 0$. Then $x \overset{w_l^f}{\equiv} y \Leftrightarrow x \overset{w_l}{\equiv} y$.*

4. $x \overset{rs_l^f}{\equiv} y-$ equivalence of rough triple sequences

Let $x = (x_{mnk})$ and $y = (y_{mnk})$ be two rough triple sequences of nonnegative real numbers. We use the notation "$x \prec y$" if $x_{mnk} \leq y_{mnk}$ holds for all $m, n, k \in \mathbb{N}$. In this section discuss the results for nonnegative real numbers with respect to the partial order on the set of real numbers.

Theorem 4.1. *Let f be an unbounded Orlicz function. If $z \prec x$ and $x - z \overset{rs_{l'}^f}{\equiv} y$ then $x \overset{rs_l^f}{\equiv} y \Longrightarrow z \overset{rs^f_{(l-l')}}{\equiv} y$.*

Proof. Suppose that $x - z \overset{rs_{l'}^f}{\equiv} y$. We need $z \prec x$ to be a rough triple sequence $x - z = x_{mnk} - z_{mnk}$ to be a rough triple sequence of nonnegative real numbers. Then

$$\left|\frac{z_{mnk}}{y_{mnk}} - \left(l - l'\right)\right| \leq \left|\frac{x_{mnk}}{y_{mnk}} - l\right| + \left|\frac{x_{mnk} - z_{mnk}}{y_{mnk}} - l'\right|$$

holds for all $m, n, k \in \mathbb{N}$. Then for a given $\beta, \epsilon > 0$ the following inequality
$\left|\left\{(m,n,k) \leq (r,s,t) : \left|\frac{z_{mnk}}{y_{mnk}} - \left(l - l'\right)\right| \geq \beta + \epsilon\right\}\right| \leq$
$\left|\left\{(m,n,k) \leq (r,s,t) : \left|\frac{x_{mnk}}{y_{mnk}} - l\right| \geq \frac{\beta+\epsilon}{2}\right\}\right| +$

$\left|\left\{(m,n,k) \leq (r,s,t) : \left|\frac{x_{mnk}-z_{mnk}}{y_{mnk}} - l^{'}\right| \geq \frac{\beta+\epsilon}{2}\right\}\right|$ is satisfied. Since f is an unbounded nondecreasing Orlicz function, we obtain

$$\frac{f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{z_{mnk}}{y_{mnk}}-\left(l-l^{'}\right)\right|\geq\beta+\epsilon\right\}\right|\right)}{f(rst)} \leq \frac{f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{x_{mnk}}{y_{mnk}}-l\right|\geq\frac{\beta+\epsilon}{2}\right\}\right|\right)}{f(rst)} + \frac{f\left(\left|\left\{(m,n,k)\leq(r,s,t):\left|\frac{x_{mnk}-z_{mnk}}{y_{mnk}}-l^{'}\right|\geq\frac{\beta+\epsilon}{2}\right\}\right|\right)}{f(rst)}.$$

Taking the limit $(r,s,t) \to \infty$. Hence the result is obtained. $\square$

Corollary 4.2. *Let f be an unbounded Orlicz function. If $y \prec z$ and $x \overset{rs^f_{l^{'}}}{\equiv} z-y$ then $x \overset{rs^f_l}{\equiv} y \Longrightarrow x \overset{rs^f_{\frac{1}{l^{"}}}}{\equiv} z$, where $l^{"} = \frac{1}{l} + \frac{1}{l^{'}}$.*

5. Competing interests

The authors declare that there is not any conflict of interests regarding the publication of this manuscript.

6. Acknowledgement

The authors are extremely grateful to the anonymous referees for their valuable suggestion and constructive comments for the improvement of the manuscript.

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