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DOUBLE PAIRWISE STRONGLY $(r,s)(u,v)$-SEMIOPEN MAPPINGS

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Abstract. We introduce the concepts of double pairwise strongly $(r,s)(u,v)$-semiopen mappings and double pairwise strongly $(r,s)(u,v)$-semiclosed mappings, and investigate some of their characteristic properties.

1. Introduction

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

We introduce the concepts of double pairwise strongly $(r,s)(u,v)$-semiopen mappings and double pairwise strongly $(r,s)(u,v)$-semiclosed mappings, and investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and

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1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$. Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.1. [5, 14] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^\mu, \mathcal{T}^\gamma)$ on X is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I(\mathcal{T}^\mu, \mathcal{T}^\gamma : I(X) \to I)$ which satisfies the following properties:

(1) $\mathcal{T}^\mu(0_\sim) = \mathcal{T}^\mu(1_\sim) = 1$ and $\mathcal{T}^\gamma(0_\sim) = \mathcal{T}^\gamma(1_\sim) = 0$.
(2) $\mathcal{T}^\mu(A \cap B) \geq \mathcal{T}^\mu(A) \wedge \mathcal{T}^\mu(B)$ and $\mathcal{T}^\gamma(A \cap B) \leq \mathcal{T}^\gamma(A) \vee \mathcal{T}^\gamma(B)$.
(3) $\mathcal{T}^\mu(\bigcup A_i) \geq \bigwedge \mathcal{T}^\mu(A_i)$ and $\mathcal{T}^\gamma(\bigcup A_i) \leq \bigvee \mathcal{T}^\gamma(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense.* Also, we call $\mathcal{T}^\mu(A)$ a *gradation of openness* of A and $\mathcal{T}^\gamma(A)$ a *gradation of nonopenness* of A.

Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu\gamma})$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r, s)*-open* set if $\mathcal{T}^\mu(A) \geq r$ and $\mathcal{T}^\gamma(A) \leq s$,
(2) a $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r, s)*-closed* set if $\mathcal{T}^\mu(A^C) \geq r$ and $\mathcal{T}^\gamma(A^C) \leq s$.

Let $(X, \mathcal{T}^{\mu\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r, s)*-closure* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)) = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$-*fuzzy* (r,s)-*interior* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r,s)\text{-open}\}.$$

A system $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space*.

Definition 2.2. [12] Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v) \in I \otimes I$. Then A is said to be

(1) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double strongly* $(r,s)(u,v)$-*semiopen* set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v)),(r,s))$,
(2) a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double strongly* $(u,v)(r,s)$-*semiopen* set if there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B in X such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B,(r,s)),(u,v))$,
(3) a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double strongly* $(r,s)(u,v)$-*semiclosed* set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set B in X such that $\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(B,(u,v)),(r,s)) \subseteq A \subseteq B$,
(4) a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double strongly* $(u,v)(r,s)$-*semiclosed* set if there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set B in X such that $\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(B,(r,s)),(u,v)) \subseteq A \subseteq B$.

3. Double pairwise strongly $(r,s)(u,v)$-semiopen mappings

Definition 3.1. Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v) \in I \otimes I$. For each $A \in I(X)$, the $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double strongly* $(r,s)(u,v)$-*semiclosure* is defined by

$$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dsscl}(A,(r,s),(u,v)) = \bigcap\{B \in I(X) \mid B \supseteq A, \\ B \text{ is } (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-double strongly } (r,s)(u,v)\text{-semiclosed}\}$$

and the $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-*double strongly* $(u,v)(r,s)$-*semiclosure* is defined by

$$(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dsscl}(A,(u,v),(r,s)) = \bigcap\{B \in I(X) \mid B \supseteq A, \\ B \text{ is } (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-double strongly } (u,v)(r,s)\text{-semiclosed}\}.$$

Definition 3.2. Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v) \in I \otimes I$. For each $A \in I(X)$, the $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-*double* *strongly* $(r,s)(u,v)$*-semiinterior* is defined by

$$(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-dssint}(A,(r,s),(u,v)) = \bigcup\{B \in I(X) \mid B \subseteq A,$$
$$B \text{ is } (\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\text{-double strongly } (r,s)(u,v)\text{-semiopen}\}$$

and the $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$*-double strongly* $(u,v)(r,s)$*-semiinterior* is defined by

$$(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-dssint}(A,(u,v),(r,s)) = \bigcup\{B \in I(X) \mid B \subseteq A,$$
$$B \text{ is } (\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})\text{-double strongly } (u,v)(r,s)\text{-semiopen}\}.$$

Definition 3.3. Let $f : (X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma}) \to (Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r,s)(u,v) \in I \otimes I$. Then f is called

(1) a *double pairwise strongly* $(r,s)(u,v)$*-semiopen* mapping if $f(A)$ is a $(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopne set of Y for each $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set A of X and $f(B)$ is a $(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})$-duoble strongly $(u,v)(r,s)$-semiopne set of Y for each $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B of X,

(2) a *double pairwise strongly* $(r,s)(u,v)$*-semiclosed* mapping if $f(A)$ is a $(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set of Y for each $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set A of X and $f(B)$ is a $(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of Y for each $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set B of X.

Theorem 3.4. Let $f : (X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma}) \to (Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a double pairwise strongly $(r,s)(u,v)$-semiopen mapping.

(2) For each intuitionistic fuzzy set A of X,

$$f(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))) \subseteq (\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(f(A),(r,s),(u,v))$$

and

$$f(\mathcal{U}^{\mu\gamma}\text{-int}(A,(u,v))) \subseteq (\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(f(A),(u,v),(r,s)).$$

(3) For each intuitionistic fuzzy set C of Y,

$$\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C),(r,s)) \subseteq f^{-1}((\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(C,(r,s),(u,v)))$$

and

$$\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C),(u,v)) \subseteq f^{-1}((\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(C,(u,v),(r,s))).$$

Proof. (1) $\Rightarrow$ (2) Let A be any intuitionistic fuzzy set of X. Clearly $\mathcal{T}^{\mu\gamma}$-int$(A,(r,s))$ is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set and $\mathcal{U}^{\mu\gamma}$-int$(A,(u,v))$ is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set of X. Since f is a double pairwise strongly $(r,s)(u,v)$-semiopen mapping, $f(\mathcal{T}^{\mu\gamma}$-int$(A,(r,s)))$ is a $(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set and $f(\mathcal{V}^{\mu\gamma}\text{-int}(A,(u,v)))$ is a $(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set of Y. Thus

$$\begin{aligned}
&f(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s)))\\
&=(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(f(\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))),(r,s),(u,v))\\
&\subseteq(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(f(A),(r,s),(u,v))
\end{aligned}$$

and

$$\begin{aligned}
&f(\mathcal{U}^{\mu\gamma}\text{-int}(A,(u,v)))\\
&=(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(f(\mathcal{U}^{\mu\gamma}\text{-int}(A,(u,v))),(u,v),(r,s))\\
&\subseteq(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(f(A),(u,v),(r,s)).
\end{aligned}$$

(2) $\Rightarrow$ (3) Let C be any intuitionistic fuzzy set of Y. Then $f^{-1}(C)$ is an intuitionistic fuzzy set of X. By (2),

$$\begin{aligned}
f(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C),(r,s)))&\subseteq(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(ff^{-1}(C),(r,s),(u,v))\\
&\subseteq(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(C,(r,s),(u,v))
\end{aligned}$$

and

$$\begin{aligned}
f(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C),(u,v)))&\subseteq(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(ff^{-1}(C),(u,v),(r,s))\\
&\subseteq(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(C,(u,v),(r,s)).
\end{aligned}$$

Then we have

$$\begin{aligned}
\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C),(r,s))&\subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}(C),(r,s)))\\
&\subseteq f^{-1}((\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(C,(r,s),(u,v)))
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C),(u,v))&\subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}(C),(u,v)))\\
&\subseteq f^{-1}((\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(C,(u,v),(r,s))).
\end{aligned}$$

(3) $\Rightarrow$ (1) Let A be any $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set and B any $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set of X. Then $A=\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))$ and $B=\mathcal{U}^{\mu\gamma}\text{-int}(B,(u,v))$. By (3),

$$\begin{aligned}
A=\mathcal{T}^{\mu\gamma}\text{-int}(A,(r,s))&\subseteq\mathcal{T}^{\mu\gamma}\text{-int}(f^{-1}f(A),(r,s))\\
&\subseteq f^{-1}((\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dssint}(f(A),(r,s),(u,v)))
\end{aligned}$$

and

$$\begin{aligned}
B=\mathcal{U}^{\mu\gamma}\text{-int}(B,(u,v))&\subseteq\mathcal{U}^{\mu\gamma}\text{-int}(f^{-1}f(B),(u,v))\\
&\subseteq f^{-1}((\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dssint}(f(B),(u,v),(r,s))).
\end{aligned}$$

Thus

$$\begin{aligned} f(A) &\subseteq ff^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r,s), (u,v))) \\ &\subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dssint}(f(A), (r,s), (u,v)) \\ &\subseteq f(A) \end{aligned}$$

and

$$\begin{aligned} f(B) &\subseteq ff^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(B), (u,v), (r,s))) \\ &\subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dssint}(f(B), (u,v), (r,s)) \\ &\subseteq f(B). \end{aligned}$$

So $f(A) = (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-dssint$(f(A), (r,s), (u,v))$ and $f(B) = (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-dssint$(f(B), (u,v), (r,s))$. Hence $f(A)$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiopen set and $f(B)$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiopen set of Y. Therefore f is a double pairwise strongly $(r,s)(u,v)$-semiopen mapping. □

THEOREM 3.5. Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s), (u,v) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a double pairwise strongly $(r,s)(u,v)$-semiclosed mapping.

(2) For each intuitionistic fuzzy set A of X,

$$(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r,s), (u,v)) \subseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r,s)))$$

and

$$(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(A), (u,v), (r,s)) \subseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u,v))).$$

Proof. (1) ⇒ (2) Let A be any intuitionistic fuzzy set of X. Clearly $\mathcal{T}^{\mu\gamma}$-cl$(A, (r,s))$ is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set and $\mathcal{U}^{\mu\gamma}$-cl$(A, (u,v))$ is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set of X. Since f is a double pairwise strongly $(r,s)(u,v)$-semiclosed mapping, $f(\mathcal{T}^{\mu\gamma}$-cl$(A, (r,s)))$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f(\mathcal{U}^{\mu\gamma}$-cl$(A, (u,v)))$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of Y. Thus

$$\begin{aligned} f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r,s))) &= (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r,s))), (r,s), (u,v)) \\ &\supseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r,s), (u,v)) \end{aligned}$$

and

$$\begin{aligned} f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u,v))) &= (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u,v))), (u,v), (r,s)) \\ &\supseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(A), (u,v), (r,s)). \end{aligned}$$

(2) ⇒ (1) Let A be any $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set of X. Then $A = \mathcal{T}^{\mu\gamma}$-cl$(A, (r,s))$ and $B =$

$\mathcal{U}^{\mu\gamma}$-cl$(B,(u,v))$. By (2),

$$\begin{aligned}&(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A),(r,s),(u,v))\\&\subseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(A,(r,s)))\\&=f(A)\\&\subseteq(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A),(r,s),(u,v))\end{aligned}$$

and

$$\begin{aligned}&(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B),(u,v),(r,s))\\&\subseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(B,(u,v)))\\&=f(B)\\&\subseteq(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B),(u,v),(r,s)).\end{aligned}$$

So $f(A)=(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$-dsscl$(f(A),(r,s),(u,v))$ and $f(B)=(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})$-dsscl$(f(B),(u,v),(r,s))$. Hence $f(A)$ is a $(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f(B)$ is a $(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of Y. Therefore f is a double pairwise strongly $(r,s)(u,v)$-semiclosed mapping. □

Theorem 3.6. Let $f:(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})\to(Y,\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})$ be a bijection and $(r,s),(u,v)\in I\otimes I$. Then f is a double pairwise strongly $(r,s)(u,v)$-semiclosed mapping if and only if for each intuitionistic fuzzy set C of Y,

$$f^{-1}((\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dsscl}(C,(r,s),(u,v)))\subseteq\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C),(r,s))$$

and

$$f^{-1}((\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dsscl}(C,(u,v),(r,s)))\subseteq\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C),(u,v)).$$

Proof. Let C be any intuitionistic fuzzy set of Y. Then $f^{-1}(C)$ is an intuitionistic fuzzy set of X. Since f is onto,

$$\begin{aligned}&(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dsscl}(C,(r,s),(u,v))\\&=(\mathcal{V}^{\mu\gamma},\mathcal{W}^{\mu\gamma})\text{-dsscl}(ff^{-1}(C),(r,s),(u,v))\\&\subseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C),(r,s)))\end{aligned}$$

and

$$\begin{aligned}&(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dsscl}(C,(u,v),(r,s))\\&=(\mathcal{W}^{\mu\gamma},\mathcal{V}^{\mu\gamma})\text{-dsscl}(ff^{-1}(C),(u,v),(r,s))\\&\subseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C),(u,v))).\end{aligned}$$

Since f is one-to-one, we have

$$\begin{aligned} & f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(C, (r,s), (u,v))) \\ & \subseteq f^{-1} f(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C), (r,s))) \\ & = \mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(C), (r,s)) \end{aligned}$$

and

$$\begin{aligned} & f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(C, (u,v), (r,s))) \\ & \subseteq f^{-1} f(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C), (u,v))) \\ & = \mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(C), (u,v)). \end{aligned}$$

Conversely, let A be any $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set of X. Then $A = \mathcal{T}^{\mu\gamma}\text{-cl}(A, (r,s))$ and $B = \mathcal{U}^{\mu\gamma}\text{-cl}(B, (u,v))$. Since f is one-to-one,

$$\begin{aligned} & f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r,s), (u,v))) \\ & \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1} f(A), (r,s)) \\ & = \mathcal{T}^{\mu\gamma}\text{-cl}(A, (r,s)) \\ & = A \end{aligned}$$

and

$$\begin{aligned} & f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u,v), (r,s))) \\ & \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1} f(B), (u,v)) \\ & = \mathcal{U}^{\mu\gamma}\text{-cl}(B, (u,v)) \\ & = B. \end{aligned}$$

Since f is onto, we have

$$\begin{aligned} & (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r,s), (u,v)) \\ & = f f^{-1}((\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r,s), (u,v))) \\ & \subseteq f(A) \\ & \subseteq (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})\text{-dsscl}(f(A), (r,s), (u,v)) \end{aligned}$$

and

$$\begin{aligned} & (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u,v), (r,s)) \\ & = f f^{-1}((\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u,v), (r,s))) \\ & \subseteq f(B) \\ & \subseteq (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})\text{-dsscl}(f(B), (u,v), (r,s)). \end{aligned}$$

Thus $f(A) = (\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-dsscl$(f(A),(r,s),(u,v))$ and $f(B) = (\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-dsscl$(f(B),(u,v),(r,s))$. Hence $f(A)$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-double strongly $(r,s)(u,v)$-semiclosed set and $f(B)$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-double strongly $(u,v)(r,s)$-semiclosed set of Y. Therefore f is a double pairwise strongly $(r,s)(u,v)$-semiclosed mapping. □

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87–96.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1968), 182–190.

[3] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness : Fuzzy topology*, Fuzzy Sets and Systems, **49** (1992), 237–242.

[4] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, **88** (1997), 81–89.

[5] D. Çoker and M. Demirci, *An introduction to intuitionistic fuzzy topological spaces in Šostak's sense*, BUSEFAL, **67** (1996), 67–76.

[6] D. Çoker and A. Haydar Eş, *On fuzzy compactness in intuitionistic fuzzy topological spaces*, J. Fuzzy Math., **3** (1995), 899–909.

[7] H. Gürçay, D. Çoker, and A. Haydar Eş, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math., **5** (1997), 365–378.

[8] A. Kandil, *Biproximities and fuzzy bitopological spaces*, Simon Stevin, **63** (1989), 45–66.

[9] E. P. Lee, *$(\mathcal{T}_i, \mathcal{T}_j)$-fuzzy α-(r,s)-semiopen sets and fuzzy pairwise α-(r,s)-semicontinuous mappings*, Bull. Korean Math. Soc., **39** (2002), 653–663.

[10] E. P. Lee and S. O. Lee, *Double pairwise $(r,s)(u,v)$-semicontinuous mappings*, Journal of the Chungcheong Mathematical Society, **27** (2014), 603–614.

[11] E. P. Lee and S. O. Lee, *Double pairwise $(r,s)(u,v)$-precontinuous mappings*, Journal of the Chungcheong Mathematical Society, **30** (2017), 1–13.

[12] E. P. Lee and S. O. Lee, *Double strongly $(r,s)(u,v)$-semiopen sets*, Journal of the Chungcheong Mathematical Society, **30** (2017), 423–433.

[13] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems, **48** (1992), 371–375.

[14] A. P. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Janos Palermo, Sr. II, **11** (1985), 89–103.

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